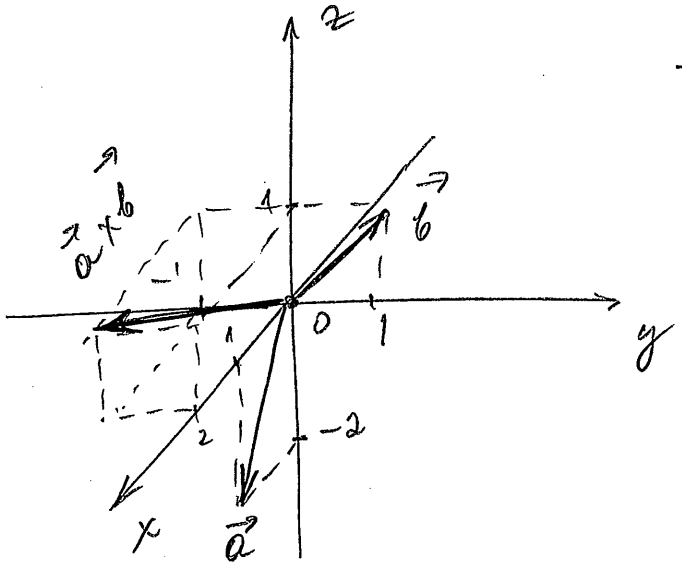
Mon. Sept 14, 2015 MATH 250, Quiz 2 Prof. V. Panferov

Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. If $\vec{a} = \vec{i} - 2\vec{k}$ and $\vec{b} = \vec{j} + \vec{k}$, find $\vec{a} \times \vec{b}$. Sketch $\vec{a}$, $\vec{b}$, and $\vec{a} \times \vec{b}$ as vectors starting at the origin.

$$\vec{a} \times \vec{b} = (\vec{i} - 2\vec{k}) \times (\vec{j} + \vec{k}) = \vec{i} \times \vec{j} - 2\vec{k} \times \vec{j} + \vec{i} \times \vec{k} - 2\vec{k} \times \vec{k}$$
$$= \vec{k} + 2\vec{i} - \vec{j} - \vec{0}$$
$$= 2\vec{i} - \vec{j} + \vec{k} = (2, -1, 1)$$

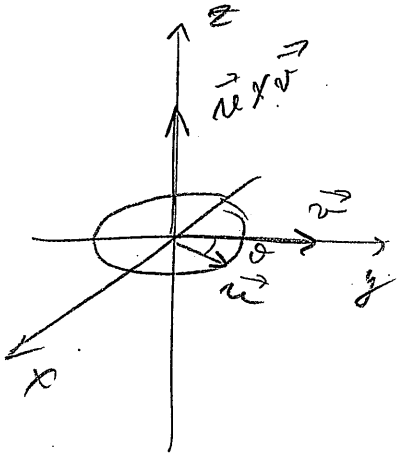


2. Let $\vec{v} = 5\vec{j}$ and let $\vec{u}$ be a vector of length 3 that starts at the origin and rotates in the xy-plane. Find the maximum and the minimum possible values of $|\vec{u} \times \vec{v}|$. In what direction does $\vec{u} \times \vec{v}$ point?

$$|\vec{u} \times \vec{v}| = u v \cdot \sin\theta = 3 \cdot 5 \cdot \sin\theta = 15 \sin\theta$$

Maximum: $|\vec{u} \times \vec{v}| = 15$ when $\vec{u} \perp \vec{v}$ ($\sin\theta = 1$)

Minimum: $|\vec{u} \times \vec{v}| = 0$ when $\vec{u} \parallel \vec{v}$. ($\sin\theta = 0$)



Direction: along the z-axis, upward or downward, depending on orientation.

Please turn over...

3. Find parametric equations of the line through the point $P(1,-1,1)$ and parallel to the line

$$\frac{x+2}{1} = \frac{1}{2}y = \frac{z-3}{1}.$$

Direction vector: $\vec{v} = (1, 2, 1)$

Parametric eqns:

$$x = 1 + t$$
$$y = -1 + 2t$$
$$z = 1 + t$$

4. Find an equation for the plane that passes through the point $A(1,2,3)$ and contains the line $x = 3t$, $y = 1 + t$, $z = 2 - t$.

Pick two points on the line:

$t = 0 \Rightarrow P(0,1,2)$; $t = 1 \Rightarrow Q(3,2,1)$.

$\vec{AP} = (-1,-1,-1)$; $\vec{AQ} = (2,0,-2)$

Normal vector:

$$\vec{AP} \times \vec{AQ} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -1 & -1 \\ 2 & 0 & -2 \end{vmatrix} = 2\vec{i} - 4\vec{j} + 2\vec{k}$$

Plane: $2(x-1) - 4(y-2) + 2(z-3) = 0$

or $(x-1) - 2(y-2) + (z-3) = 0$

or $x - 2y + z = 0$.