Name: (print) ____

This exam includes 10 questions, on 12 pages, with the total of 56 points + 10 bonus points. The duration of the exam is 2 hours.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	8	9	10	total

Important: The exam is closed books/notes. A basic scientific calculator is allowed, no graphing calculators or other electronic devices. Show all your work.

1. (6 points) The pressure P, volume V and temperature T of a unit mass of ideal gas are related by the equation $PV = RT$, where R is the gas constant. At a certain instant the temperature is $T = 300$ K and increasing at a rate of 0.1 K/s, and the volume is $V = 100$ L and decreasing at a rate of 0.2 L/s. Assuming the value of R given find an expression for the rate at which the pressure is changing.

Continued...

2. (6 points) (a) Find a vector perpendicular to the plane through the points $A(1, 0, 1)$, $B(2, 1, 0)$ and $C(3, 2, 1)$.

   (b) Find the area of the triangle ABC.

Continued...

3. (6 points) (a) Show that the curve

$$\vec{r}(t) = (\sin t, t, 1 - \cos t).$$

lies on the circular cylinder with the axis parallel to the y-axis.

(b) Use the previous fact to sketch the curve. *[Hint: When $t = 0$, $\vec{r}(t) = (0, 0, 0)$.]*

(c) Find the expressions for the unit tangent vector $\vec{T}(t)$ for the curve in (a)-(b).

(d) Bonus: (2 points) find the principal normal vector $\vec{N}(t)$ and the curvature $\kappa(t)$.

4. (6 points) (a) Determine whether the following function is continuous at $(0,0)$:

$$f(x,y) = \begin{cases} \dfrac{x^2 - 4y^2}{x^2 + 2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

(b) Find the limit if it exists:

$$\lim_{(x,y)\to(1,0)} \ln\left(\frac{1+y^2}{x^2+xy}\right).$$

Continued...

5. (6 points) Find all critical points of the function and determine their type (local maximum/minimum, or saddle point):

$$f(x, y) = xy + \frac{4}{x} + \frac{2}{y}.$$

6. (8 points) (a) Determine whether the vector field $\vec{F}(x,y)$ is conservative:

$$\vec{F}(x,y) = e^{-y}\,\vec{i} - xe^{-y}\,\vec{j}.$$

If it is, find the function f such that $\vec{F}(x,y) = \vec{\nabla} f(x,y)$.

Continued...

(b) Evaluate the line integral

$$\int_C e^{-y}\,dx - xe^{-y}\,dy,$$

where C is the line segment from $(-1, 0)$ to $(1, 1)$, either directly, or using the potential function. (Bonus: 2 points) for evaluating it both ways.

7. (6 points) Find the volume of the solid below the paraboloid $z = 18 - 2x^2 - 2y^2$ and above the xy-plane.

8. (6 points) (a) Sketch the region of integration and change the order of integration:

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$$

(b) Evaluate the integral in part (a).

9. (6 points) A particle starts at the point $(-1, 0)$, moves along the x-axis to $(1, 0)$, then along the semicircle $y = \sqrt{1 - x^2}$ to the starting point. Use Green's theorem to find the work done on this particle by the force field

$$\vec{F}(x, y) = (x, x^3 + 3xy^2).$$

10. (bonus: 6 points) The helix $\vec{r_1}(t) = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$ intersects the curve $\vec{r_2}(t) = (1+t)\,\vec{i} + t^2\,\vec{j} + t^3\,\vec{k}$ at the point $(1,0,0)$. Find the angle of intersection of these curves.

The end.