Lecture#4 Chapter 4: **Probability**

In this chapter, we use sample data to make conclusions about the population. Many of these conclusions are based on probabilities of the events.

4-2 Fundamentals

Definitions: **Probability** is a measure of the chances of the possible outcomes for a random phenomenon. The result of a single trial in a probability experiment is an **outcome**. An **even**t is any collection of results of outcomes of a procedure. A **simple event** is an outcome or an event that cannot be further broken down into simpler components. The **sample space** for a procedure consists of all possible simple events.

Example 1: Rolling one die: sample space={1,2,3,4,5,6}

Rolling two dice: sample space: {1-1, 1-2,1-3,…6-5,6-6} 36 simple events.
Tossing a coin and then rolling a die:

Example2: What is the sample space for the correctness of a student's answers on the three multiple-choice questions?

This technique for listing the outcomes in a sample space is called the three diagrams. Note that the number of braches doubles at each stage.

Rounding off rule for probabilities: Round off final decimal results to three significant digits.

Notation for probabilities:
P denotes a probability.
A, B, and C denote specific events.
P(A) denotes the probability of event A occurring.

Types of probability:

Relative Frequency Probability is based on observations obtained from probability experiments. Based on these actual results,

P(E)=$\frac{the\ number\ \ ot\ times\ \ E\ occured}{the\ number\ \ of\ times\ trials\ \ was\ repeated\ .} = \frac{frequency\ \ of\ event\ \ E}{total\ \ frequency} = \frac{f}{n}$

As the total of observation increases, our approximation gets closer to the actual probability.

Example 3: An insurance company determined that in every 100 claims, 4 are fraudulent. What is the probability that the next claim the company processes is fraudulent?

Classical (theoretical) probability is used when each outcome in a sample space is equally likely to occur. An experiment is said to have equally likely outcome when each outcome has the same probability of occurring.

$$p(E) = \frac{\text{the number of ways } E \text{ can occur}}{\text{the number of different simple events}}$$

Example 4: You roll a die. Find the probability of the following events.

a) Event A: rolling a 3 b) Event B: rolling a 7 c) Event C: rolling a number less than 5.

Properties of probability: Every event is either impossible, certain, or somewhere in between. Therefore the probability of any event is 0, 1, or a number between 0 and 1.

Example 5: You select a card from a standard deck. Find the probability of the following events.

a) Event D: selecting a seven of diamonds

b) Event E: selecting a diamond

Law of Large Numbers: As an experiment is repeated over and over, the relative frequency probability of an event approaches the theoretical (actual) Probability of the event.

Example 6: Find the probability of having only one girl among three children.
Example 7: Find the probability of getting a total of 3 when a pair of dice is rolled.

Subjective Probabilities: Guess or estimate of a probability based on knowledge of relevant circumstances.

Example 8: A study of 400 randomly selected American airlines flights showed that 344 arrived on time. What is the estimated probability of an American airlines flight arriving late?

If the probability is a simple fraction such as 1/3, or 4/9, express it as a fraction; otherwise express it as a decimal.

Complementary Events: $\bar{A}$ =complement of an event A: all outcomes in which the original event, A, does not occur. $P(A) + P(\bar{A}) = 1$

Example 9: a) If P(A)=0.05, find P($\bar{A}$). b) Based on data from the U. S. National center for Health statistics, the probability of a baby being a boy is 0.513. Find the probability of a baby being a girl.

Odds: Actual like hood of some event can be described as the following:

a) Actual odds against b) Actual odds in favor

The actual odds against event A are the ratio of $\frac{P(\bar{A}}{P(A)}$. Usually expressed in the form a : b or "a to b". This ratio should be in reduced form.

The actual odd in favor of event A are the reciprocal of the actual odds against that event.

Payoff odds against event A describes the ratio of net profit (if you win) to the amount bet.

Payoff odds = $\frac{net\ profit}{amount\ \ bet}$, Or net profit= (payoff odds). (amount bet)

Example 10: If you bet $10 on the number 17 in roulette, your probability of winning is 1/38 and the payoff odds are given by the casino as 35:1. a) Find the actual odds against the outcome 17. b) How much net profit would you make if you win by getting 17? c) Let B be an event that an odd number occurs. What is the probability of winning? What are the actual odds against winning?

4-3 Addition Rule

In this section we like to find out what is the probability that either event A occurs or event B occurs or they both occur as the single outcome of a procedure.

P(A or B) = P(event A occurs or event B occurs or they both occur)=P(A)+P(B)-P(A and B) where P(A and B) denotes the probability that A and B both occur at the same time as an outcome in a trial of a procedure. Or $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Event A and B are **mutually exclusive** (disjoint) if they cannot occur at the same time. That is, they have no outcomes in common. Then, $P(A \cup B) = P(A) + P(B)$

Example 11: You select a card from a standard deck. Find the probability that the card is a 4 or an ace.

Example 12: You roll a die. Find the probability of rolling a number less than three or rolling an odd number.

Example 13: a college has an undergraduate enrollment of 3500. Of these, 860 are business majors and 1800 are women. Of the business majors, 425 are women. If a college newspaper conducts a poll and selects students at random to answer a survey, find the probability that a selected student is a woman or a business major.

4-4 Multiplication Rule: Basics

In this section we like to come up with a rule for P(A and B). The probability that A occurs in a 1^{st} trial and event B occurs in a 2^{nd} trial.

P(A and B)= P(event A occurs in a 1^{st} trial and event B occurs in a 2^{nd} trial).

Def: Two events A and B are i**ndependent** if the occurrence of event A in a probability experiment does not affect the probability of event B. Two events are **dependen**t if the occurrence of event A in a prob. Experiment affects the prob. of event B. Suppose you flip a coin twice, the outcome from the 1st flip is indep. of the outcome from the 2nd flip.

Conditional Probability: $P(A|B)$= P (event A occurring after it is assumed that event B has already occurred).

$P(A|B)$= P(A) and P(B|A)=P(B), if A and B are two independent events.

Example 14: Suppose that a single die is rolled. a) What is the prob. that the die comes up 3? b) Now suppose the die is rolled a second time, but we are told the outcome will be an odd number. What is the prob. that the die comes up 3?

Multiplication Rule: P(A and B)= P(A). P(B|A)

If events A and B are independent, then the rule can be simplified to P(A and B)=P(A). P(B)

Example 15: Two cards are selected, without replacement, from a standard deck. a) Find the probability of selecting a king and the selecting a queen. b) Find the probability of selecting a king and then selecting another king. c) A pool of potential jurors consists of 10 men and 15 women. If two different people are randomly selected from this pool, find the probability that they are both women.

Example 16: The prob. that a driver who is speeding gets pulled over is 0.8. The prob. that a driver gets a ticket given he/she is pulled over is 0.9. What is the prob. that a randomly selected driver who is speeding gets pulled over and gets a ticket?

Treating Dependent events as independent:

If small random samples are taken from large populations without replacement, it is reasonable to assume independence of the events. As a rule of thumb, if the sample size is less than 5% of the population size, we treat the events as independent.

4-5 Multiplication Rule: complements and conditional Probability

Complements: The probability of "at least one"

To find the probability of at least of something, calculate the probability of none, then subtract that result from 1.

Example 17: Find the probability of a couple having at least 1 girl among 3 children.

Example 18: A quick quiz consists of three multiple-choice questions, each with five possible answers, only one of which is correct. If you make random guesses for each answer, what is the probability that all three of your answers are wrong?

Conditional probability $\boldsymbol{P(A|B)} = \frac{\boldsymbol{P(Band\ A)}}{\boldsymbol{P(A)}}$

Example 19: Use the following data from the 100 Senators from the 108th Congress of the United States.

	Republican	Democrat	Independent
Male	46	39	1
Female	5	9	0

a) If we randomly select one senator, what is the probability of getting a Republican, given that a male was selected?
b) If we randomly select one Senator, what is the probability of getting a female, given that an Independent was selected?

Common mistake: Note that P(A|B) is not as the same as P(B|A)

4-7 Counting

In this section we want to find numbers of different possible outcomes without listing them.

Fundamental counting Rule: If a task consists of a sequence of choices in which there are m selections for the 1st choice, n selections for the 2nd choice, r selections for the 3rd choice, and so on, then the take of making these selections can be done in m.n.r… different ways.

Example 20: The international Airline Transportation Association assigns 3-letter codes to represent airport locations. How many different airport codes are possible?

Example 21: Three members from a 14-member Committee are to be randomly selected to serve as a chair, vice-chair, and secretary. The 1st person selected is the chair, the 2nd person selected, the vice-chair, and the 3rd, the secretary. How many different committee structures are possible?

Notation: the **factorial** symbol ! 0!=1 1!=1 n!=n(n-1)(n-2)…(3)(2)(1)

6! =? $\frac{6!}{3!.3!}$

A **permutation** is an ordered arrangement in which r objects are chosen from n distinct (different) objects and repetition is not allowed. Read as the number of permutations of r objects selected from n objects. Note that, there are n different items available, we select r of n items (without replacement), the order is important. ABC is different from ACB.

Example 22: In how many ways can horses in a 10-horse race finish 1st, 2nd, and 3rd?

Permutation Rule (when some items are identical to others) Sometimes we wish to arrange objects in order, but some of objects are identical to others. The number of permutation of all items selected without replacement is

Example 23: How many different vertical arrangements are thereof 10 flags if 5 are white, 3 are blue, and 2 are red?

Permutation of distinct items with replacement : The selection of r objects from a set of n different objects when the order in which the objects are selected matters, and an object may be selected more than once (repetition allowed) n^r

See example 20

Combination: A combination is a collection, without regard to order, of n distinct objects without repetition. Note that the n objects are different. Once an object is used, it cannot be repeated. Order is not important.

Example 24: How many different simple random samples of size 4 can be obtained from a population whose size is 20?

Example 25: Pg 188, #32