Wed, Sept 23, 2015 **MATH 250, Quiz 3** Prof. V. Panferov

Name (print): *Solution.*

Each problem is worth 2 points. Show all your work.

1. (a) Find and identify traces of the quadric surface $-x^2 - y^2 + z^2 = 1$ and explain why the graph looks like that of a hyperboloid of two sheets.

   (b) If the equation is changed to $x^2 - y^2 - z^2 = 1$, what happens to the graph? Sketch the new graph.

(a): $x^2 + y^2 - z^2 = -1$

Traces by $\underline{z = k}$:

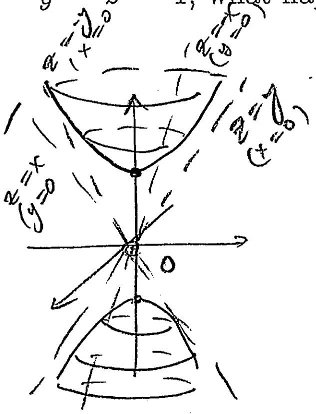
$x^2 + y^2 = k^2 - 1$

– empty if $|k| < 1$

$(x, y) = (0, 0)$ if $k = \pm 1$

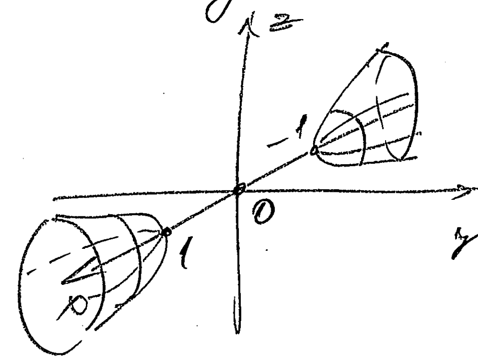
circles of radius $\sqrt{k^2 - 1}$ if $|k| > 1$.

$\Rightarrow$ Surface has circular symmetry; z-axis – axis of sym.



(b) Same geometry, but with the x-axis being the axis of symmetry.

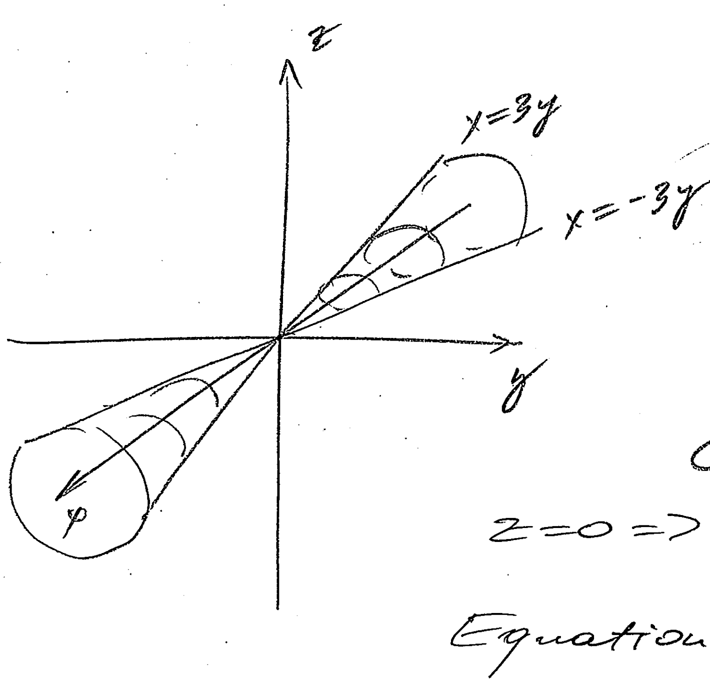
$y^2 + z^2 - x^2 = -1$



Traces by $x = 0$ $z^2 - x^2 = 1$ hyperbolas

$y = 0$ $z^2 - y^2 = 1$ with asymptotes $z = \pm x$, $z = \pm y$

2. Find an equation of the surface obtained by rotating the line $x = 3y$ about the x-axis.



Cone with x-axis being the axis of the cone.

$$x^2 = \frac{y^2}{b^2} + \frac{z^2}{c^2}$$

Circular cone $\Rightarrow b = c$

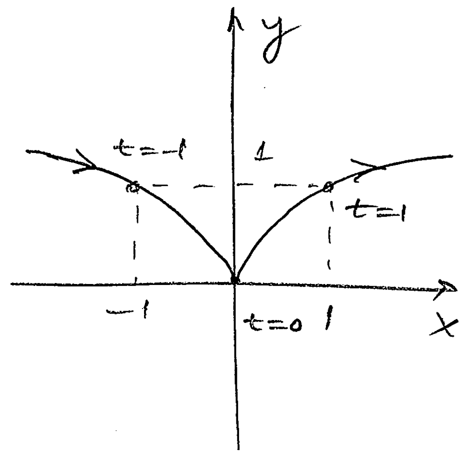
$z = 0 \Rightarrow x^2 = (3y)^2 \Rightarrow b = \frac{1}{3}$

Please turn over...

Equation: $x^2 = 9y^2 + 9z^2$.

3. Sketch the 2D curve given by the vector equation $\vec{r}(t) = (t^3, t^2)$. Indicate with an arrow the direction of increasing t.

$$\begin{aligned} x &= t^3 \\ y &= t^2 \end{aligned} \implies \begin{aligned} t &= x^{1/3} \\ y &= (x^{1/3})^2 \end{aligned} \implies y = x^{2/3}$$



Graph
$y = x^{2/3}$

4. At what points does the curve $\vec{r}(t) = t\,\vec{\imath} + (2t - t^2)\,\vec{\jmath}$ intersect the paraboloid $z = x^2 + y^2$? (Give 3D coordinates of the points.)

[Problem has a typo: textbook has $\vec{r}(t) = t\vec{\imath} + (2t - t^2)\vec{k}$]

Solve as-is:

$$x = t, \quad y = 2t - t^2, \quad z = 0$$

$$t^2 + (2t - t^2)^2 = 0$$

$$\implies t = 0, \quad 2t - t^2 = 0 \implies t = 0$$

Only point of intersection: $\vec{r}(0) = (0, 0, 0)$

Alternative solution: The curve $\vec{r}(t)$ lies in the xy-plane. The surface is an elliptic (circular) paraboloid; its only point in the xy-plane is the origin. Since the curve passes through the origin, it is the only pt. of int.

Solve as-in-textbook:

$$x = t, \quad y = 0, \quad z = 2t - t^2 \implies 2t - t^2 = t^2$$

$$2t^2 = 2t$$
$$t^2 = t$$
$$t(t-1) = 0$$
$t = 0$ or $t = 1$

$t = 0 \implies \vec{r}(0) = (0, 0, 0)$

$t = 1 \implies \vec{r}(1) = (1, 1, 0)$

— 2 points of intersection.