13-2 Sign Test

Many of the hypothesis tests require normal distributed populations or some tests require that population variances be equal. What if, for a given test, such requirements cannot be met? For these cases, statisticians have developed hypothesis tests that are "distribution free." Such tests are called nonparametric tests.

A **nonparametric** test is a hypothesis test that does not require any specific conditions concerning the shape of populations or the value of any population parameters.

Nonparametric tests are easier to perform (they do not require normally distributed populations).

They can be applied to categorical data (such as genders of survey responds).

They are less efficient than parametric tests. Stronger evidence is required to reject a null hypothesis.

One of the easiest nonparametric tests to perform is the **sign test**.

The **sign test** is a nonparametric test that can be used to test either a claim involving matched pairs of sample data, a claim involving nominal data with two categories, or a claim about the population median against a hypothesized value k.

Note that the **Nominal data** to be data that consist of names, labels, or categories only. The data cannot be arranged in an order scheme (such as low to high). For example the number 24, 28, 18, … on the shirts of the LA Lakers are substitutes for names. They don't count or measure anything, so they are categorical data.

To use the sign test, first we convert data values to plus and minus signs. Then we test for disproportionately more of either sign.

Claims involving matched pairs

In section 9-4, we learned how to use a t-test for the difference between means of dependent samples. That test required both populations to be normally distributed. If the condition of normality cannot be satisfied, we can use the paired-sample sign test to test the difference between two population medians, the following conditions must be met.

1. A sample must be randomly selected from each population.
2. The samples must be dependent (paired).

We find the difference between corresponding data entries by subtracting the entry representing the second variable from the entry representing the first variable, and record the sign of the difference. Then compare the number of + and – signs. (the 0s are ignored.) If the number of + signs is approximately equal to the number of – signs, the null hypothesis should not be rejected. If, however, there is a significant difference between the number of + signs and the number of – signs, the null hypothesis should be rejected.

Guidelines for performing a paired-sample sign test

1. State the claim. Identify the null and alternative hypotheses.
   H_0: There is no difference. (The median of the differences is not equal to 0.)
   H_1: there is a difference. (The median of the differences is not equal to 0.)
2. Specify the level of significance.
3. Determine the sample size n by finding the difference for each data pair. Assign a + sign for a positive difference, a – sign for a negative difference, and a 0 for no difference.
4. Determine the critical value. Use table A-4.
5. Find the test statistic. x = smaller number of + and – signs.
6. Make a decision to reject or fail to reject he null hypothesis. If the test statistic is less than or equal to the critical value, reject null hypothesis. Otherwise, fail to reject the null hypothesis.
7. Interpret the decision in the context of the original claim.

Example 1)

A psychologist claims that the number of repeat offenders will decrease if first-time offenders complete a particular rehabilitation course. You randomly select 10 prisons and record the number of repeat offenders during a two-year period. Then, after first-time offenders complete the course, you record the number of repeat offenders at each prison for another two-year period. The results are shown in the following table. At 0.05 significance level, can you support the psychologist's claim?

Prison	1	2	3	4	5	6	7	8	9	10
Before	21	34	9	45	30	54	37	36	33	40
After	19	22	16	31	21	30	22	18	17	21

Claims involving Nominal Data with Two Categories:

The nature of nominal data limits the calculations that are possible, but we can identify the proportion of the sample data that belong to a particular category, and we can test claims about the corresponding population proportion *p*.

Guidelines:

1. State the claim. Identify the null and alternative hypotheses.

   Left-tailed test:

   H_0: p = k and H_1: p <k

Right-tailed test:

H_0: p = k and H_1: p >k

   Two-tailed test:

   H_0: p = k and H_1: $p \neq$k

2. Specify the level of significance.
3. Determine the sample size n by assigning + signs and – signs to the sample data. n = total number of + and – signs. Discard any zeros.
4. Determine the critical value: if $n \leq 25$, use table A-7.
   If n > 25, use table A-2.

5. Calculate the test statistic.
   If $n \leq 25$, use x.

   Where x is the smaller number of + and – signs and n is the sample size.

   If n > 25, use $z = \frac{(x+0.5)-\left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$

6. Make a decision to reject or fail to reject the null hypothesis. If the test statistic is less than or equal to the critical value, reject null H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.

Example 2)

The Genetics and IVF Institute conducted a clinical trial of its methods for gender selection. As of the writing, 172 of 211 babies born to parents using the YSORT method were boys. Use a 0.01 significance level to test the claim that the YSORT method is effective in increasing the likelihood of boy.

Claims about the Median of a single population

To use the sign test for a population median, if the entry is below the median, assign it a – signs; if the entry is above the median, assign it a + sign; and if the entry is equal to the median, assign it a 0. Then compare the number of + and – signs. (The 0s are ignored.) If there is a large difference between the number of +

signs and the number of – signs, it is likely that the median is different from the hypothesized value and the null hypothesis should be rejected.

The sign test for a population median can be left-tailed, right tailed, or two-tailed. The null and alternative hypotheses for each type of test are as follows.

Left-tailed test:

H_0: median = k and H_1: median <k

Right-tailed test:

H_0: median = k and H_1: median >k

Two-tailed test:

H_0: median = k and H_1: median $\neq$k

Guidelines:

1.State the claim. Identify the null and alternative hypotheses.

2.Specify the level of significance.

3.Determine the sample size n by assigning + signs and – signs to the sample data. n = total number of + and – signs.

4.Determine the critical value: if n$\leq$ 25, use table A-7.

If n > 25, use table A-2.

5.Calculate the test statistic.

If n$\leq$ 25, use x.

Where x is the smaller number of + and – signs and n is the sample size.

If n > 25, use $z = \frac{(x+0.5)-\left(\frac{n}{2}\right)}{\frac{\sqrt{n}}{2}}$

6. Make a decision to reject or fail to reject the null hypothesis. If the test statistic is less than or equal to the critical value, reject null H_0. Otherwise, fail to reject H_0.
7. Interpret the decision in the context of the original claim.

Example 3)

A bank manager claims that the median number of customer per day is no more than 750. A teller doubts the accuracy of this claim. The number of bank customers per day for 16 randomly selected days are listed below. At 0.05 significance level, can the teller reject the bank manager's claim?

775	765	801	742	754	753
739	751	745	750	777	769
756	760	782	789		

Example 4)

A car dealership claims to give customers a median trade-in offer of at least \$6000. A random sample of 103 transactions revealed that the trade-in offer for 60 automobiles was less than \$6000 and the trade-in offer for 40 automobiles was more than \$6000. At 0.01 significance level, can you reject the dealership's claim?