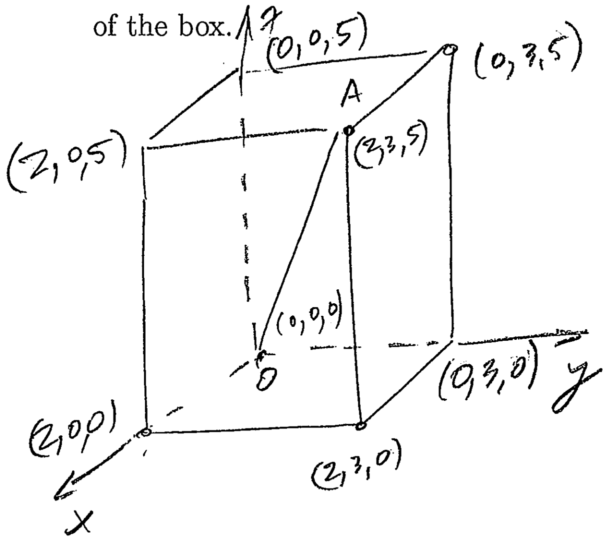
Name (print): Solutions.

Each problem is worth 2 points. Show all your work.

1. What are the projections of the point $(2,3,5)$ on the xy-, yz-, and xz-planes? Draw a rectanglular box with the origin and $(2,3,5)$ as opposite vertices and with its faces parallel to coordinate planes. Label all verices of the box. Find the length of the diagonal of the box.



xy-plane: $(2,3,0)$
yz-plane: $(0,3,5)$
xz-plane: $(2,0,5)$

Diagonal

$$|OA| = \sqrt{2^2+3^2+5^2} = \sqrt{4+9+25} = \sqrt{38}.$$

2. Show that the equation $3x^2+3y^2+3z^2 = 10+6y+12z$ represents a sphere, and find its center and radius.

$$3x^2 + 3(y^2-2y) + 3(z^2-4z) = 10$$
$$3x^2 + 3(y^2-2y+1) + 3(z^2-4z+4) - 3 - 12 = 10$$
$$3x^2 + 3(y-1)^2 + 3(z-2)^2 = 25$$
$$x^2 + (y-1)^2 + (z-2)^2 = \frac{25}{3}$$

Sphere with center @ $C(0,1,2)$
of radius $R = \sqrt{\frac{25}{3}} = \frac{5\sqrt{3}}{3}$.

Please turn over...

3. Use vectors to decide whether the triangle with vertices $P(1,-3,-2)$, $Q(2,0,-4)$ and $R(6,-2,-5)$ is right-angled.

$\vec{PQ} = (1, 3, -2)$

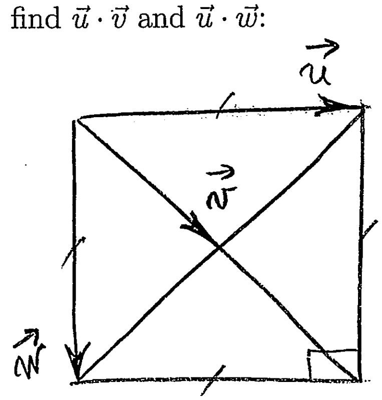
$\vec{QR} = (4, -2, -1)$

$\vec{PQ} \cdot \vec{QR} = 4 - 6 + 2 = 0$

Yes, it is, since the angle between PQ and QR is 90°.

(Remark: Of course, if you don't get a correct pair right away, you may try $\vec{PQ} \cdot \vec{PR}$ and/or $\vec{PR} \cdot \vec{RQ}$ before finding a pair of vectors that is orthogonal.)

4. If $\vec{u}$ is a unit vector, find $\vec{u} \cdot \vec{v}$ and $\vec{u} \cdot \vec{w}$:



$\vec{u}, \vec{w}$ – unit vectors;

The figure is a square

$\vec{u} \cdot \vec{W} = 0$

since the vectors are perpendicular.

$$\vec{u} \cdot \vec{v} = uv \cdot \cos\theta = 1 \cdot \frac{\sqrt{2}}{2} \cdot \cos 45^\circ$$

$$= 1 \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{1}{2}$$

Alternatively, $\vec{u}$ – unit vector $\Rightarrow$

$\vec{u} \cdot \vec{v}$ = the length of the projection of $\vec{v}$ onto $\vec{u}$ $\Rightarrow \vec{u} \cdot \vec{v} = \frac{1}{2}$.