Mon, Nov 19, 2012 **MATH 210, Midterm 2** Prof. V. Panferov

Name: (print) Solutions.

This test includes 10 questions on 6 pages; the last question is a bonus. The perfect score is 54 points; the bonus question is an extra 6 points. The duration of the test is 50 minutes.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	8	9	10	total

Important: The test is closed books/notes. No calculators or any other electronic devices. Show all your work.

1. (6 points) (a) Write algebraic expressions for the following:

   (i) The value in cents of x nickels, y dimes and z quarters.

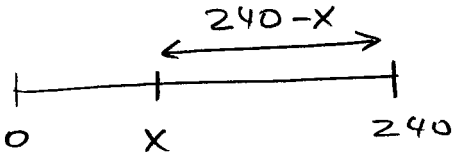
   $5x + 10y + 25z$

   (ii) The sum of three consecutive even numbers, the first of which is $2n$.

   $2n + (2n+2) + (2n+4)$

   $= 6n + 6$

   (iii) The remaining traveling time in the following problem: *We are on a roadtrip from city A to city B, which are 240 miles apart. We have traveled x miles, going at the speed of* 50 *miles per hour. Find the remaining traveling time.*



$240 - x$ miles, at 50 mph.

$240 - x = 50 \cdot T$

$T = \dfrac{240 - x}{50}$

2. (6 points) (a) Use an algebraic identity for $a^2 - b^2$ to compute the difference $51^2 - 49^2$.

$a^2 - b^2 = (a-b)(a+b)$

$51^2 - 49^2 = (51-49)(51+49)$

$= 2 \cdot 100 = \underline{\underline{200}}.$

(b) Compute the following mentally and show how you did it:

(i) $1\frac{1}{2} + \frac{2}{3} - \frac{1}{6}$

$= \frac{1}{2}$

$1\frac{1}{2} + \frac{1}{2} = 2$

(ii) $44 \cdot \frac{3}{8} + 44 \cdot \frac{7}{8}$

$= 44 \cdot \left(\frac{3}{8} + \frac{7}{8}\right)$

$= 44 \cdot \frac{10}{8}$

$= 44 \cdot \frac{5}{4}$

$= 11 \cdot 5 = 55$

3. (6 points) Prove that the sum of any three consecutive whole numbers is divisible by 3. Show your reasoning in a clear way. *Hint:* Call the first number x.

Sum $= x + (x+1) + (x+2)$

$= 3x + 3 = 3(x+1)$

divisible by 3, since it is 3 times a whole number.

Continued...

4. (6 points) Find <u>all possible digits</u> (and show how you found them) that can be used to replace $\square$ so that

   (a) 9 is a divisor of $756, \square 32$

   $7+5+6+3+2 = 23$

   $23 + \square$ must be divisible by 9

   $\Rightarrow 23 + \square = 27 \Rightarrow \square = 4$

   (b) 6 is a divisor of $375, 87\square$

   $\square$ must be even, and sum of the digits must be divisible by 3.

   $3+7+5+8+7 = 30 \Rightarrow \square$ can be 0 or 6

5. (6 points) Use Euclid's algorithm to find $GCF(1001, 208)$ and $LCM(1001, 208)$. Hint: $1001 \div 208 = 4\,R\,169$.

   $GCF(1001, 208) = GCF(208, 169)$

   $= GCF(169, 39)$

   $= GCF(39, 13) = 13$

$$\begin{array}{r} 1 \\ 169 \overline{)\,208} \\ 169 \\ \hline 39 \end{array} \qquad \begin{array}{r} 4 \\ 39 \overline{)\,169} \\ 156 \\ \hline 13 \end{array} \qquad \begin{array}{r} 3 \\ 13 \overline{)\,39} \\ 39 \\ \hline 0 \end{array}$$

$LCM(1001, 208)$

$$= \frac{1001 \cdot 208}{13} = 1001 \cdot 16$$

$= 16,016.$

$208 = 13 \cdot 16$

$1001 = 1000 + 1$

Continued...

6. (6 points) Simplify the following expressions with exponents:

(a) $\dfrac{2^5 \cdot 3^2 \cdot 12^2}{6^3 \cdot 18}$

$= \dfrac{2^5 \cdot 3^2 \cdot 4^2 \cdot 3^2}{2^3 \cdot 3^3 \cdot 2 \cdot 3^2}$

$= \dfrac{2^5 \cdot 3^2 \cdot 2^4}{2^4 \cdot 3^{3+1}}$

$= \dfrac{2^5}{3} = \dfrac{32}{3}$

(b) $\dfrac{a^3 \cdot (bc^2)^3 \cdot (ab)^0}{c^5 \cdot (abc)^2}$

$\dfrac{a^3 \cdot b^3 \cdot c^6}{c^5 \cdot a^2 \cdot b^2 \cdot c^2}$ $= c^7$

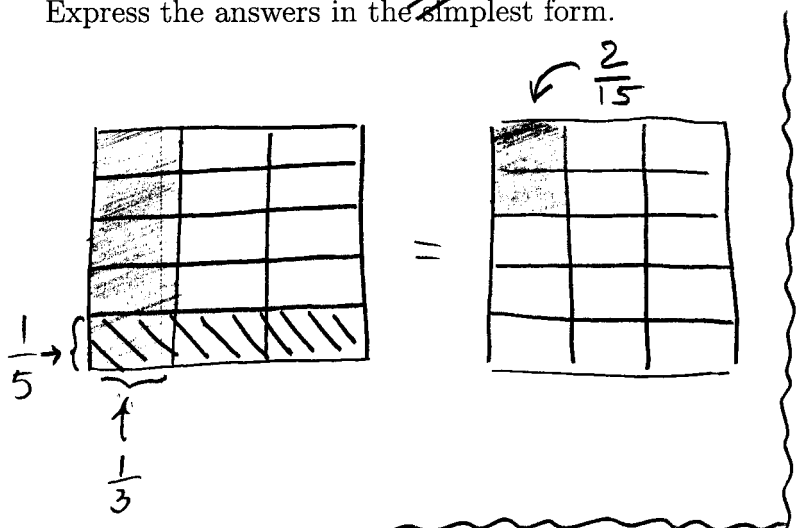
$= \dfrac{ab}{c}$

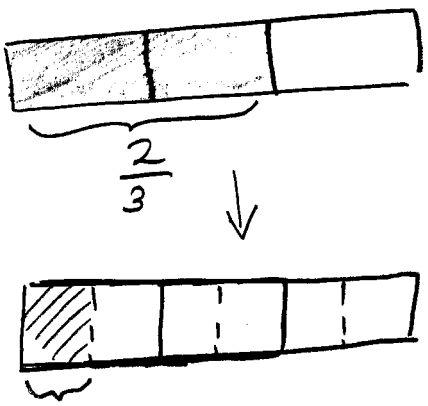
7. (6 points) Use an area (rectangular array) model to demonstrate how to find the following:

(a) $\frac{1}{3} - \frac{1}{5} = \frac{2}{15}$

(b) $\frac{2}{3} \div 4 = \frac{1}{6}$ (or $\frac{2}{12}$)

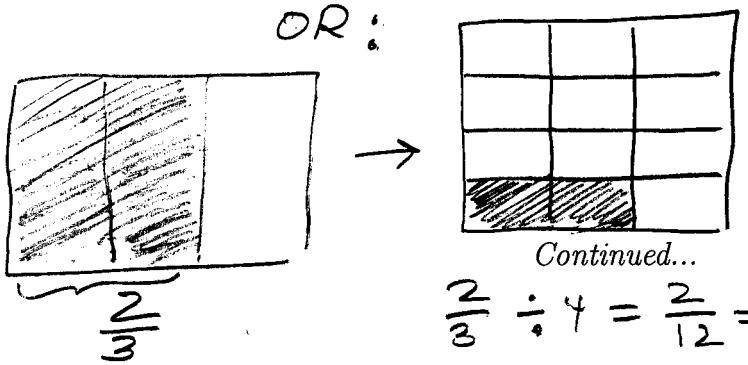
Express the answers in the simplest form.





$\frac{1}{6} = \frac{2}{3} \div 4$

OR:



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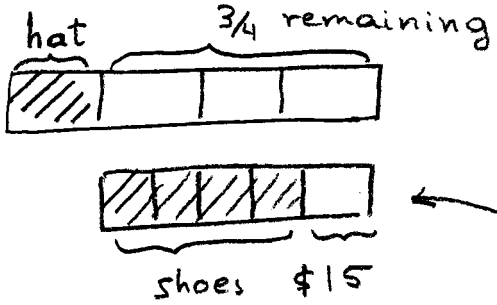
$\frac{2}{3} \div 4 = \frac{2}{12} = \frac{1}{6}$.

8. (6 points) Give a *teacher's solution* using bar diagrams – do not use algebra:

   Matthew spent $\frac{1}{4}$ of his money on a hat. He then spent $\frac{5}{6}$ of the remainder on a pair of shoes. If he had \$15 left, how much money did he have in the beginning?



1 unit = 15 dollars.

"remainder" = 6 units

= 6 · 15 = 90 dollars

$90 \div \frac{3}{4} = 90 \times \frac{4}{3}$

$= 30 \times 4$

$= 120$ dollars

had originally

9. (6 points) Give a *teacher's solution* using algebra:

   The ratio of white marbles to red marbles in a bag is 2 : 3. After 2 white marbles are removed from the bag, the ratio becomes 3 : 5. How many red marbles are there?

before: white = $2x$, red = $3x$ after: $2x-2$, $3x$

$(2x-2) : 3x = 3 : 5$

$\frac{2x-2}{3x} = \frac{3}{5}$

$5(2x-2) = 3 \cdot 3x$

$10x - 10 = 9x$

$x - 10 = 0$

$x = 10$

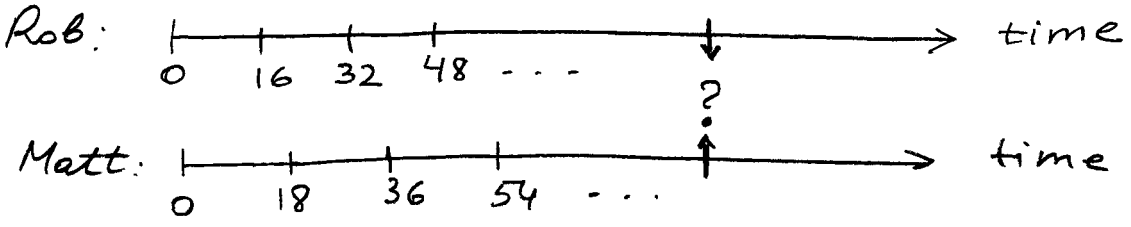
red marbles

$3x = 30$

Continued...

10. (bonus: 6 points) Two runners run around a circular track. Rob completes one lap in 16 minutes. Matt completes one lap in 18 minutes. They both start at the same time, and go in the same direction.

    (a) After how many minutes will they meet again at the starting place? Show work.



$LCM(18,16) = LCM(2\cdot 3^2, 2^4) = 2^4 \cdot 3^2 = 16 \cdot 9 = 144$ (minutes.)

    (b) At that time, how many laps has Rob completed? 9

    how many laps has Matt completed? 8

$144 = 9 \cdot 16 = 8 \cdot 18$

↑ Rob ↑ Matt

The end.