College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 501A
Seminar in Engineering Analysis

Fall 2017 Number: 15966 Instructor: Larry Caretto

December 6 Homework Problems

1. Use the Euler method, $\mathbf{y}_{n+1} = \mathbf{y}_n + hf(x_n, \mathbf{y}_n)$, for a system of equations to solve $y_1' = 2y_1 - 4y_2$, $y_2' = y_1 - 3y_2$, $y_1(0) = 3$, $y_2(0) = 0$. Solve for ten steps with h = 0.1 and plot the solution in the y_1-y_2 plane.
2. Use the fourth-order Runge-Kutta method for the system of equations solved in problem 1: $y_1' = 2y_1 - 4y_2$, $y_2' = y_1 - 3y_2$, $y_1(0) = 3$, $y_2(0) = 0$. Solve for two steps with h = 0.5 and compare the results to those in problem 1.
3. Consider the model stiff ODE: y' = -1000[y –(t + 2)] + 1, y(0) = 1. Solve this ODE by the explicit Euler method from t = 0 to t = 0.01 using h = 0.0005, 0.0001, 0.002, and 0.0025. Compare the solution with the exact solution: $y = -e^{-1000t} + t + 2$
4. Solve y' = -1000[y – (t + 2)] + 1, y(0) = 1 by the second-order Gear method from t = 0 to t = 0.1 using h = 0.01, and 0.02. The equation for the second-order Gear method is

$$y_{n+1} = \frac{2}{3}hf_{n+1} + \frac{4}{3}y_n - \frac{1}{3}y_{n-1}$$

   Note that for this textbook problem you can write an equation for f_{n+1} and substitute it into the Gear algorithm to obtain an equation that is linear in y_{n+1}. You can solve that equation for y_{n+1} so that you will not have to iterate each step of the solution. Use the exact solution, $y = -e^{-1000t} + t + 2$, to get the necessary starting values.
5. Solve the following ODE y'' + (1+x)y' +(1+x)y = 1 with y(0) = 1 and y'(1) = 0, by the shooting (shoot-and-try) method using the second-order modified Euler method, given by the equations below, with h = 0.125.

$$y_{n+1}^P = y_n + hf(x_n, y_n) \qquad y_{n+1}^C = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1}^P)\right]$$