November 22 Homework Problem

Consider the following differential equation: $\frac{dy}{dx} = \frac{2\sqrt{y - \ln x}}{x} + \frac{1}{x}$ **with the initial condition that y = 0 at x = 1.**

Verify that the exact solution is y = (ln x)^2 + ln x.

Compute the numerical solution of this equation from the initial condition to a final point at x = 1.8. Find the difference between the numerical and exact solution at the final point. Do the numerical solutions and find the error at the final point (x = 1.8) for each of the following method/step-size combinations.

- **Euler's method with h = 0.1**
- **Heun's method with h = 0.2**
- **Fourth-order Runge-Kutta method with h = 0.4**
- **Fourth-order Runge-Kutta method with h = 0.1**