Wed, Sept 30, 2015 **MATH 250, Quiz 4** Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. Consider the 2D parametric curve given by the vector equation $\vec{r}(t) = (t-2, t^2+1)$.

   (a) Sketch the curve.

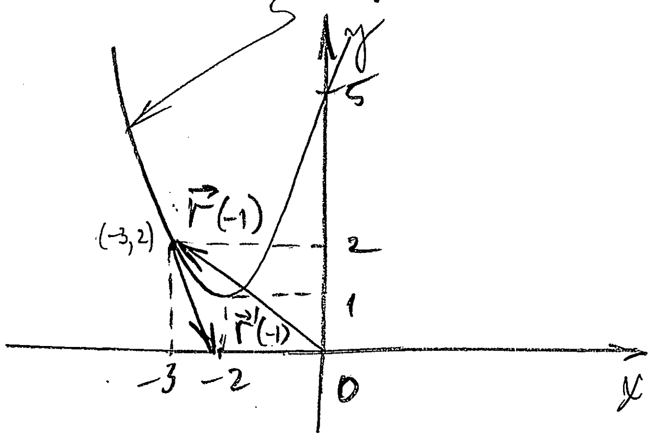
   (b) Find the velocity vector $\vec{r}'(t)$.

   (c) Show the position vector $\vec{r}(t)$ and the velocity vector $\vec{r}'(t)$ for $t=-1$ on the graph.

$x = t-2 \Rightarrow t = x+2$

$y = t^2+1 \qquad y = (x+2)^2+1$

(parabola, vertex $(-2,1)$
vert. intercept $y=5$)



$\vec{r}(-1) = (-3, 2)$

$\vec{r}'(t) = (1, 2t)$

$\vec{r}'(-1) = (1, -2)$

2. Find $f'(2)$, where $f(t) = \vec{u}(t)\cdot\vec{v}(t)$, $\vec{u}(2) = (1,2,-1)$, $\vec{u}'(2) = (3,0,4)$, and $\vec{v}(t) = (t, t^2, t^3)$.

$\vec{v}(2) = (2,4,8)$

$\vec{v}'(t) = (1, 2t, 3t^2)\,; \quad \vec{v}'(2) = (1,4,12)$

$f'(2) = \vec{u}'(2)\cdot\vec{v}(2) + \vec{u}(2)\cdot\vec{v}'(2)$

$= (3,0,4)\cdot(2,4,8) + (1,2,-1)\cdot(1,4,12)$

$= (6+0+32) + (1+8-12) = 35$

Please turn over...

3. Find the length of the curve:

$$\vec{r}(t) = \sqrt{2}\,t\,\vec{i} + e^t\,\vec{j} + e^{-t}\,\vec{k}, \quad 0 \le t \le 1.$$

$$\vec{r}'(t) = (\sqrt{2},\ e^t,\ -e^{-t})$$

$$|\vec{r}'(t)| = \sqrt{2 + (e^t)^2 + (-e^{-t})^2} = e^t + e^{-t}.$$

$$\ell(C) = \int_0^1 |\vec{r}'(t)|\,dt = \int_0^1 e^t + e^{-t}\,dt$$

$$= \left[e^t - e^{-t}\right]_0^1 = (e-1) + (-e^{-1}+1)$$

$$= e - e^{-1} \approx 2.3504$$

4. Find the velocity vector $\vec{r}'(t)$ and the unit tangent vector $\vec{T}(t)$:

$$\vec{r}(t) = (t, \tfrac{1}{2}t^2, t^2)$$

$$\vec{r}'(t) = (1,\ t,\ 2t)$$

$$|\vec{r}'(t)| = \sqrt{1 + t^2 + (2t)^2} = \sqrt{1+5t^2}$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \left(\frac{1}{\sqrt{1+5t^2}},\ \frac{t}{\sqrt{1+5t^2}},\ \frac{2t}{\sqrt{1+5t^2}}\right).$$