Thu, Oct 14, 2010 **MATH 150A, Midterm 1** Prof. V. Panferov

Name: (print) ____________________

CSUN ID No. : Solutions

This test includes 7 questions (54 points in total), on 7 pages. The duration of the test is 1 hour 5 minutes.

Your scores: (do not enter answers here)

1	2	3	4	5	6	7	total

Important: The test is closed books/notes. Graphing calculators are not permitted. Show all your work.

1. (6 points) Find an equation of the tangent line to the curve $2x^2 + y^2 = 3$ (an ellipse) at the point with coordinates $(1, 1)$.

Implicit differentiation:

$$2 \cdot 2x + 2yy' = 0$$

$$2x + yy' = 0$$

$$y' = -\frac{2x}{y}$$

when $x = 1$, $y = 1$, $y' = -2$

Tangent line:

$$y - 1 = -2(x - 1)$$

$$y = -2x + 3.$$

2. (8 points) Find the constant b such that the function

$$f(x) = \begin{cases} 1+\sqrt{-x}, & x \le 0 \\ x+b, & x > 0 \end{cases}$$

is continuous on $(-\infty, \infty)$. Sketch the graph of the function. Based on the graph, is the function $f(x)$ differentiable at $x = 0$? Explain.

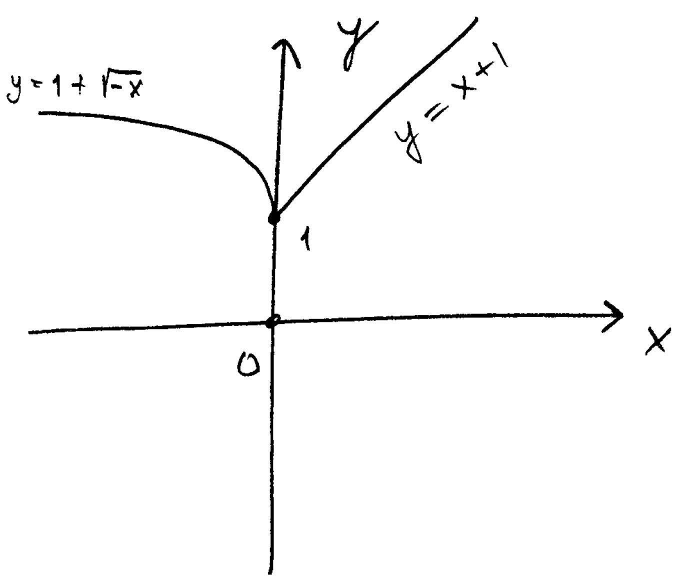
$f(x)$ is continuous for $x<0$ and for $x>0$.

To make it continuous at 0 choose b such that $\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0+} f(x)$.

$$\lim_{x\to 0-} f(x) = \lim_{x\to 0-} 1+\sqrt{-x} = 1.$$

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} x+b = b$$

$$\Rightarrow \quad b = 1$$



The graph is not smooth at $x=0$ (the slopes of tangent lines from the left and from the right are different: graph has a "corner")

$\Rightarrow$ $f(x)$ is not differentiable.

3. (8 points) Use the ε–δ definition of the limit to show that

$$\lim_{x\to 2}(x^2-4x+5)=1$$

$f(x)=x^2-4x+5, \quad L=1;$

Then

$|f(x)-L| = |x^2-4x+5-1| = |x^2-4x+4|$

$= (x-2)^2 < \delta^2$

if $|x-2|<\delta$.

Given $\varepsilon>0$ Choose $\delta=\sqrt{\varepsilon}$; then

$|f(x)-L| < \delta^2 = \varepsilon$

if $|x-2|<\delta$.

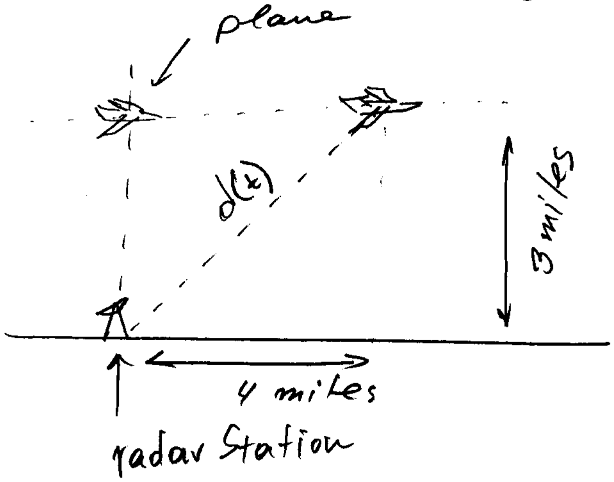
4. (8 points) For the function $f(x) = x \sin\left(2x + \frac{\pi}{2}\right)$ find the derivative $f'(x)$.
Then find $f'(\pi/4)$.

$$f'(x) = \sin\left(2x + \frac{\pi}{2}\right) + x\cos\left(2x + \frac{\pi}{2}\right) \cdot 2$$

$$f'\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{2}\right) + 2 \cdot \frac{\pi}{4} \cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= 0 + \frac{\pi}{2}(-1) = -\frac{\pi}{2}.$$

5. (8 points) A plane flying north to south at an altitude of 3 miles and a speed of 400 miles/hr passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 4 miles south from the station.



$x(t)$ – distance traveled by the plane

$x(0) = 0$ – point over the radar station.

$x'(t) = 400$ [miles/hr].

$d(t) = \sqrt{3^2 + x^2(t)}$ – distance from the radar station to the plane.

Find $d'(t)$ when $x(t) = 4$ [miles].

$$d^2(t) = 3^2 + x^2(t)$$

$$2d(t)d'(t) = 2x(t)x'(t)$$

$$d' = \frac{xx'}{d}$$

when $x = 4$ $\quad d = \sqrt{3^2+4^2} = 5$ [miles]

$$\Rightarrow d' = \frac{4}{5} \cdot 400 = 320 \text{ [mph]}.$$

6. (8 points) Find the linear approximation of the function $y = \sqrt{1+x}$ valid near $x = 0$. Use this approximation to find an approximate value of $\sqrt{0.99}$.

$$f(x) = \sqrt{1+x}$$

Linear approximation

$$y = f(0) + f'(0) \cdot x$$

$$f(0) = 1; \quad f'(x) = \frac{1}{2\sqrt{1+x}}; \quad f'(0) = \frac{1}{2}$$

$$y = 1 + \frac{1}{2}x$$

For $\sqrt{0.99}$, $x = -0.01$

$$y = 1 - 0.005 = 0.995.$$

Compare with the exact value

$$\sqrt{0.99} = 0.994987 \ldots$$

7. (8 points) Find the limits

(a) $\lim_{h\to 0} \frac{(1+h)^3-1}{h}$

$$= \lim_{h\to 0} \frac{\cancel{1}+3h+3h^2+h^3-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{3h+3h^2+h^3}{h}$$

$$= \lim_{h\to 0} (3+3h+h^2)$$

$$= 3.$$

(b) $\lim_{x\to 0-} \frac{x-|x|}{x}$

if $x<0$ then $|x| = -x$, so

$$\frac{x-|x|}{x} = \frac{2x}{x} = 2$$

$$\lim_{x\to 0-} \frac{x-|x|}{x} = \lim_{x\to 0-} 2 = 2.$$