Chapter 3: Solutions of Homework Problems
Vectors in Physics

12. **Picture the Problem:** The given vector components correspond to the vector \( \vec{r} \) as drawn at right.

   (a) Use the inverse tangent function to find the distance angle \( \theta \):
   \[
   \theta = \tan^{-1}\left(\frac{-9.5 \text{ m}}{14 \text{ m}}\right) = -34^\circ \text{ or } 34^\circ \text{ below the } +x \text{ axis}
   \]

   (b) Use the Pythagorean Theorem to determine the magnitude of \( \vec{r} \):
   \[
   r = \sqrt{r_x^2 + r_y^2} = \sqrt{(14 \text{ m})^2 + (-9.5 \text{ m})^2} = 17 \text{ m}
   \]

   (c) If both \( r_x \) and \( r_y \) are doubled, the direction will remain the same but the magnitude will double:
   \[
   \theta = \tan^{-1}\left(\frac{-9.5 \text{ m} \times 2}{14 \text{ m} \times 2}\right) = -34^\circ
   \]
   \[
   r = \sqrt{(28 \text{ m})^2 + (-19 \text{ m})^2} = 34 \text{ m}
   \]

15. **Picture the Problem:** The two vectors \( \vec{A} \) (length 50 units) and \( \vec{B} \) (length 120 units) are drawn at right.

   **Solution:**
   1. (a) Find \( B_x \):
      \[
      B_x = (120 \text{ units}) \cos 70^\circ = 41 \text{ units}
      \]
   2. Since the vector \( \vec{A} \) points entirely in the \( x \) direction, we can see that \( A_x = 50 \) units and that vector \( \vec{A} \) has the greater \( x \) component.
   3. (b) Find \( B_y \):
      \[
      B_y = (120 \text{ units}) \sin 70^\circ = 113 \text{ units}
      \]
   4. The vector \( \vec{A} \) has no \( y \) component, so it is clear that vector \( \vec{B} \) has the greater \( y \) component. However, if one takes into account that the \( y \)-component of \( B \) is negative, then it follows that it smaller than zero, and hence \( \vec{A} \) has the greater \( y \)-component.

20. The two vectors \( \vec{A} \) (length 40.0 m) and \( \vec{B} \) (length 75.0 m) are drawn at right.

   (a) A sketch (not to scale) of the vectors and their sum is shown at right.

   (b) Add the \( x \) components: \[
   C_x = A_x + B_x = (40.0 \text{ m}) \cos (-20.0^\circ) + (75.0 \text{ m}) \cos (50.0^\circ) = 85.8 \text{ m}
   \]

   Add the \( y \) components: \[
   C_y = A_y + B_y = (40.0 \text{ m}) \sin (-20.0^\circ) + (75.0 \text{ m}) \sin (50.0^\circ) = 43.8 \text{ m}
   \]

   Find the magnitude of \( \vec{C} \): \[
   C = \sqrt{C_x^2 + C_y^2} = \sqrt{(85.8 \text{ m})^2 + (43.8 \text{ m})^2} = 96.3 \text{ m}
   \]

   Find the direction of \( \vec{C} \): \[
   \theta_C = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{43.8 \text{ m}}{85.8 \text{ m}}\right) = 27.0^\circ
   \]
24. The vectors involved in the problem are depicted at right.

Set the length of $\mathbf{A} + \mathbf{B}$ equal to 37 units:

$$37 = \sqrt{A^2 + B^2}$$

$$37^2 = A^2 + B^2$$

Solve for $B$:

$$B = \sqrt{37^2 - A^2} = \sqrt{37^2 - (22)^2} = 30 \text{ units}$$

29. The vector $\mathbf{A}$ has a length of 6.1 m and points in the negative $x$ direction.

Note that in order to multiply a vector by a scalar, you need only multiply each component of the vector by the same scalar.

(a) Multiply each component of $\mathbf{A}$ by $-3.7$:

$$\mathbf{A} = (-6.1 \text{ m})\hat{x}$$

$$-3.7\mathbf{A} = \left[(-3.7)(-6.1 \text{ m})\right]\hat{x} = (23 \text{ m})\hat{x} \text{ so } A_x = 23 \text{ m}$$

(b) Since $\mathbf{A}$ has only one component, its magnitude is simply $23 \text{ m}$.

Picture the Problem: The vectors involved in the problem are depicted at right.

(a) Find the direction of $\mathbf{A}$ from its components:

$$\theta_A = \tan^{-1}\left(\frac{-2.0 \text{ m}}{5.0 \text{ m}}\right) = -22^\circ$$

Find the magnitude of $\mathbf{A}$:

$$A = \sqrt{(5.0 \text{ m})^2 + (-2.0 \text{ m})^2} = 5.4 \text{ m}$$

(b) Find the direction of $\mathbf{B}$ from its components:

$$\theta_B = \tan^{-1}\left(\frac{5.0 \text{ m}}{-2.0 \text{ m}}\right) = -68^\circ + 180^\circ = 110^\circ$$

Find the magnitude of $\mathbf{B}$:

$$B = \sqrt{(-2.0 \text{ m})^2 + (5.0 \text{ m})^2} = 5.4 \text{ m}$$

(c) Find the components of $\mathbf{A} + \mathbf{B}$:

$$\mathbf{A} + \mathbf{B} = (5.0 - 2.0 \text{ m})\hat{x} + (-2.0 + 5.0 \text{ m})\hat{y} = (3.0 \text{ m})\hat{x} + (3.0 \text{ m})\hat{y}$$

Find the direction of $\mathbf{A} + \mathbf{B}$ from its components:

$$\theta_{A+B} = \tan^{-1}\left(\frac{3.0 \text{ m}}{3.0 \text{ m}}\right) = 45^\circ$$

Find the magnitude of $\mathbf{A} + \mathbf{B}$:

$$|\mathbf{A} + \mathbf{B}| = \sqrt{(3.0 \text{ m})^2 + (3.0 \text{ m})^2} = 4.2 \text{ m}$$