Learning Objectives:
1. Solve radical equations containing one radical.
2. Solve radical equations containing two radicals.
3. Solve for a variable in a radical equation.
4. Application and Graph of a Radical function.

1. Solve Radical Equations Containing One Radical

Rules:
1. If \( \sqrt{x} = a \) then
2. If \( x = a \) then

Steps for Solving Radical Equation Containing One Radical.
1. Isolate the radical on one side of the equation and everything else on the other side.
2. Raise both sides of the equation to the power of the index to eliminate the radical from the equation.
3. Solve the result equation from step 2.
4. Check your answer for extraneous solutions by substituting the result in step 3 into the original equation.

Example 1. Solve each equation and simplify completely.

1. \( \sqrt{4x + 1} - 2 = 3 \)

2. \( \sqrt{6p - 5} = -5 \)

3. \( \frac{1}{3}\sqrt{7m + 20} = 5 \)
2. **Solve Radical Equations Containing Two Radical**

**Steps for Solving Radical Equation Containing Two Radical.**

1. Isolate one of the radical on one side of the equation and the other radical and everything else on the other side.
2. Raise both sides of the equation to the power of the index to eliminate one of the radical or both radicals from the equation.
3. If a radical remains in the equation, then follow the steps for solving a radical equation containing one radical. Otherwise, solve the equation that results.
4. Check your answer for **extraneous solutions** by substituting the result in step 3 into the original equation.

**Example 2.** Solve each equation and simplify completely.

1. \(\sqrt{2x-1} - \sqrt{x-1} = 1\)
2. \((2x + 5)^{\frac{1}{2}} - (x - 2)^{\frac{1}{2}} = 3\)

3. **Solve for a variable in a radical equation**

**Example 3.** Solve for a variable.

1. Volume of a Sphere: Solve \(r = \sqrt[3]{\frac{3V}{4\pi}}\) for \(V\).

2. Coulomb’s Law: Solve \(r = \sqrt{\frac{4F\pi e_0}{q_1q_2}}\) for \(F\).

4. **Application and Graph of a Radical function**

**Example 4.** Suppose that \(g(x) = \sqrt{x + 3}\).

a. Solve \(g(x) = 0\). What point is on the graph of \(g\)?
b. Solve $g(x) = 1$. What point is on the graph of $g$?

c. Solve $g(x) = 2$. What point is on the graph of $g$?

d. Use the information obtained in problems 1-3 to graph $g(x) = \sqrt{x + 3}$.

e. Use the graph and the concept of the range of a function to explain why the equation $g(x) = -1$ has no solution.

Example 5. Birth Rates: A plural birth is a live birth to twins, triplets, and so forth. The function $R(t) = 26 \cdot 10^t$ models the plural birth rate $R$ (live births per 1000 live births), where $t$ is the number of years since 1995. Use the model to predict the year in which the plural birth rate will be 39.