SOLUTIONS TO CHAP 4-6 REVIEW

1) (a) $n,\ n+2,\ n+4$

(b) $(25x + y + 100z)$ cents

(c) $(2k + 2s)$ cm

2)(a) Method 1 Let x = James' amount (in \$)
So $x+60$ = Peter's amt
and $2(x+60)$ = John's amt

$$x + (x+60) + 2(x+60) = 600$$
$$x + x + 60 + 2x + 120 = 600$$
$$4x + 180 = 600$$
$$4x = 420$$
$$x = 105$$

Solution James has \$105
Peter has 105 + 60 or \$165
John has 2(105+60) or \$330

Method2 Let t = Peter's amt (in \$)
So $2t$ = John's amt
and $t-60$ = Jame amt

$$t + 2t + (t-60) = 600$$
$$4t - 60 = 600$$
$$4t = 660$$
$$t = 165$$

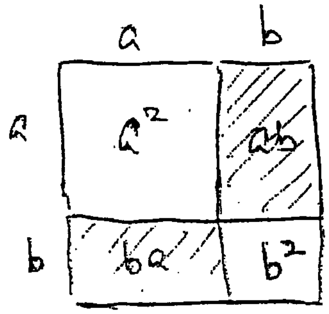
Solution Peter has \$165
John has 2(165) or \$330
James has 165 − 60 or \$105

(b) Let x = Jane's amt (in \$)
So $3x$ = Mary's amt

$$3x - 125 = x - 35$$
$$3x = x + 90$$
$$2x = 90$$
$$x = 45$$

Solution
Jane had \$45 to start
Mary had 3(\$45) or \$135 to start

3) (a)



AREA of a square with side a+b

$$(a+b)^2 = (a+b)(a+b)$$
$$= a^2 + ab + ba + b^2$$
$$= a^2 + ab + ab + b^2$$
$$= a^2 + 2ab + b^2$$

[ba = ab by Commutative property of mult]

(b) Using DISTRIB PROP,

$$(a+b)^2 = (a+b)(a+b) = (a+b)a + (a+b)b = a^2 + ba + ab + b^2$$
$$= a^2 + 2ab + b^2$$

since ba = ab by comm prop of mult

4) (a) $31^2 = (30+1)^2 = 30^2 + 2(30)(1) + 1^2 = 900 + 60 + 1 = 961$

(b) $23 \times 17 = (20+3)(20-3) = 20^2 - 3^2 = 400 - 9 = 391$

(c) $58^2 = (60-2)^2 = 60^2 - 2(60)(2) + 2^2 = 3600 - 240 + 4$
$= 3360 + 4 = 3364$

(d) $66^2 - 34^2 = (66+34)(66-34) = 100 \cdot 32$
$= 3200$

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5) (a) 55,264

ones digit is 4 → DIV BY 2, NOT DIV BY 5, NOT DIV BY 10

$4 \mid 64$ → DIV BY 4

number represented by last two digits

$8 \mid 264$ → DIV BY 8

number represented by last three digits

sum of digits $= 5 + 5 + 2 + 6 + 4 = 22$ $\quad 3 \nmid 22$ → NOT DIV BY 3

$9 \nmid 22$ → NOT DIV BY 9

NOT DIV BY 3 → NOT DIV BY 6

sum of digits in even powers of 10 spots $= \underbrace{11}_{5+2+4}$

sum of digits in odd powers of 10 spots $= \underbrace{11}_{5+6}$

Does $11 \mid (11-11)$? Yes $11 \mid 0$ → DIV BY 11

(b) 2,357,220

ones digit is 0 → DIV BY 2, DIV BY 5, DIV BY 10

$4 \mid 20$ → DIV BY 4

$8 \nmid 220$ → NOT DIV BY 8

sum of digits $= 2 + 3 + 5 + 7 + 2 + 2 + 0 = 21$ $\quad 3 \mid 21$ → DIV BY 3

$9 \nmid 21$ → NOT DIV BY 9

DIV BY BOTH 2 and 3 → DIV BY 6

$11 \nmid (\underbrace{12}_{3+7+2} - \underbrace{9}_{2+5+2+0})$ (that is, $11 \nmid 3$) → NOT DIV BY 11

(c) 5445

ones digit is 5 → DIV BY 5, NOT DIV BY 2, NOT DIV BY 10

$4 \nmid 45$ → NOT DIV BY 4

$8 \nmid 445$ → NOT DIV BY 8

sum of digits $= 5 + 4 + 4 + 5 = 18$ $\quad 3 \mid 18$ → DIV BY 3

$9 \mid 18$ → DIV BY 9

$11 \mid (\underbrace{9}_{4+5} - \underbrace{9}_{5+4})$ (that is, $11 \mid 0$) → DIV BY 11

(d) 11,111 NOT DIV BY 2, 3, 4, 5, 6, 8, 9, 10, 11 (give reasons as above)

6) (a) 4 must divide the last two digits; so we need $4 \mid 6\square$

The largest digit that works is 8 (that is, $4 \mid 68$)

<u>Note</u>: if the question asked for ALL the digits that work, give 0, 4, 8 ($4 \mid 60$, $4 \mid 64$, $4 \mid 68$)

(b) Since the ones digit is even, 2 divides the number. In order to make it divisible by 6, it must be divisible by 3. So 3 must divide the sum of the digits, namely $26 + \square$ The largest digit that works is 7. (That is $3 \mid (26+7)$ or $3 \mid 33$)

<u>Note</u>: if the question asked for ALL the digits that work, give 1, 4, 7 ($3 \mid \underset{26+1}{27}$, $3 \mid \underset{26+4}{30}$, $3 \mid \underset{26+7}{33}$)

7) Since 2 and 9 have no common factors other than 1 and $2 \cdot 9 = 18$, we can say a number is divisible by 18 if and only if it is divisible by both 2 and 9.

Consider 4536 ones digit is a 6, so $2 \mid 4536$

sum of digits $= 4+5+3+6 = 18$ $9 \mid 18$, so $9 \mid 4536$

Since both 2 and 9 divide 4536, we can conclude $18 \mid 4536$ (without actually dividing)

8) (a) FALSE for ex 4 and 8 are both even, but $GCF(4,8) = 4$ (NOT 2)

(b) FALSE for ex $GCF(8,9) = 1$ but 8 and 9 are not primes

(c) TRUE if $12 \mid a$, then we can certainly conclude $2 \mid a$ and $6 \mid a$

why? $12 \mid a$ means $a = 12 \cdot n$ for some whole number n

Then $a = \underline{2} \cdot \underline{6} \cdot n$ showing $a = 2(6n)$, so $2 \mid a$,

and $a = 6(2n)$, so $6 \mid a$

(d) FALSE for ex $2 \mid 18$ and $6 \mid 18$, but $12 \nmid 18$

(note: 2 and 6 have a common factor other than 1)

(e) FALSE for ex $9 \nmid 6$, but $3 \mid 6$

(f) FALSE for ex $4 \nmid 3$ and $3 \nmid 8$, but $4 \mid 8$

(g) TRUE If m is a <u>multiple</u> of n and 5 is a <u>factor</u> of n, then we can conclude m is a <u>multiple</u> of 5.

(CONTINUED →)

WHY? m is a multiple of n, so $m = n \cdot t$ for some whole number t

5 is a factor of n, so $n = 5 \cdot K$ for some whole number K

So $m = n \cdot t = (5 \cdot K) \cdot t = 5(Kt)$, showing m is a multiple of 5

(Basically m is a multiple of n and n is a multiple of 5, so we can conclude m is a multiple of 5. Alternatively, $5|n$ and $n|m$ implies $5|m$)

9) (a) 1 (b) ab (c) 1 (d) c^3d^2 (e) a^2b^2 (f) $a^3b^4c^4d$

10) (a) 29 (b) 23

largest prime satisfying $p^2 \leq 859$ — largest prime satisfying $p^2 \leq 671$

(b) 859 is <u>not</u> divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 $\rightarrow$ 859 is <u>PRIME</u>

What about 671?

$2 \nmid 671$, $3 \nmid 671$, $5 \nmid 671$, $7 \nmid 671$, $11 | 671$ $\rightarrow$ 671 is <u>COMPOSITE</u> (in fact $671 = 11 \cdot 61$)

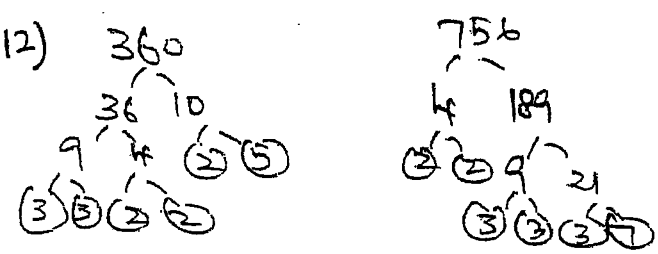
11) The <u>FACTORS</u> of 210 are:

1, 2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105, 210

Notice how each factor has a partner

$210 = 1 \times 210$
$= 2 \times 105$
$= 3 \times 70$
$= 5 \times 42$
$= 6 \times 35$
$= 7 \times 30$
$= 10 \times 21$
$= 14 \times 15$

12)



$360 = 2^3 \cdot 3^2 \cdot 5$

$756 = 2^2 \cdot 3^3 \cdot 7$

$GCF(360, 756) = 2^2 \cdot 3^2 = 36$

$LCM(360, 756) = 2^3 \cdot 3^3 \cdot 5 \cdot 7 = 7560$

13) $GCF(1988, 1065)$

$= GCF(1065, 923)$

$= GCF(923, 142)$

$= GCF(142, 71)$

$= GCF(71, 0) = 71$

1065) 1988 = 1, − 1065, remainder 923
923) 1065 = 1, − 923, remainder 142
142) 923 = 6, − 852, remainder 71
71) 142 = 2, − 142, remainder 0

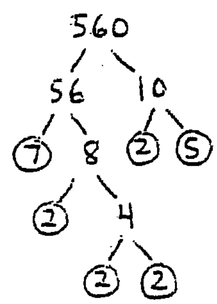
last nonzero remainder turns out to be GCF

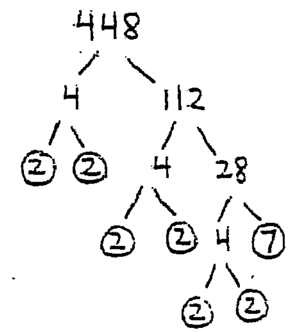
$$LCM(1988, 1065) = \frac{1988 \times 1065}{GCF(1988, 1065)} = \frac{(1988)(1065)}{71} = 29{,}820$$

14) (a) $560 = 2^4 \cdot 5 \cdot 7$ $\quad$ $GCF(560, 448)$

$448 = 2^6 \cdot 7$ $\quad$ $= 2^4 \cdot 7 = 112$

greatest length possible is 112 ins each





(b) RED: $\frac{560}{112} = 5$ pieces

BLUE: $\frac{448}{112} = 4$ pieces

15) $LCM(30, 15, 20) = 60$

Packages of plates $= \frac{60}{30} = 2$

Packages of cups $= \frac{60}{15} = 4$

Package of napkins $= \frac{60}{20} = 3$

$$\left[\begin{array}{l} 30 = 2 \cdot 3 \cdot 5 \\ 15 = 3 \cdot 5 \\ 20 = 2^2 \cdot 5 \\ \text{So } LCM(30,15,20) \\ \quad = 2^2 \cdot 3 \cdot 5 = 60 \end{array}\right]$$

16) $LCM(12, 16) = 48$ The least amount Jackie could have spent is \$48 on each

Number of cassette tapes $= \frac{48}{12} = 4$ $\quad$ Number of CDs $= \frac{48}{16} = 3$

17) 266 egg bagels, 112 onion bagels

SAME NUMBER of bagels in each box, NOT MIXING the bagels

To use the least number of boxes, we have to put the largest possible number of bagels in each box. A common divisor is the number of bagels per box; so the GCF(266,112) will give us the greatest number we could put per box.

Method 1 PRIME FACTORIZATION $\left.\begin{array}{l} 266 = 2^1 \cdot 7^1 \cdot 19^1 \\ 112 = 2^4 \cdot 7^1 \end{array}\right\}$ So $GCF(266,112) = 2^1 \cdot 7^1 = 14$

Method 2 EUC ALG

$GCF(266, 112) = GCF(112, 42) = GCF(42, 28)$

$= GCF(28, 14)$ ← answer is obvious here

$= GCF(14, 0)$

$= 14$

$$\begin{array}{r} 2 \\ 112\overline{)266} \\ \underline{224} \\ 42 \end{array} \qquad \begin{array}{r} 2 \\ 42\overline{)112} \\ \underline{84} \\ 28 \end{array} \qquad \begin{array}{r} 1 \\ 28\overline{)42} \\ \underline{28} \\ 14^{*} \end{array} \qquad \begin{array}{r} 2 \\ 14\overline{)28} \\ \underline{28} \\ 0 \end{array}$$

LAST NONZERO REMAINDER is the GCF

So we should put 14 BAGELS PER BOX

Now to get the minimum number of boxes:

We'll need 19 boxes for the egg bagels $(19 \cdot 14 = 266)$

We'll need 8 boxes for the onion bagels $(8 \cdot 14 = 112)$

The least number of boxes to pack all the bagels is $19 + 8$ or 27

18) $\text{LCM}(2,3,\underset{2^2}{\underset{\shortparallel}{4}},5,\underset{2\cdot 3}{\underset{\shortparallel}{6}}) = 2^2\cdot 3\cdot 5 = 60$ ↩ smallest multiple the numbers have in common

Since there is always one cupcake left over, the smallest number of cupcakes John could have is $60+1$ or 61

19) (a) (i) $\frac{5}{6} = \frac{25}{30}$ $\frac{4}{5} = \frac{24}{30}$

Since $25 > 24$, $\frac{5}{6} > \frac{4}{5}$

(ii) $\frac{73}{90} = \frac{146}{180}$ $\frac{51}{60} = \frac{153}{180}$

Since $146 < 153$, $\frac{73}{90} < \frac{51}{60}$

(iii) $\frac{13}{36} < \frac{13}{27}$ if two (pos) fractions have the same num, the one with the bigger denom is the smaller number

(iv) $\frac{243}{317} = \frac{2673}{3487}$ since

$(243)(3487) = (317)(2673)$

(both prods equal 847,341)

(b) $\frac{1}{2} < \frac{11}{20} < \frac{2}{3} < \frac{29}{36} < \frac{5}{6} < \frac{8}{9} < \frac{13}{12}$

20) (a) $\frac{1}{4}+\frac{1}{5}+\frac{1}{6} = \frac{15}{60}+\frac{12}{60}+\frac{10}{60} = \frac{15+12+10}{60} = \frac{37}{60}$

(b) Method 1 $3\frac{1}{6} - 1\frac{3}{4} = \frac{19}{6} - \frac{7}{4} = \frac{38}{12} - \frac{21}{12} = \frac{17}{12} = 1\frac{5}{12}$

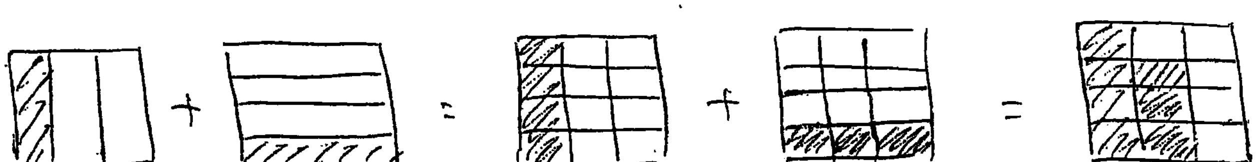
Method 2 Rewrite $3\frac{1}{6}$ as $2\frac{7}{6}$

$3\frac{1}{6} - 1\frac{3}{4} = 2\frac{7}{6} - 1\frac{3}{4} = 2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$

(c) $1\frac{1}{2}\cdot 1\frac{1}{3}\cdot 1\frac{1}{4} = \frac{\cancel{3}}{2}\cdot\frac{\cancel{4}}{\cancel{3}}\cdot\frac{5}{\cancel{4}} = \frac{5}{2} = 2\frac{1}{2}$

(d) $6\frac{2}{3} \div 1\frac{5}{6} = \frac{20}{3} \div \frac{11}{6} = \frac{20}{\cancel{3}}\cdot\frac{\cancel{6}\,2}{11} = \frac{40}{11} = 3\frac{7}{11}$

21) (a)

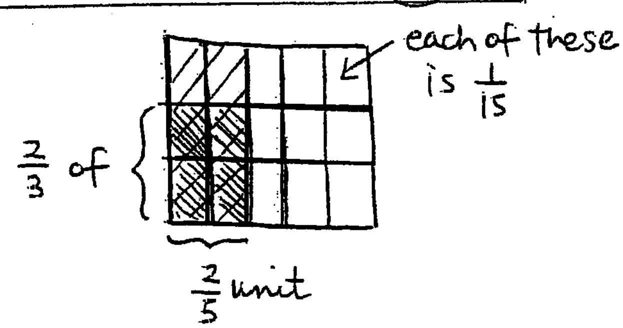


$\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

(b) $\frac{2}{3}\times\frac{2}{5}$ means "$\frac{2}{3}$ of $\frac{2}{5}$"

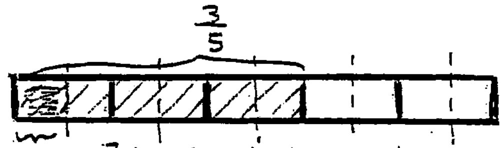
$\frac{2}{3}\times\frac{2}{5} = \frac{2}{3}$ of $\frac{2}{5}$ unit

$= \frac{4}{15}$



→

(c) $\frac{3}{5} \div 6$ Method 1

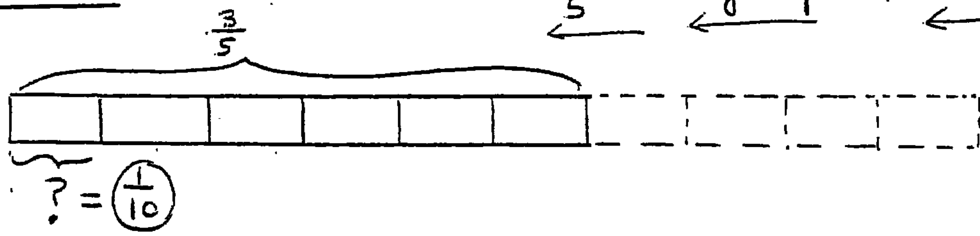


$\frac{3}{5} \div 6 = \frac{1}{10}$

? $\frac{3}{5}$ is divided up into 6 equal pieces

Method 2

$\frac{3}{5}$ is 6 groups of what? PD



6 units $= \frac{3}{5}$

So 1 unit $= \frac{3}{5} \div 6$

$= \frac{1}{6}$ of $\frac{3}{5}$

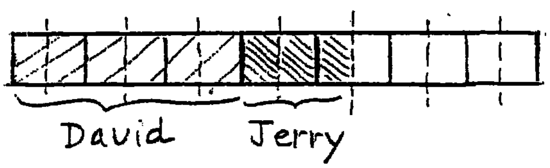
$= \frac{1}{6} \cdot \frac{3}{5} = \frac{1}{10}$

22) (a) David ate $\frac{3}{7}$ of the box (THINK OF ENTIRE BOX as 1)

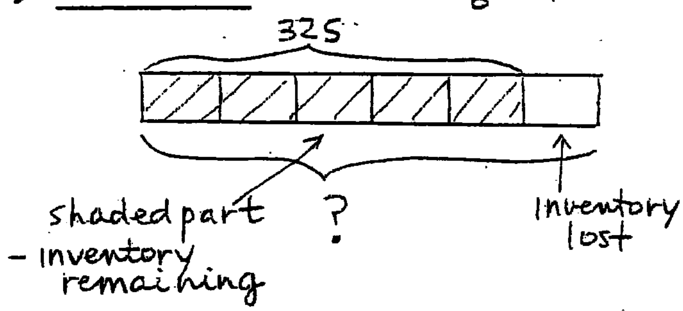
Jerry ate $\frac{1}{2}$ of $\frac{3}{7}$ $\quad \frac{1}{2} \cdot \frac{3}{7} = \frac{3}{14}$

Together they ate $\frac{3}{7} + \frac{3}{14} = \frac{6}{14} + \frac{3}{14} = \frac{9}{14}$

What fraction of the box remains? $1 - \frac{9}{14} = \frac{14}{14} - \frac{9}{14} = \frac{5}{14}$



(b) Method 1 325 is $\frac{5}{6}$ of what? PD



5 units = 325

1 unit = 325 ÷ 5 = 65

6 units = 6 x 65 = 390

So there were 390 boxes originally

(alt writing as a division $325 \div \frac{5}{6} = 325 \cdot \frac{6}{5} = 390$)

Method 2 using algebra

lost $\frac{1}{6}$ of inventory → $\frac{5}{6}$ of inventory remains (and we're told there are 325 boxes left)

Let x = original number of boxes $\quad \frac{5}{6}x = 325$

Mult both sides by $\frac{6}{5}$

$\frac{6}{5} \cdot \frac{5}{6}x = \frac{6}{5} \cdot 325$

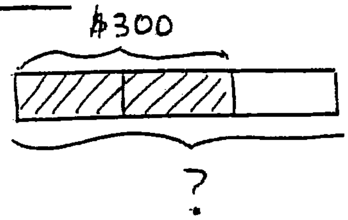
$x = 390$

note: can also do

$$\underset{\text{total number}}{x} - \underset{\text{number lost}}{\tfrac{1}{6}x} = \underset{\text{number remaining}}{325}$$

This is the same as $\frac{5}{6}x = 325$

(c) Method 1 $\$300$ is $\frac{2}{3}$ of what? PD



2 units = 300
1 unit = $300 \div 2 = 150$
3 units = $3 \times 150 = 450$

The washing machine originally cost $\$450$

(alt
$300 \div \frac{2}{3}$
$= 300 \times \frac{3}{2}$
$= 450$)

Method 2 Let x = orig price $\frac{2}{3}$ of x is $\$300$

$\frac{2}{3}x = 300$ Mult thru by $\frac{3}{2}$ $\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 300 \rightarrow x = \$450$

(d) $\frac{3}{4} \div 9 = \frac{\cancel{3}}{4} \cdot \frac{1}{\cancel{9}\,3} = \frac{1}{12}$ gal Each cat gets $\frac{1}{12}$ gal.

$\frac{3}{4}$ gals is 9 groups of what? PD

(Alternatively If $\frac{3}{4}$ gallon feeds 9 cats,
then $\frac{1}{4}$ gallon feeds 3 cats
So $\frac{1}{3}$ of $\frac{1}{4}$ gallon or $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ gallon feeds 1 cat)

(e) $18 \div 2\frac{1}{2}$ How many $2\frac{1}{2}$ cups are there in 18 cups? MD

$18 \div 2\frac{1}{2} = 18 \div \frac{5}{2} = 18 \cdot \frac{2}{5} = \frac{36}{5} = 7\frac{1}{5}$

The question asks for how many <u>entire</u> cakes we can make? 7 cakes

How much flour is left over?

We use up $7 \times 2\frac{1}{2} = 7 \cdot \frac{5}{2} = \frac{35}{2} = 17\frac{1}{2}$ cups making the 7 cakes

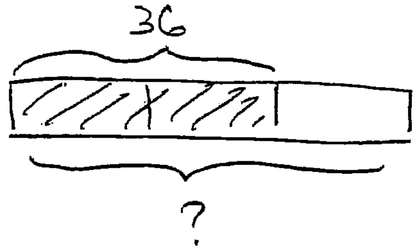
What's left? $18 - 17\frac{1}{2} = \frac{1}{2}$ cup

[another way to look at second part of question:
Since we're dividing by $\frac{5}{2}$, the amount left over is
$\frac{1}{5}$ of $\frac{5}{2}$ or $\frac{1}{5} \cdot \frac{5}{2} = \frac{1}{2}$ cup

Note $18 = 7\frac{1}{5} \cdot \frac{5}{2} = (7 + \frac{1}{5}) \cdot \frac{5}{2} = \underbrace{7 \cdot \frac{5}{2}}_{\text{flour used for 7 cakes}} + \underbrace{\frac{1}{5} \cdot \frac{5}{2}}_{\text{amt left over}} = 17\frac{1}{2} + \frac{1}{2}$]

23) (a) PD $36 \div \frac{2}{3}$

"36 is $\frac{2}{3}$ of what?"

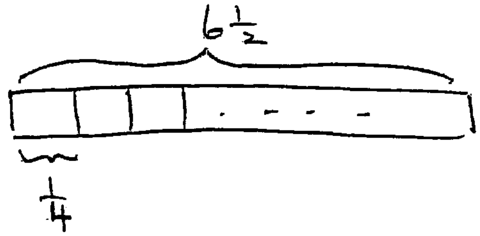


After spending $\frac{1}{3}$ of her money on a gift, Jane had \$36 left. How much money did she have at first?

answer: \$54

(b) MD $6\frac{1}{2} \div \frac{1}{4}$

"$6\frac{1}{2}$ is how many $\frac{1}{4}$s?"



I have $6\frac{1}{2}$ yards of material. If I need $\frac{1}{4}$ yard to make an apron, how many aprons can I make?

answer: 26 aprons

24) If x = total number of stars, then $\frac{1}{4}x = 9$ (there are 9 stars drawn)

$x = 36$ total stars

So what is $\frac{5}{9}$ of 36? $\frac{5}{9}(36) = 20$ stars

25) $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

So J got $1 - \frac{7}{12}$ or $\frac{5}{12}$ of the bag

If x = total number of marbles, then $\frac{5}{12}x = 35$

Mult both sides by $\frac{12}{5}$

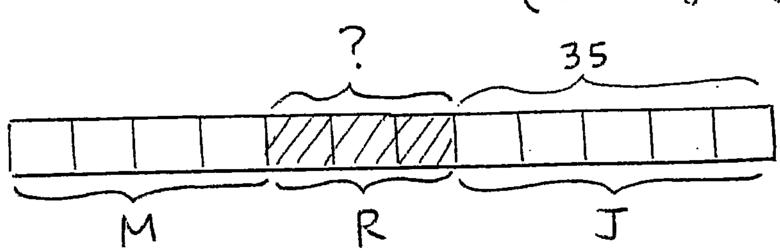
$x = \frac{12}{5} \cdot 35 = 84$ total marbles

So R got $\frac{1}{4}(84)$ or 21

(and M got $\frac{1}{3}(84)$ or 28)

check
$$\begin{array}{r} 21 \\ 28 \\ +35 \\ \hline 84 \end{array} \checkmark$$

alt



5 units = 35

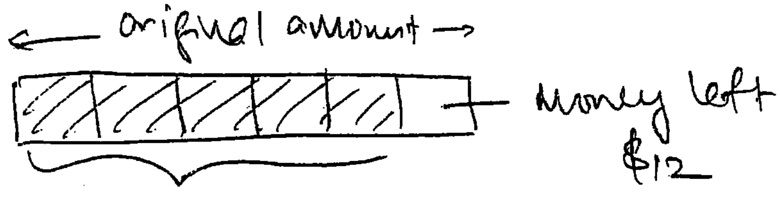
1 unit = 7

So 3 units = 3 x 7 = 21

R got 21 marbles

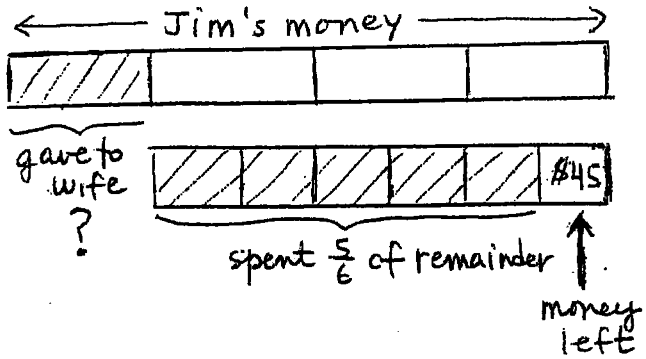
(and M got 4 x 7 or 28)

26) (a)



original amount = \$12 × 6 = \$72

(b)

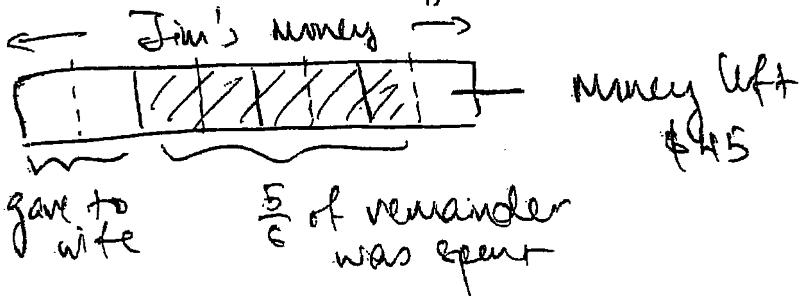


Based on picture he gave

2 × \$45 = \$90 to his wife

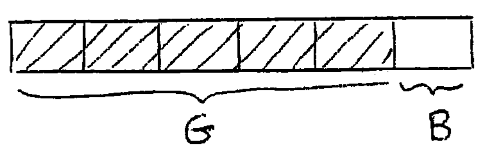
(note: he started with 4 × \$90 = \$360)

alt can draw on one bar diagram

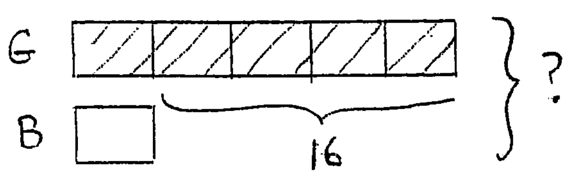


Amount given to wife = \$45 × 2 = \$90

(c)



Next comparing G and B (we're told there are 16 more G than B)



4 units = 16

1 unit = 16 ÷ 4 = 4

Number of children altogether

= 6 units = 6 × 4 = 24

(There are 20 G, 4 B)

27) (a) $630 = \frac{7}{30} n$

$\frac{30}{7} \cdot 630 = \frac{\cancel{30}}{\cancel{7}} \cdot \frac{\cancel{7}}{\cancel{30}} n$ So $n = \frac{30}{\cancel{7}} \cdot \frac{\cancel{630}^{90}}{1} = \underline{\underline{2700}}$

The number is 2700.

(b) EASIEST APPROACH!

$\frac{2}{5}$ of lawn ~ $\frac{1}{3}$ hour) divide by 2

$\frac{1}{5}$ of lawn ~ $\frac{1}{6}$ hour) mult by 5

entire lawn = $\frac{5}{5}$ of lawn ~ $\frac{5}{6}$ hour

takes $\frac{1}{3}$ hr, takes $\frac{1}{3}$ hr, takes $\frac{1}{2}$ of $\frac{1}{3}$ or $\frac{1}{6}$ hr

so total lawn takes $\frac{1}{3} + \frac{1}{3} + \frac{1}{6} = \frac{5}{6}$ hr

alt. Can let n = time (in hrs) to mow entire lawn

$\frac{2}{5}$ of $n = \frac{1}{3}$ hr

$\frac{2}{5} n = \frac{1}{3}$

$\frac{5}{2} \cdot \frac{2}{5} n = \frac{1}{3} \cdot \frac{5}{2}$

$n = \frac{5}{6}$ hr

same as

$\frac{5}{6}$(60 mins) = 50 mins

Note: if you used minutes instead ($\frac{1}{3}$ hr = 20 mins)

$\frac{2}{5}$ of lawn ~ 20 minutes) divide by 2

$\frac{1}{5}$ of lawn ~ 10 minutes) mult by 5

entire lawn = $\frac{5}{5}$ of lawn ~ 50 mins