MATH 210, Chapters 4-6, 7.1 Review

1. Write an algebraic expression for the following:

   (a) Three consecutive odd numbers, the smaller of which is n.

   (b) The value in cents of x quarters, y pennies, and z dollars.

   (c) The perimeter of a rectangle with length k cm and width s cm.

2. Solve the following word problems algebraically:

   (a) John has twice as much money as Peter, and Peter has \$60 more than James. If all three boys have a total of \$600, how much money does each boy have?

   (b) Mary had three times as much money as Jane. After Mary spent \$125 and Jane spent \$35, the two girls had the same amount of money left. How much did each have at the beginning?

3. Prove the identity $(a+b)^2 = a^2 + 2ab + b^2$:

   (a) using rectangular array diagram (b) using algebra (distributive property)

4. Identities: $(a+b)^2 = a^2 + 2ab + b^2$, $(a+b)^2 = a^2 + 2ab + b^2$,
   $a^2 - b^2 = (a-b)(a+b)$.

   Use the above identities to calculate the following:

   (a) 31^2 (b) 23×17 (c) 58^2 (d) $66^2 - 34^2$

5. Determine if the following numbers are divisible by 2, 3, 4, 5, 6, 8, 9, 10, or 11, and justify your answers:

   (a) $55,264$ (b) $2,357,220$ (c) 5445 (d) $11,111$

6. Find the larges possible single digit that can replace the $\square$ so that

   (a) 4 is a divisor of $3,600,874,56\square$

   (b) 6 is a divisor of $102,\square 03,004,952$

7. Without actually dividing how would you determine if 4536 is divisible by 18? (State a test of divisibility by 18 and use it on 4536.)

8. Answer true or false; if false, provide a counter-example:

   (a) If a and b are both even, then $\text{GCD}(a,b) = 2$.

   (b) If $\text{GCD}(a,b) = 2$ then a and b are both even.

(c) If $\text{GCD}(a, b) = 1$ then a and b are both prime numbers.

(d) If a number is divisible by 12 then it is divisible by 2 and by 6.

(e) If a number is divisible by 2 and by 6 then it is divisible by 12.

(f) If m is a multiple of n and 5 is a factor of n, then m is a multiple of 5.

9. If a, b, c, d are distinct prime numbers, find the following:

(a)	$\text{GCD}(a, b)$	(b)	$\text{LCM}(a, b)$
(c)	$\text{GCD}(c^3, d^2)$	(d)	$\text{LCM}(c^3, d^2)$
(e)	$\text{GCD}(a^3b^2c^4, a^2b^4)$	(f)	$\text{LCM}(a^3b^2c^4, a^2b^4d)$.

10. (a) For each of the following, determine the largest prime number that must be checked to see if the number is prime (using the test for primeness):

    (i) 859 (ii) 671

    (b) Is 859 prime or composite? Is 671 prime or composite? Why?

11. List all the factors of 210.

12. Write the prime factorization of 360 and 756 in exponential form. Use your result to determine $\text{GCD}(360, 756)$ and $\text{LCM}(360, 756)$.

13. Using Euclid's algorithm, find $\text{GCD}(1988, 1065)$. Use your result to find $\text{LCM}(1988, 1065)$.

14. A red ribbon has length 560 inches, and a blue ribbon has length 488 inches. Both ribbons are to be divided into pieces of the same length, and so that these pieces are as long as possible.

    (a) What is the length of each piece?

    (b) How many pieces of each color will there be?

15. At the Party store, paper plates come in packages of 30, paper cups in packages of 15, and napkins in packages of 20. What is the least number of packages of plates, cups and napkins that can be purchased so that there is an equal number of each?

16. Jackie spent the same amount of money on cassette tapes as she did on compact disks. If tapes cost \$12, and CDs \$16, what is the least amount she could have spent on each? How many cassette tapes were bought and how may CDs?

17. A bakery has 266 egg bagels and 112 onion bagels. The bagels are to be packaged in boxes so that each box will have the same number of bagels. However, each box may contain only one type of bagel. What is the minimum number of boxes that can be used to pack all the bagels?

18. John is serving cupcakes at a school party. If he arranges the cupcakes in groups of 2, 3, 4, or 6, he always has exactly one cupcake left over. What is the smallest number of cupcakes he could have?

19. (a) Fill in the blank with $<$, $=$, $>$

    (i) $\frac{5}{6} \square \frac{4}{5}$ (ii) $\frac{73}{90} \square \frac{51}{60}$ (iii) $\frac{13}{30} \square \frac{13}{27}$

    (b) order from smallest to largest: $\frac{2}{3}$, $\frac{5}{6}$, $\frac{29}{36}$, $\frac{8}{9}$, $\frac{11}{20}$, $\frac{13}{12}$, $\frac{1}{2}$.

20. Write each of the following as a single fraction (in simplest form) or as a mixed number:

    (a) $\frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ (b) $3\frac{1}{6} - 1\frac{3}{4}$ (c) $1\frac{1}{2} \cdot 1\frac{1}{3} \cdot 1\frac{1}{4}$ (d) $6\frac{2}{3} \div 1\frac{5}{6}$

21. Use an area model to illustrate each of the following:

    (a) $\frac{1}{3} + \frac{1}{4}$ (b) $\frac{2}{3} \times \frac{2}{5}$ (c) $\frac{3}{5} \div 6$

22. Solve the following short problems involving fractions:

    (a) David ate $\frac{3}{7}$ of a box of cookies. Jerry ate <u>half</u> as much as David. What fraction of the box still remains?

    (b) Due to a fire, a company lost $\frac{1}{6}$ of its inventory. There are 325 boxes left. How many boxes were there originally?

    (c) A washing machine is on sale for $\frac{2}{3}$ of its original price. The sale price is \$300. What was the original price?

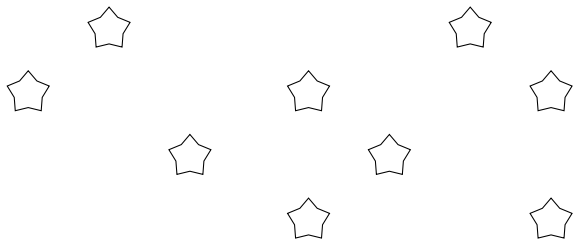
    (d) Mary had $\frac{3}{4}$ gallon of milk to feed her 9 cats. How much milk does each cat get?

    (e) A recipe of chocolate cake calls for $2\frac{1}{2}$ cups of flour. How many <u>entire</u> cakes can be made using 18 cups of flour? How much flour is left over?

23. Make up a short word problem of the type specified:

    (a) partitive division for $36 \div \frac{2}{3}$ (b) measurement division for $6\frac{1}{2} \div \frac{1}{4}$

24. The stars in this picture represent $\frac{1}{4}$ of a certain amount. How many stars would represent $\frac{5}{9}$ of the same amount?



25. Matt, Rob and Jen shared a bag of marbles. Matt got $\frac{1}{3}$ of the marbles, Rob got $\frac{1}{4}$ of the marbles, and Jen got the remaining 35 marbles in the bag. How many marbles did Rob get?

26. Give a full teacher's solution to each using a bar diagram – do not use algebra.

    (a) Wendy spent $\frac{5}{6}$ of her money on a pair of shoes. If she had \$12 left, how much money did she have to start?

    (b) Jim gave $\frac{1}{4}$ of his money to his wife. He then spent $\frac{5}{6}$ of the remainder. If he had \$45 left, how much did he give to his wife?

    (c) $\frac{5}{6}$ of the class are girls. There are 16 more girls than boys. How many children are in the class altogether?

27. Write out clear solutions of the following (you can use any approach)

    (a) 630 is $\frac{7}{30}$ of which number?

    (b) A gardener takes $\frac{1}{3}$ of an hour to mow $\frac{2}{5}$ of the lawn. At that rate, how many hours will it take to mow the entire lawn?

28. The ratio of the weight of an object on Jupiter to its weight on Earth is $8 : 3$. If a rock weighs 112 lbs on Jupiter, how much would it weigh on Earth?

29. On a map, $\frac{1}{2}$ cm represents 70 miles. If city A and city B are $3\frac{2}{5}$ cm apart on the map, what is the actual distance between them?

30. The ratio of men-to-women at a dance is 3-to-4, and there are 315 people present. How many are men?

Answers to 28, 29 and 30: 42 lbs, 476 miles, 135 men.