College of Engineering and Computer Science
Mechanical Engineering Department

*Mechanical Engineering 501A*
*Seminar in Engineering Analysis*

Fall 2017 Number: 15966 Instructor: Larry Caretto

# November 8 Homework Solutions

**1. Find the Laplace transform of a plot which is a straight line from (0,1) to (1,0.5). Show the details of your work.**

The line has a slope of -0.5 and an intercept of 1 so its equation is f(t) = -t/2 + 1. If we plug this function into the definition of the Laplace transform we obtain the following (after using an integral table for the integral of $te^{-st}dt$.)

$$F(s)=\int_0^\infty f(t)e^{-st}dt=\int_0^1\left(1-\frac{t}{2}\right)e^{-st}dt+\int_1^\infty (0)e^{-st}dt=\int_0^1 e^{-st}dt-\frac{1}{2}\int_0^1 te^{-st}dt$$
$$=-\frac{e^{-st}}{s}\bigg|_0^1-\frac{1}{2}\left[\frac{e^{-st}}{(-s)^2}(-st-1)\right]_0^1=\frac{1}{s}-\frac{e^{-s}}{s}-\frac{1}{2}\left[\frac{e^{-s}}{(-s)^2}(-s-1)-\frac{1}{(-s)^2}(0-1)\right]$$

Rearranging to collect common terms gives the following result.

$$F(s)=\frac{1}{s}-\frac{e^{-s}}{s}+\frac{e^{-s}}{2s}+\frac{e^{-s}}{2s^2}-\frac{1}{2s^2}=\frac{1}{s}-\frac{e^{-s}}{2s}+\frac{e^{-s}}{2s^2}-\frac{1}{2s^2}$$

To check this, we can get the inverse transform. Using a transform table such as tables 6.8 and 6.9 in Kreyszig (p 248-249 in tenth edition). The inverse transforms of 1/s and $1/2s^2$ are 1, and t/2, respectively. For the terms with $e^{-s}$, we use the second shifting theorem that $\mathcal{L}$[f(t – a)u(t – a)] = $e^{-as}$F(s). Applying this result, we see that the inverse transform of $e^{-s}/2s$ is f(t – 1)u(t – 1)/2 where f(t) is the inverse transform, f(t) = $\mathcal{L}^{-1}$[1/s] = 1. Since t does not appear in this inverse transform we leave f(t – 1) = $\mathcal{L}^{-1}$[1/s] = 1. Similarly, the inverse transform of $e^{-s}/2s^2$ = is f(t – 1)u(t – 1)/2 where f(t) is the inverse transform, f(t) = $\mathcal{L}^{-1}[1/s^2]$ = t. In this case f(t – 1) = t – 1. We use these four transforms on our solution for F(s) to obtain the following result.

$$\mathcal{L}^{-1}\left(\frac{1}{s}-\frac{e^{-s}}{2s}+\frac{e^{-s}}{2s^2}-\frac{1}{2s^2}\right)=1-\frac{u(t-1)}{2}+\frac{(t-1)u(t-1)}{2}-\frac{t}{2}=1-\frac{t}{2}-\left(1-\frac{t}{2}\right)u(t-1)$$

We can rearrange this final term as follows.

$$\mathcal{L}^{-1}\left(\frac{1}{s}-\frac{e^{-s}}{2s}+\frac{e^{-s}}{2s^2}-\frac{1}{2s^2}\right)=-\left(\frac{t}{2}-1\right)+\left(\frac{t}{2}-1\right)u(t-1)$$

We see that the first term, –(t/2 – 1) is just our original line from (0,1) to (1, 0.5) and the second term is the same equation of a line, but with a positive sign, multiplied by the unit step function at t = 1. Thus the net result of the inverse transform is the same as the f(t) we used by splitting the Laplace transform integral at t = 1 with the equation of the line for t <=1 and 0 for t >= 1.

**2. Given F(s) = $\mathcal{L}$ [f(t)] = (–s – 10)/($s^2$ – s – 2), find f(t). Show the details.**

We can factor the denominator of F(s) and rearrange it into two fractions using the two components in the numerator to obtain transforms that are in the transform table.

$$F(s) = \frac{-s-10}{s^2 - s - 2} = \frac{-s}{(s-2)(s+1)} - 10\frac{1}{(s-2)(s+1)}$$

The fractions on the right-hand side are found as transforms 11 and 12 in Table 6.9 of Kreyszig (on page 249 of the 10th edition.) Using these transforms, with a = 2 and b = -1 gives the following result for f(t),

$$f(t) = -\frac{1}{2-(-1)}\left(2e^{2t} - (-1)e^{-t}\right) - 10\frac{1}{2-(-1)}\left(e^{2t} - e^{-t}\right) = \frac{-12e^{2t} + 9e^{-t}}{3} = 3e^{-t} - 4e^{2t}$$

**3. Find the Laplace transform of $5e^{2t}$sinh 2t. (Show the details.)**

Here we can use the first shifting theorem which lets us write the Laplace transform of $e^{at}$ f(t) in terms of the Laplace transform, F(s), of f(t). This theorem tells us that $\mathcal{L}[e^{at}f(t)] = F(s - a)$. Using this theorem, we take the original Laplace transform of f(t) and replace s by s – a everywhere in that transform to get the Laplace transform of $e^{at}f(t)$. Since the Laplace transform of sinh at is $a/(s^2 - a^2)$, the Laplace transform of $5e^{2t}$sinh 2t is $5\frac{2}{(s-2)^2 - 2^2} = \frac{10}{s(s-4)}$

**4. If $\mathcal{L}$[f(t)] = F(s) and c is any positive constant, show that $\mathcal{L}$[f(ct)} = F(s/c)/c. Use this result to obtain $\mathcal{L}$(cos ωt) from $\mathcal{L}$(cos t).**

We can start with the definition of the Laplace transform and use this definition to write the transform of f(ct).

$$F(s) = \Im[f(t)] = \int_0^\infty f(t)e^{-st}dt \quad \Rightarrow \quad \Im[f(ct)] = \int_0^\infty f(ct)e^{-st}dt$$

We can integrate this equation by the substitution y = ct so that t = y/c and dt = dy/c. Because c is a positive constant the limits of t = 0 and t = ∞ are equivalent to y = 0 and y = ∞. Making these substitutions in the integral and defining a new parameter k = s/c gives.

$$\Im[f(ct)] = \int_0^\infty f(ct)e^{-st}dt = \int_0^\infty f(y)e^{-sy/c}d(y/c) = \frac{1}{c}\int_0^\infty f(y)e^{-ky}dy$$

The final integral in this step is simply the definition of the Laplace transform except the dummy variable of integration is changed from t to y and the parameter s is replaced by the parameter k = s/c. Thus, this integral gives the Laplace transform F(k) = F(s/c). Since the integral is divided by c, we have the desired result that $\mathcal{L}$[f(ct)] = F(s/c)/c.

To apply this result to obtain $\mathcal{L}$(cos wt) from $\mathcal{L}$(cos t), we start with the transform in the table for cos ωt and set ω = 1. As we see below, this gives the correct result for $\mathcal{L}$(cos ωt).

$$\Im(\cos t) = F(s) = \frac{s}{s^2+1} \quad \Rightarrow \quad \Im(\cos \omega t) = \frac{1}{\omega}F\left(\frac{s}{\omega}\right) = \frac{1}{\omega}\frac{\frac{s}{\omega}}{\left(\frac{s}{\omega}\right)^2 + 1} = \frac{s}{s^2 + \omega^2}$$

**5. Solve the following differential equation y'' + 9y = 8 sin t if 0 < t < π and 0 if t > π. y(0) = 0, y'(0) = 4. Use Laplace transforms. (Show the details.)**

Here we can use the Heaviside function (the unit step function), u(t – a) to solve one equation that includes the discontinuity on the right-hand side.

$$\frac{d^2y}{dt^2} + 9y = [1 - u(t-\pi)]8\sin t$$

Taking Laplace transforms of this differential equation gives

$$s^2Y(s) - sy(0) - y'(0) + 9Y(s) = 8\Im[\sin t - u(t-\pi)\sin t]$$

It looks like we can use the second shifting theorem for the product of the sine and the unit step function, but we can only do this if the argument of the sine function is the same as the argument of the unit step function, t – π. We can use the periodicity of the sine to write sin t as –sin(t – π). Doing this, using the second shifting theorem to get the Laplace transform of the right side and substituting the initial conditions, y(0) = 0, y'(0) = 4, to get an following equation for Y(s).

$$s^2Y(s) - 4 + 9Y(s) = \frac{8(1)}{s^2+1^2} + 8\Im[u(t-\pi)\sin(t-\pi)] = \frac{8(1)}{s^2+1^2} + \frac{8(1)e^{-\pi s}}{s^2+1^2}$$

$$Y(s) = \frac{1}{s^2+9}\left[4 + \frac{8}{s^2+1} + \frac{8e^{-\pi s}}{s^2+1}\right] = \frac{4}{s^2+9} + \frac{1}{s^2+9}\frac{8}{s^2+1} + \frac{1}{s^2+9}\frac{8e^{-\pi s}}{s^2+1}$$

To get the inverse, we first notice that the term $4/(s^2+9)$ has the form of the Laplace transform of the sine, $\omega/(s^2+\omega^2)$ with ω = 3. So the inverse of the first term is (4/3) sin 3t. The second term can be treated by the method of partial fractions. Here we have repeated complex factors so we write the partial fractions as follows.

$$\frac{1}{s^2+9}\frac{8}{s^2+1} = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+1}$$

Multiplying this equation by $(s^2 + 9)(s^2 + 1)$ gives.

$$8 = (As+B)(s^2+1) + (Cs+D)(s^2+9)$$
$$As^3 + Bs^2 + As + B + Cs^3 + Ds^2 + 9Cs + 9D$$

Equating coefficients of like terms in this equation gives.

$$\begin{array}{ll} s^3 & A + C = 0 \\ s^2 & B + D = 0 \\ s^1 & A + 9C = 0 \\ s^0 & B + 9D = 8 \end{array}$$

The first and third equations are satisfied only if A = C = 0. The second and fourth equations are satisfied if D = –B = 1. We then have the following result and inverse transform for the second term.

$$\Im^{-1}\left(\frac{1}{s^2+9}\frac{8}{s^2+1}\right) = \Im^{-1}\left(-\frac{1}{s^2+9} + \frac{1}{s^2+1}\right) = -\frac{1}{3}\sin 3t + \sin t$$

We see that the third term in our equation for Y(s) is simply the second term multiplied by the same term whose inverse we just found above. We can obtain the inverse of such a term from the second shifting theorem, shown below, where F(s) is the Laplace transform of f(t).

$$\mathfrak{I}^{-1}[e^{-as}F(s)] = u(t-a)\,f(t-a)$$

Applying this theorem to the final term in the Y(s) equation gives

$$\mathfrak{I}^{-1}\left(\frac{1}{s^2+9}\frac{8e^{-\pi s}}{s^2+1}\right) = u(t-\pi)\left[-\frac{1}{3}\sin 3(t-\pi) + \sin(t-\pi)\right]$$

Combining the three inverse transforms for the terms in the Y(s) equation gives the following result for y(t).

$$y(t) = \frac{4}{3}\sin 3t - \frac{1}{3}\sin 3t + \sin t + u(t-\pi)\left[-\frac{1}{3}\sin 3(t-\pi) + \sin(t-\pi)\right]$$

For $t < \pi$, the term multiplied by $u(t - \pi)$ is zero and we can combine the first two terms in the solution, giving.

$$y(t) = \sin 3t + \sin t \qquad 0 < t < \pi$$

For $t > \pi$, $u(t - \pi) = 1$, and the solution becomes

$$y(t) = \sin 3t + \sin t - \frac{1}{3}\sin 3(t-\pi) + \sin(t-\pi) \qquad t > \pi$$

From the periodicity of the sine function, $\sin(t - \pi) = -\sin t$, and $\sin 3(t - \pi) = \sin(3t - 3\pi) = -\sin 3t$. Making these substitutions gives

$$y(t) = \sin 3t + \sin t + \frac{1}{3}\sin 3t - \sin t = \frac{4}{3}\sin 3t \qquad t > \pi$$

**6. Solve the differential equation $y'' + \omega_0^2 y = K \sin pt$ with $y(0) = y'(0) = 0$ and $p^2 \neq \omega_0^2$. Find the solution by Laplace transforms. (Show the details of your work.)**

Using the transform table for the second derivative and the sine gives the following transformation of our differential equation.

$$s^2Y(s) - sy(0) - y'(0) + \omega_0^2 Y(s) = \frac{Kp}{s^2+p^2}$$

Setting the initial conditions $y(0) = 0$ and $y'(0) = 0$ and solving for Y(s) gives the following.

$$\left(s^2+\omega_0^2\right)Y(s) = \frac{Kp}{s^2+p^2} \quad \Rightarrow \quad Y(s) = \frac{Kp}{\left(s^2+\omega_0^2\right)\left(s^2+p^2\right)}$$

We can use the method of partial fractions to find the transforms for Y(s). Since we have repeated complex factors we use the following method for partial fractions.

$$\frac{Kp}{\left(s^2+\omega_0^2\right)\left(s^2+p^2\right)} = \frac{A+Bs}{\left(s^2+p^2\right)} + \frac{C+Ds}{\left(s^2+\omega_0^2\right)}$$

Multiplying both sides of the equation by the denominator on the left side and rearranging terms to group common powers of s gives.

$$Kp = (A+Bs)\left(s^2+\omega_0^2\right) + (C+Ds)\left(s^2+p^2\right)$$
$$Kp = s^3(B+D) + s^2(A+C) + s\left(B+Dp^2\right) + A\omega_0^2 + Cp^2$$

Equating coefficients of like powers of s on both sides of the equation gives the following set of four equations that can be solved for A, B, C, and D.

$$B + D = 0 \quad A + C = 0 \quad B\omega_0^2 + Dp^2 = 0 \quad A\omega_0^2 + Cp^2 = Kp$$

$$B = D = 0 \qquad C = \frac{Kp}{p^2 - \omega_0^2} \qquad A = -\frac{Kp}{p^2 - \omega_0^2}$$

$$Y(s) = \frac{Kp}{\left(s^2 + \omega_0^2\right)\left(s^2 + p^2\right)} = -\frac{Kp}{p^2 - \omega_0^2}\frac{1}{\left(s^2 + p^2\right)} + \frac{Kp}{p^2 - \omega_0^2}\frac{1}{\left(s^2 + \omega_0^2\right)}$$

The expressions on the right are almost the sine transformations. However, the sine transformation requires the frequency in the numerator. We can obtain this for the $\omega_0$ term by multiplying top and bottom by the frequency to obtain.

$$Y(s) = -\frac{K}{p^2 - \omega_0^2}\frac{p}{\left(s^2 + p^2\right)} + \frac{Kp}{p^2 - \omega_0^2}\frac{1}{\omega_0}\frac{\omega_0}{\left(s^2 + \omega_0^2\right)}$$

Taking the inverse transformations gives the final solution for y(t).

$$y(t) = -\frac{K \sin pt}{p^2 - \omega_0^2} + \frac{\frac{Kp}{\omega_0}\sin \omega_0 t}{p^2 - \omega_0^2}$$

We can show that this solution matches the initial conditions and satisfies the differential equation. Simply substituting t = 0 into the solution for y(t) shows that we match the initial condition that y(0) = 0. The first two derivatives of the solution are.

$$y'(t) = -\frac{Kp \cos pt}{p^2 - \omega_0^2} + \frac{Kp \cos \omega_0 t}{p^2 - \omega_0^2} \qquad y''(t) = \frac{p^2 K \sin pt}{p^2 - \omega_0^2} - \frac{\omega_0 Kp \sin \omega_0 t}{p^2 - \omega_0^2}$$

Setting t = 0 shows that y'(0) = 0 matching the other initial condition and substituting the second derivative and the solution into the differential equation shows, after some algebra, that the differential equation is satisfied.

$$\frac{p^2 K \sin pt}{p^2 - \omega_0^2} - \frac{\omega_0 Kp \sin \omega_0 t}{p^2 - \omega_0^2} + \omega_0^2\left[-\frac{K \sin pt}{p^2 - \omega_0^2} + \frac{\frac{Kp}{\omega_0}\sin \omega_0 t}{p^2 - \omega_0^2}\right] = K \sin pt$$

**7. Solve the initial value problem y'' + 4y = u(t – 3), y(0) = 1, y'(0) = 0, using Laplace transforms. Show the details of your work.**

Taking the Laplace transforms of the differential equation gives the following result.

$$s^2 Y(s) - sy(0) - y'(0) + 4Y(s) = \frac{e^{-3s}}{s}$$

We can substitute the initial conditions, y(0) = 1 and y'(0) = 0, and solve for Y(s).

$$s^2Y(s) - (1)s - 0 + 4Y(s) = \frac{e^{-3s}}{s} \qquad \Rightarrow \qquad Y(s) = \frac{\frac{e^{-3s}}{s} + s}{s^2 + 4} = \frac{e^{-3s}}{s\left(s^2+4\right)} + \frac{s}{\left(s^2+4\right)}$$

The final term in this equation has the form of the transform for cos $\omega$t, s/(s$^2$ + $\omega^2$), with $\omega$ = 2. The term with e$^{-3s}$ can be inverted using the second shifting theorem, provided that the remainder of the term is a recognized transform. We can obtain such a transform by using partial fractions for the 1/[s(s$^2$+4)] term. We do this as shown below, using the special form for the imaginary factors in (s$^2$+4) = (s + 2i)(s – 2i).

$$\frac{1}{s\left(s^2+4\right)} = \frac{A}{s} + \frac{B + Cs}{\left(s^2+4\right)} \qquad \Rightarrow \qquad 1 = A\left(s^2+4\right) + Bs + Cs^2$$

Setting the coefficients of like powers of s on both sides of the second equation gives A + C = 0 for the s$^2$ terms, B = 0 for the s terms and 4A = 1 for the s$^0$ terms. This gives A = ¼ and C = -A = -¼, giving the following expression for Y(s).

$$Y(s) = \frac{1}{4}\frac{e^{-3s}}{s} - \frac{1}{4}\frac{se^{-3s}}{\left(s^2+4\right)} + \frac{s}{\left(s^2+4\right)}$$

The first term in this equation is simply the transform of the unit step function that we used to obtain the original transform of the differential equation. We can use the second shifting theorem to obtain the inverse of the second term. According to this theorem, the inverse of e$^{-as}$F(s), where F(s) is the known Laplace transform of some function of time, f(t), is f(t – a) u(t – a). We have already noted that inverse of s/(s$^2$ + $\omega^2$) is cos $\omega$t. Thus the inverse of e$^{-3s}$s/(s$^2$ + 2$^2$) is given by the expression u(t – 3) cos $\omega$(t – 3). We now know how to transform each term in our equation for Y(s) so that we can find the following result for y(t).

$$y(t) = \frac{1}{4}u(t-3) - \frac{1}{4}u(t-3)\cos\left[2(t-3)\right] + \cos 2t$$

Since u(t – 3) = 0 at t = 0, we see that this solution satisfies the initial conditions that y(0) = 1 and y'(0) = 0. To show that our solution satisfies the differential equation, we can write the differential equation and the solution in two regions, one for each side of the discontinuity at t = 3. For t < 3, our differential equation and solution are y'' + 4y = 0 and y = 2 cos t. We see that this solution satisfies the differential equation. For t > 3, the differential equation and solution are y'' + 4y = 1 and y = 0.25 – 0.25 cos[2(t – 3)] + cos 2t. In this case we obtain the following result when we differentiate the solution two times and substitute the solution and its second derivative into the differential equation, y'' + 4y = 1.

$$y'' + 4y = \left\{\frac{4}{4}\cos\left[2(t-3)\right] - 4\cos 2t\right\} + 4\left\{\frac{1}{4} - \frac{1}{4}\cos\left[2(t-3)\right] + \cos 2t\right\} = 1$$

We see that this solution satisfies the differential equation for t > 3.[1] To get the solution shown in the text we have to use the trigonometric identity for the cosine of a double angle given in equation (10) on page A64 of Kreyszig (10$^{th}$ edition).

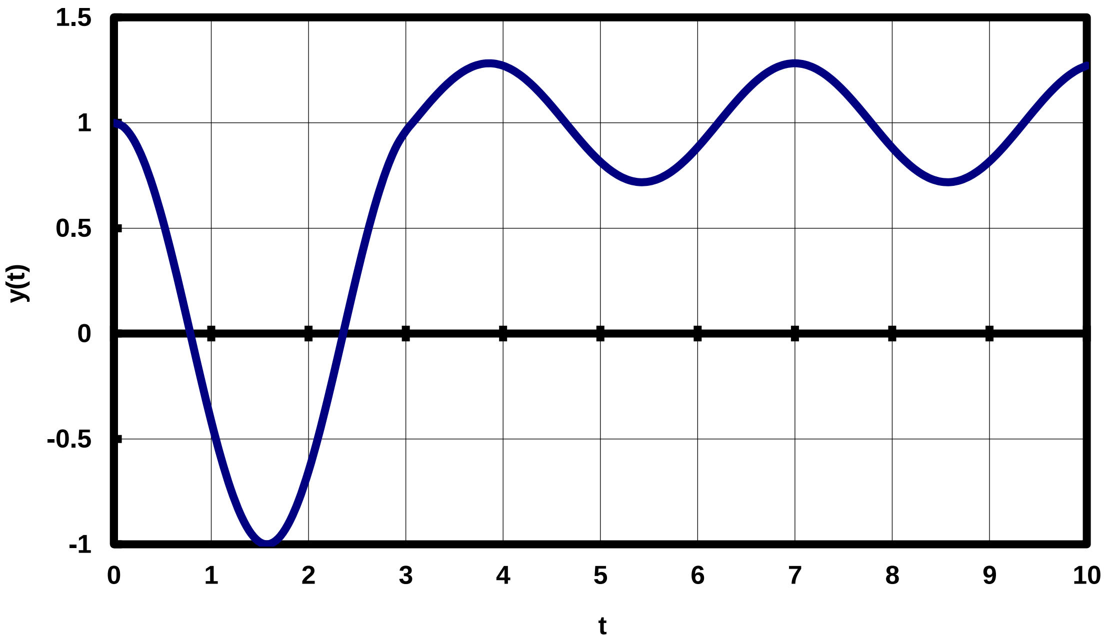
$$1 - \cos 2x = 2\sin^2 x$$

Applying this formula to our solution gives the text solution shown below.

[1] We could differentiate the solution with the unit function, but to show that this satisfies the differential equation requires arguments about the definition of the unit function (which is the delta function or impulse function) that we have not covered in the class.

$$y(t) = \frac{1}{4}u(t-3)\{1-\cos[2(t-3)]\} + \cos 2t = \frac{1}{4}u(t-3)\{2\sin^2[(t-3)]\} + \cos 2t$$

The following figure shows a plot of the solution to this problem.




**8. Obtain the solution to the coupled spring-mass system governed by the following equations**

$$\frac{d^2y_1}{dt^2} = -\frac{k_1+k_2}{m_1}y_1 + \frac{k_2}{m_1}y_2 \qquad \frac{d^2y_2}{dt^2} = \frac{k_2}{m_2}y_1 - \frac{k_3+k_2}{m_2}y_2$$

**Use the following properties and intimal conditions: $m_1 = m_2 = 10$ kg; $k_1 = k_3 = 20$ kg/s$^2$; $k_2 = 40$ kg/s$^2$, $y_1(0) = y_2(0) = 0$; $y_1'(0) = 1$ m/s, and $y_2'(0) = -1$ m/s. Solve the problem using Laplace transforms; show your work.**

We see that the units given for the properties are consistent with distance measured in meters and time measured in seconds. With these units, each term in both differential equations has units of m/s$^2$. Substituting the property data given in the problem into the differential equations gives the following numerical problem to solve.

$$\frac{d^2y_1}{dt^2} = -\frac{20+40}{10}y_1 + \frac{40}{10}y_2 = -6y_1 + 4y_2 \qquad \frac{d^2y_2}{dt^2} = \frac{40}{10}y_1 - \frac{20+40}{10}y_2 = 4y_1 - 6y_2$$

Taking the Laplace transforms of the two differential equations, $Y_1(s)$ and $Y_2(s)$ gives the following results.

$$s^2Y_1(s) - sy_1(0) - y_1'(0) = -6Y_1(s) + 4Y_2(s) \qquad s^2Y_2(s) - sy_2(0) - y_2'(0) = 4Y_1(s) - 6Y_2(s)$$

Substituting the initial conditions, $y_1(0) = y_2(0) = 0$; $y_1'(0) = 1$ m/s, and $y_2'(0) = -1$ m/s, into this equation gives the final pair of algebraic equations that we have to solve for $Y_1(s)$ and $Y_2(s)$.

$$s^2Y_1(s) - s0 - 1 = -6Y_1(s) + 4Y_2(s) \qquad s^2Y_2(s) - 0 - (-1) = 4Y_1(s) - 6Y_2(s)$$

Rearranging these equations gives the two equations below that we have to solve simultaneously. If we multiply the top equation on the left by 4 and the bottom equation on the left by $s^2 + 6$, we get the pair of equations on the right

$$\begin{aligned}(s^2+6)Y_1(s) \qquad\qquad -4Y_2(s) &= 1\\ -4Y_1(s) + (s^2+6)Y_2(s) &= -1\end{aligned}$$

If we multiply the top equation by 4 and the bottom equation by $s^2 + 6$, we get the equations shown below that we can add to obtain an equation that contains only the $Y_2(s)$ term.

$$\begin{aligned}4(s^2+6)Y_1(s) \qquad\qquad -16Y_2(s) &= 4\\ -4(s^2+6)Y_1(s) + (s^2+6)^2 Y_2(s) &= -(s^2+6)\\ (s^2+6)^2 Y_2(s) - 16Y_2(s) &= 4 - (s^2+6)\end{aligned}$$

We can solve the final equation for $Y_2(s)$ and rearrange the result as follows.

$$Y_2(s) = \frac{4-(s^2+6)}{(s^2+6)^2 - 16} = \frac{-s^2-2}{s^4+12s^2+36-16} = \frac{-s^2-2}{s^4+12s^2+20} = -\frac{s^2+2}{(s^2+10)(s^2+2)} = \frac{-1}{(s^2+10)}$$

From transform pair 13 on page 249 of Kreyszig (10th edition), we find that the inverse transform of $1/(s^2 + \omega^2) = (\sin \omega t)/\omega$. This is the form that we have with $\omega^2 = 10$. Thus, the solution for the motion of the second mass is $\boxed{y_2 = -\sin\sqrt{10}t \big/ \sqrt{10}}$.

We can find the solution for $y_1$ by solving the second differential equation for $y_1$ and substituting the solution that we just found for $y_2$. (Note that $y_2'' = \sqrt{10}\ \sin\sqrt{10}\ t$.)

$$y_1 = \frac{1}{4}\left[6y_2 + \frac{d^2y_2}{dt^2}\right] = \frac{1}{4}\left[6\left(-\frac{\sin\sqrt{10}t}{\sqrt{10}}\right) + \sqrt{10}\sin\sqrt{10}t\right] = \frac{\sin\sqrt{10}t}{4\sqrt{10}}(-6+10)$$

$$\boxed{y_1 = \frac{\sin\sqrt{10}t}{\sqrt{10}}}$$

We see that the proposed solutions satisfy the initial conditions $y_1(0) = y_2(0) = 0$; $y_1'(0) = 1$ m/s, and $y_2'(0) = -1$ m/s. Since we found the solution for $y_1$ from the second differential equation, we can check to see if our solutions satisfy the first differential equation. Substituting the solutions and $d^2y_1/dt^2$ into the first differential equation gives the following result.

$$\frac{d^2y_1}{dt^2} + 6y_1 - 4y_2 = -\sqrt{10}\sin\sqrt{10}t + 6\frac{\sin\sqrt{10}t}{\sqrt{10}} - 4\left(-\frac{\sin\sqrt{10}t}{\sqrt{10}}\right) = \frac{\sin\sqrt{10}t}{\sqrt{10}}(-10+6+4) = 0$$

So our solution satisfies the differential equations and boundary conditions as required.