Wed, Oct 21, 2015 **MATH 250, Quiz 6** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. (a) Find the linear approximation $L(x, y)$ for $f(x, y) = x/(x + y)$ near $(x_0, y_0) = (2, 1)$.
   (b) Explain why the function $f(x, y)$ is differentiable at that point.

$f = \frac{x}{x+y}$; $f(2,1) = \frac{2}{3}$

$f_x = \frac{1}{x+y} - \frac{x}{(x+y)^2} = \frac{y}{(x+y)^2}$

$f_y = -\frac{x}{(x+y)^2}$

continuous at $(2,1)$ – elementary functions defined at $(2,1)$

$\Rightarrow$ f is differentiable at $(2,1)$ by the Thm on continuity of first partials.

$f_x(2,1) = \frac{1}{9}$

$f_y(2,1) = -\frac{2}{9}$

$L(x,y) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1) = \frac{2}{3} + \frac{1}{9}x - \frac{2}{9}y$

2. The length and width of a rectangle are measured as 30 cm and 24 cm, respectively, with an error of measurement at most 0.1 cm in each. Use differentials to estimate the maximum error in the calculated area of the rectangle.

$A = lw$; $|dw| \le 0.1$; $|dl| \le 0.1$

$dA = l\,dw + w\,dl = 30\,dw + 24\,dl$

$|dA| \le 30 \cdot 0.1 + 24 \cdot 0.1 = 5.4 \text{ cm}^2$

Please turn over...

3. The pressure of 1 mole of an ideal gas is increasing at a rate 0.05 kPa/s and the temperature is increasing at a rate of 0.15 K/s. Use the equation of the ideal gas, $PV = RT$, $R = 8.31$, to find the rate of change of the volume when the pressure is 20 kPa and the temperature is 320 K.

$$PV = RT. \Rightarrow V = \frac{RT}{P} \; ; \quad \frac{dV}{dt} = \frac{R}{P}T' + \left(-\frac{RT}{P^2}\right)P'$$

$$= \frac{RT}{P}\left(\frac{T'}{T} - \frac{P'}{P}\right)$$

$$\frac{RT}{P} = \frac{8.31 \cdot 320}{20} = 8.31 \cdot 16 = 132.96$$

$$\frac{dV}{dt} = 132.96\left(\frac{0.15}{320} - \frac{0.05}{20}\right) \approx -0.27 \text{ L/s.}$$