

November 29 Homework Problems

1. Solve the problem $y' = -0.2xy$ with $y(0) = 1$ for ten steps, with $h = 0.1$, using the Adams-Moulton method. Compare the errors with the exact solution. Use the Runge-Kutta method to get the starting values.
2. Solve the problem $y' = ty^2$ with $y(0) = 1$ from $t = 0$ to $t = 1$ using $h = 0.2$ and $h = 0.1$. Use the exact solution $y = -2/(t^2 - 2/y_0)$ to compute the ratio of the errors for the two step sizes at $t = 1$. Use the implicit Euler method with time linearization which is given by the following

$$\text{equation } y_{n+1} - y_n = \left[hf_n + h^2 \frac{\partial f}{\partial t} \Big|_n \right] / \left[1 - h \frac{\partial f}{\partial y} \Big|_n \right]$$

3. Solve the problem $y' = ty^2$ with $y(0) = 1$ from $t = 0$ to $t = 1$ using $h = 0.2$ and $h = 0.1$. Use the exact solution $y = -2/(t^2 - 2/y_0)$ to compute the ratio of the errors for the two step sizes at $t = 1$. Use the implicit Euler method with Newton iteration in which the implicit Euler method is written as one equation in one unknown, $g(y_{n+1}) = 0$. One then applies the usual formula for Newton iteration, $y_{n+1}^{(m+1)} = y_{n+1}^{(m)} - g(y_{n+1}^{(m)}) / g'(y_{n+1}^{(m)})$, where g' denotes dg/dy_{n+1} . This iteration is used at each new time step, $n + 1$, to advance from iteration m to iteration $m+1$, until a satisfactory convergence criterion is reached. Use a convergence criterion that $|y_{n+1}^{(m+1)} - y_{n+1}^{(m)}| \leq 10^{-5} |y_{n+1}^{(m+1)}|$ for this problem.