Sec. 10.1 Adding and Subtracting Polynomials

Learning Objectives:
1. Add or subtract Polynomials
2. Evaluate Polynomials at given replacement values.

1. Add Polynomials

Definitions:
Polynomial—is a finite sum of terms of the form $ax^n$, where $a$ is a real number and $n$ is a whole number.
Term—is a number or the product of a number and variables raised to powers separated by plus or minus signs.
Numerical Coefficient (coefficient)—is the numerical factor of each term.
Constant term—is the term that contains only a number.

Types of Polynomials
1. Monomial—is a polynomial with one term.
2. Binomial—is a polynomial with two terms.
3. Trinomial—is a polynomial with three terms.
4. Polynomial—is a polynomial with four or more terms.

To Add or to subtract Polynomials is to combine like terms.
Like Terms—are terms that contain exactly the same variables raised to exactly the same powers.
To Combine Like Terms—is to combine the coefficient of the like term.

Example 1. Perform the indicated operation

1. Add: $5x + 8x^2 - 3$ and $-3x^2 + 8 - 5x$

Answer:________________________________

2. $(2a^2 - ab + 6b^2) + (-3a^2 + ab - 7b^2)$

Answer:________________________________

3. Subtract $3x + 1$ from the sum of $4x - 3$ and $5x + 2$.

Answer:________________________________
4. \((5x^2y^2 + 3 - 9x^2y + y^2) - (-x^2y^2 + 7 - 8xy^2 + 2y^2)\)

Answer: __________________________

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2. Evaluate Polynomials at given replacement values

Example 2. Find the value of each polynomial when \(y = 3\).

1. \(2y^3 + y^2 - 6\)

Answer: __________________________

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2. \(\frac{4y^2}{2} - 14\)

Answer: __________________________
Sec. 10.2 Multiplications Properties of Exponents

**Learning Objectives:**
1. Use the product rule for exponents.
2. Use the power rule for exponents.
3. Use the power of a product rule for exponents.
4. Key Vocabulary: exponential expression, power, raised, product rule, same base, simplifying an exponential expression, power rules, quotient rule.

### 1. Use the product rule for exponents

**Definition:**

**Recall:**

**Exponent**—is an expression of the form: $x^n = x \cdot x \cdot \cdots \cdot x$ (n times)

**Example:**

$x^5 = x \cdot x \cdot x \cdot x \cdot x$ and $(-x)^5 = (-x) \cdot (-x) \cdot (-x) \cdot (-x) \cdot (-x)$

**Product Property for Exponents.** If $m$ and $n$ are positive integers, and $x$ are any real number, $x \neq 0$, then

$x^m \cdot x^n = x^{m+n}$

**Example 1.** Use the product rule to simplify each expression. Write the results using exponents.

1. $x^5 \cdot x^3$  
   Answer: ________________

2. $-3x^3 y^2 \cdot 5x^4 y^6$  
   Answer: ________________

3. $9ab^2 c^4 \cdot 11a^3 b \cdot 2b^2 c^5$  
   Answer: ________________

### 2. Use the power rule for exponents

**Power Property for Exponents.** If $m$ and $n$ are positive integers, and $x$ are any real number, $x \neq 0$, then

$(x^m)^n = x^{mn}$

**Example 2.** Use the power rules to simplify each expression.

1. $(x^7)^3$  
   Answer: ________________
2. \((m^4)^5 \cdot (m^3)^3\)

Answer: ______________________________

3. **Use the power of a product rule for exponents**

**Power of a Product Property for Exponents.** If \(m\) and \(n\) are positive integers, and \(x, y\) are any real number, \(x \neq 0; y \neq 0\), then

\[(xy)^m = \]

**Example 3.** Use the power of a product rule to simplify each expression.

1. \((2x)(-2x^3y^2)^2\)

Answer: ______________________________

2. \((3a^2b^4)(2a^6b^9)^3\)

Answer: ______________________________
Sec. 10.3 Multiplying Polynomials

**Learning Objectives:**
1. Multiply a Monomial and Any Polynomial.
2. Multiply Two Binomials.
3. Square a Binomial.
4. Use the FOIL Order to Multiply Binomials.
5. Multiply Any Two Polynomials.

1. **Multiply a Monomial and Any Polynomial**

   **Distributive Property:** 
   \[ a(b + c) = ab + ac \quad \text{and} \quad -a(b + c) = -ab - ac \]

   **Example 1.** Multiply:
   
   1. \[ 4(a - 3b) \]
      
      Answer: __________________________________
   
   2. \[ -2x^3(3x^2 + 3x - 4) \]
      
      Answer: __________________________________

2. **Multiply Two Binomials**

   \[(a + b)(c + d) = a(c + d) + b(c + d) = ac + ad + bc + bd\]

   **Example 2.** Multiply.
   
   1. \[(a + 4)(a - 3) \]
      
      Answer: ________________________________
   
   2. \[(2a - 3)(4a - 1) \]
      
      Answer: ________________________________

3. **Square a Binomial**

   \[(a + b)^2 = (a + b)(a + b) = a^2 + 2ab + b^2 \quad \text{and} \quad (a - b)^2 = (a - b)(a - b) = a^2 - 2ab + b^2\]
Example 3. Multiply.

1. \((x + 3)^2\)

Answer:___________________________

2. \((6y - 1)^2\)

Answer:___________________________

4. Use the FOIL Order to Multiply Binomials

\[ (a + b)(c + d) = ac + ad + bc + bd \]

Example 4. Use FOIL order to multiply.

1. \((10x + 7)(2x + 3)\)

Answer:___________________________

2. \((3x + 2)^2\)

Answer:___________________________

5. Multiply Any Two Polynomials

Example 5. Multiply. \((2x + 5)(x^2 + 4x - 1)\)

Answer:___________________________
Sec. 10.4 Introduction to Factoring Polynomials

Learning Objectives:
1. Find the greatest common factor of a list of integers.
2. Find the greatest common factor of a list of terms.
3. Factor the greatest common factor from the terms of a polynomial.
4. Key Vocabulary: factors, factored form, factoring, factoring out, greatest common factor, GCF.

1. Find the greatest common factor of a list of integers

Greatest common Factor (GCF)—is the largest common factor of the integers in the list.

Steps to find the GCF.
1. Write each of the numbers as a product of prime number using exponent for repeated number.
2. Choose the number that has the lowest exponent, then find their product.
3. For the common variable, choose the smallest exponent

Example 1. Find the GCF of:

1. 90, 60 and 108

Answer: __________________________

2. Find the greatest common factor of a list of terms

Steps to find the GCF of a list of terms.
1. Write each of the numbers as a product of prime number using exponent for repeated number.
2. Choose the number that has the lowest exponent, then find their product.
3. For the common variable, choose the smallest exponent

Example 2. Find the GCF for each list.

1. 12x^8, 18x^5 and –30x^8

Answer: __________________________
3. **Factor the greatest common factor from the terms of a polynomial**

**Example 3.** Factor out the GCF from each polynomial.

1. \(-6x^2 - 36x^4\)
   
   Answer: _____________________

2. \(15x^7 - 20x^6 + 10x^5 + 25x^3\)
   
   Answer: _____________________

3. \(5(y - 2) - x(y - 2)\)
   
   Answer: _____________________

4. \(6a^8b^9 - 8a^3b^4 + 2a^2b^3 + 4a^5b^3\)
   
   Answer: _____________________