

14.1 Simplifying Rational Expressions

Learning Objectives:

1. Find the value of a rational expression given a replacement number.
2. Identify values for which a rational expression is undefined.
3. Simplify or write rational expressions in lowest terms.
4. Write equivalent forms of rational expressions.
5. Key Vocabulary: *rational expressions, simplifying rational expressions.*

Rational Expression—is an expression of the form $\frac{P}{Q}$; P and Q are any polynomials; $Q \neq 0$

A. Evaluating Rational Expressions

Standard Form of a Fraction: For $b \neq 0$

$$1. \quad -\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b} = -\frac{-a}{-b}$$

$$2. \quad \frac{-a}{-b} = -\frac{-a}{b} = -\frac{a}{-b} = \frac{a}{b}$$

Example 1. Find the value of $\frac{x^2 + 5x - 2}{x^2 - x - 2}$; $x = -3$

B. Identifying When a Rational Expression is Undefined

A rational expression is **undefined** when the **denominator is 0**.

Example 2. Find values for which the rational expression is undefined.

$$1. \quad \frac{m}{2m + 3}$$

$$2. \quad \frac{y + 1}{y^2 - 4y - 5}$$

$$3. \quad \frac{n + 3}{n^2 + 9}$$

C. Simplifying Rational Expressions

1. **Fundamental Rule of Fractions:** $\frac{AC}{BC} = \frac{A}{B}; \quad B \neq 0, C \neq 0$

2. **Quotient of Additive Inverses ("-1 trick")**

a. $\frac{a+b}{b+a} = \frac{b+a}{a+b} = 1$

b. $\frac{a-b}{b-a} = \frac{b-a}{a-b} = -1$

Step to Simplify a Rational Expression

1. Completely factor the numerator and denominator.
2. Cancel the common factor in the numerator and denominator by replace the quotient of the common factors by the number 1, since $\frac{a}{a} = 1$.
3. Rewrite the expression in simplified form.

Example 3. Simplify completely.

1. $\frac{-3x^2y}{6xy^4}$

2. $\frac{-6(m^2 - n^2)}{-3(m - n)}$

3. $\frac{3x - 12y}{12x - 48y}$

4. $\frac{25 - y^2}{y - 5}$

5. $\frac{y^2 + y - 12}{3 - y}$