Wed, Oct 14, 2015 **MATH 250, Quiz 5** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Draw a contour map of the function, showing several level curves:

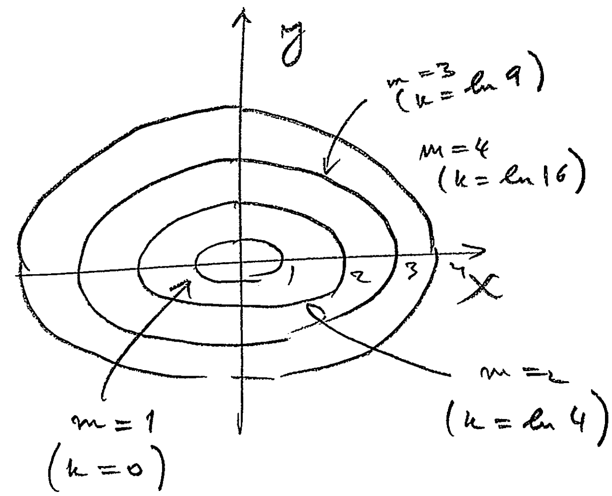
$$f(x,y) = \ln(x^2 + 4y^2).$$

$\ln(x^2+4y^2) = k$

$x^2 + 4y^2 = e^k =: m^2$

$\frac{x^2}{m^2} + \frac{y^2}{(m/2)^2} = 1$

– ellipse that extends from $x=-m$ to $x=m$ and from $y = -\frac{m}{2}$ to $y = \frac{m}{2}$



2. Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}.$$

[Preliminary: try $y = cx \Rightarrow f(x, cx) = \frac{cx^2}{\sqrt{1+c^2}\,|x|} = \frac{c}{\sqrt{1+c^2}}|x| \xrightarrow[x\to 0]{} 0$

(straight lines through the origin)

Thus, the limits along straight lines through $(0,0)$ are $= 0$]

To show that $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$, use polar coord.:

$x = r\cos\theta, \ y = r\sin\theta$

$$\frac{xy}{\sqrt{x^2+y^2}} = \frac{r\cos\theta \, r\sin\theta}{r} = r\cos\theta\sin\theta = \frac{1}{2} r\sin(2\theta) \xrightarrow[r\to 0]{} 0$$

since $\frac{1}{2}r \le \frac{1}{2} r\sin(2\theta) \le \frac{1}{2} r$

$\downarrow r\to 0$ $\downarrow r\to 0$

0 0

Please turn over...

3. Determine the set of points at which the function is continuous:

$$F(x,y) = \frac{1+x^2+y^2}{1-x^2-y^2}.$$

[Give the answer in the form $S = \{(x,y) : \ldots$ conditions on $(x,y)\ldots\}$, and describe the set geometrically. Justify your answer.]

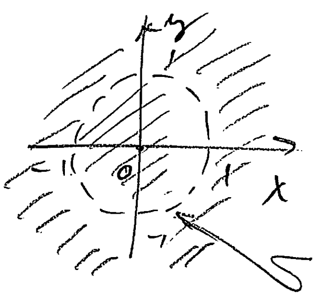
$F(x,y)$ – elementary function (rational)

$\Rightarrow$ continuous at every pt. of its domain.

$$\text{dom}(F) = \{(x,y): \ 1-x^2-y^2 \neq 0\}$$

$$= \{(x,y): \ x^2+y^2 \neq 1\}$$

— the complement of the circle $x^2+y^2=1$



(circle not included)

4. Find the indicated partial derivative:

$$f(x,y) = \ln(x+\sqrt{x^2+y^2}); \qquad f_x(3,4) = ?$$

$$f_x = \frac{1+\frac{2x}{2\sqrt{x^2+y^2}}}{x+\sqrt{x^2+y^2}} = \frac{1}{\sqrt{x^2+y^2}} \frac{x+\sqrt{x^2+y^2}}{x+\sqrt{x^2+y^2}} = \frac{1}{\sqrt{x^2+y^2}}$$

$$f_x(3,4) = \frac{1}{\sqrt{3^2+4^2}} = \frac{1}{5}.$$