Wed, Oct 28, 2015 **MATH 250, Quiz 7** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Find the directional derivative of the function in the direction of the vector $\vec{v}$ at the given point:

$$f(x,y) = \frac{x}{x^2+y^2}, \quad (x_0,y_0) = (1,2), \quad \vec{v} = (3,5).$$

$$f_x = \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$f_y = -\frac{2yx}{(x^2+y^2)^2}$$

$$\vec{\nabla} f(1,2) = \left(\frac{3}{25}, -\frac{4}{25}\right);$$

$$\frac{\partial f}{\partial \vec{u}}(1,2) = \frac{1}{25}\cdot\frac{1}{\sqrt{34}}(3,-4)\cdot(3,5) = \frac{-11}{25\sqrt{34}}.$$

$$\vec{v} = (3,5)$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{(3,5)}{\sqrt{3^2+5^2}} = \frac{(3,5)}{\sqrt{34}}$$

2. Find the equation of the tangent plane to the surface at the given point:

$$y = x^2 - z^2, \quad (x_0,y_0,z_0) = (4,7,3).$$

$$F = y - x^2 - z^2; \quad \vec{\nabla} F = (-2x, 1, 2z)$$

$$\vec{\nabla} F(4,7,3) = (-8, 1, 6)$$

Tangent plane:

$$-8(x-4) + (y-7) + 6(z-3) = 0$$

$$8x - y + 6z = 7$$

Please turn over...

3. Find all critical points of the function $f(x,y)$ and determine their type (local maximum / minimum / saddle point...):

$$f(x,y) = (x-y)(1-xy).$$

$$\left.\begin{aligned} f_x &= 1-xy-y(x-y) = 1-2xy+y^2 = 0 \\ f_y &= xy-1-x(x-y) = -1+2xy-x^2 = 0 \end{aligned}\right\} \Rightarrow x^2=y^2, \quad x=\pm y$$

$x=y \Rightarrow \quad 1-x^2=0, \quad x=\pm 1 \Rightarrow (1,1), (-1,-1)$ — crit. pts.

$x=-y \Rightarrow \quad 1+3x^2=0$ — no solution.

$$f_{xx} = -2y; \quad f_{xy} = -2x+2y; \quad f_{yy} = 2x$$

$$D^2 f = \begin{pmatrix} -2y & -2(x-y) \\ -2(x-y) & 2x \end{pmatrix}$$

$$D^2 f(1,1) = \begin{pmatrix} -2 & 0 \\ 0 & 2 \end{pmatrix}; \quad \Delta < 0 \text{ — saddle point}$$

$$D^2 f(-1,-1) = \begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}; \quad \Delta < 0 \text{ — saddle pt.}$$