

Sec 6.8  The Complex Number System

**Learning Objectives:**
1. Evaluating the square root of negative real numbers.
2. Adding or subtracting complex numbers.
3. Multiplying complex numbers.
4. Complex Conjugates
5. Dividing complex numbers.
6. Evaluating the powers of $i$.

1. **Evaluating the Square Root of Negative Real Numbers**

**Definitions:**

1. **Nonnegativity Property of Real Numbers:** For any real number $a$,

2. **Imaginary Unit**, $i$ is the number whose square is $-1$. That is

3. **Complex Numbers**—are numbers of the form $a + bi$, where $a$ and $b$ are real numbers. The real number $a$ is called **real part** and the real number $b$ is called the **imaginary part** of $a + bi$.

4. **Evaluate the square root of Negative Real Numbers:** If $N$ is a positive real number, we defined the principal square root of $-N$, denoted by $\sqrt{-N}$, as

   where $i = \sqrt{-1}$

**Example 1.** Write each expression as a pure imaginary number.

1. $\sqrt{-25}$
2. $\sqrt{-48}$

**Example 2.** Write each expression as a complex number in standard form.

1. $3 - \sqrt{-16}$
2. $5 + \sqrt{-12}$
3. $\frac{15 - \sqrt{-75}}{5}$

2. **Adding or Subtracting Complex Numbers**

**Properties:**

1. **Sum of Complex Numbers:**

2. **Difference of Complex Numbers:**
Example 3. Performed the indicated operation: \((-6 + 4i) - (2 - i)\)

3. Multiplying Complex Numbers

Example 4. Multiply the following.

1. \(2i(5 - 3i)\)

2. \((5 - 2i)(-1 + 3i)\)

3. \(\sqrt{-49} \cdot \sqrt{-4}\)

4. \((3 + \sqrt{-25})(1 - \sqrt{-9})\)

4. Complex Conjugates

Definition.

Complex Conjugate: 1. If \(a + bi\) is a complex number, then its conjugate is defined by \(a - bi\).

2. If \(a - bi\) is a complex number, then its conjugate is defined by \(a + bi\).

Product of Complex Number and Its Conjugate:

\((a + bi)(a - bi) =\)
Example 5. a) Find the conjugate of the complex number, and
b) Multiply the complex number by its conjugate.

1. \(5 + 2i\)

2. \(-1 - 4i\)

5. Dividing complex numbers

Steps for dividing complex numbers:

1. Write the numerator and denominator in standard form, \(a + bi\).
2. Multiply the numerator and denominator by the complex conjugate of the denominator.
3. Simplify by writing the quotient in standard form, \(a + bi\).

Example 6. Divide.

1. \(\frac{6 + 5i}{3i}\)

2. \(\frac{2 - i}{4 + 3i}\)

6. Evaluating the powers of \(i\)

Steps for simplifying the powers of \(i\):

1. Divide the exponent of \(i\) by 4. Rewrite \(i^n\) as \((i^4)^q \cdot i^r\), where \(q\) is the quotient and \(r\) is the remainder of the division.
2. Simplify the product in Step 1 to \(i^r\) since \(i^4 = 1\)
Example 7. Simplify.

1. $i^{27}$
2. $i^{38}$
3. $i^{-43}$
4. $i^{-98}$

Example 8. (a) Find the reciprocal of the complex number. (b) Write each number in standard form.

1. $7i$
2. $-6 + 2i$

Example 9. Suppose that $f(x) = x^2 + x - 1$; find

1. $f(2i)$
2. $f(2 + i)$