

Name (print): Solutions

Each problem is worth 2 points. Show all your work.

1. (a) Find a rational number between
- $\frac{4}{13}$
- and
- $\frac{5}{13}$
- .

— One way:

$$\frac{4}{13} = \frac{8}{26}; \quad \frac{5}{13} = \frac{10}{26}; \quad \frac{9}{26} \text{ is rational and is between } \frac{4}{13} \text{ and } \frac{5}{13}.$$

— Another way:
(harder)

$$\frac{4}{13} = 0.\overline{307692}$$

$$\frac{5}{13} = 0.\overline{384615}$$

Pick a terminating decimal between these two.

Examples: $0.31, 0.32, \dots, 0.35, \dots, 0.38, \dots$
or $0.\overline{3} = \frac{1}{3}$ - easy fraction.

— Somewhat more advanced:

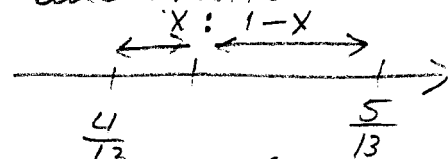
Take any number of the form $\frac{4}{13}(1-x) + \frac{5}{13}x$

- (b) Find two irrational numbers whose sum is rational.

Examples:

$$\underbrace{\sqrt{2}}_{\text{irr.}} + \underbrace{(2-\sqrt{2})}_{\text{irr.}} = \underbrace{2}_{\text{rat.}}$$

$$\left(\sqrt{2} + (-\sqrt{2}) = 0 \right) \text{ also works, but the second number is negative.}$$

where $0 < x < 1$
and is irrational.

For example:

$$\frac{4}{13} \cdot \frac{2}{3} + \frac{5}{13} \cdot \frac{1}{3} = \frac{13}{39} = \frac{1}{3}.$$

$$\begin{array}{r} 0.101001000100001\dots\dots \\ + 0.898998999899998\dots\dots \\ \hline 0.99999\dots\dots \end{array}$$

both
infinite,
non-repeating
decimals
 $\Rightarrow$ irrational

infinite repeating decimal $\Rightarrow$ rational.

(In fact $0.\overline{9} = 3 \times 0.\overline{3} = 3 \times \frac{1}{3} = 1$; see also Exercise 5 in 9.2) Please turn over...

2. Ryan took 15 minutes to walk to school at an average speed of 70 m/min. His brother took 20 min to walk the same distance. Find his brother's average speed.

One way: Ask how far is the school:

$$70 \frac{\text{m}}{\text{min}} \cdot 15 \text{ min} = 1050 \text{ m}$$

(m = meters!)

Find the speed for his brother:

$$\frac{1050 \text{ m}}{20 \text{ min}} = \frac{105}{2} \frac{\text{m}}{\text{min}} = 52.5 \text{ m/min.}$$

Another way: Distance = speed $\times$ time

$$70 \text{ m/min} \cdot 15 \text{ min} = X \text{ m/min} \cdot 20 \text{ min}$$

$$20 X = 70 \cdot 15$$

$$X = \frac{70 \cdot 15}{20} = \frac{7 \cdot 15}{2} = \frac{105}{2} = 52.5 \text{ m/min}$$

3. One hose can fill a swimming pool in 30 hours, while the other hose fills the swimming pool in 70 hours. How long would it take to fill up the swimming pool using both hoses together?

The rate for 1st hose: $\frac{1}{30} \frac{\text{pool}}{\text{hr}}$

The rate for 2nd hose: $\frac{1}{70} \frac{\text{pool}}{\text{hr}}$

Add rates, since the total amount per hour is the sum of the above two:

$$\text{Together: } \frac{1}{30} + \frac{1}{70} = \frac{7}{210} + \frac{3}{210} = \frac{10}{210} = \frac{1}{21}$$

$\frac{\text{pool}}{\text{hr}}$

The pool will be filled in 21 hours.