

### November 8 Homework Problems

1. Find the Laplace transform of a plot which is a straight line from (0,1) to (1,0.5). Show the details of your work.
2. Given  $F(s) = \mathcal{L}[f(t)] = (-s - 10)/(s^2 - s - 2)$ , find  $f(t)$ . Show the details.
3. Find the Laplace transform of  $5e^{2t}\sinh 2t$ . (Show the details.)
4. If  $\mathcal{L}[f(t)] = F(s)$  and  $c$  is any positive constant, show that  $\mathcal{L}[f(ct)] = F(s/c)/c$ . Use this result to obtain  $\mathcal{L}(\cos \omega t)$  from  $\mathcal{L}(\cos t)$ .
5. Solve the following differential equation  $y'' + 9y = 8 \sin t$  if  $0 < t < \pi$  and  $0$  if  $t > \pi$ .  $y(0) = 0$ ,  $y'(0) = 4$ . Use Laplace transforms. (Show the details.)
6. Solve the differential equation  $y'' + \omega_0^2 y = K \sin pt$  with  $y(0) = y'(0) = 0$  and  $p^2 \neq \omega_0^2$ . Find the solution by Laplace transforms. (Show the details of your work.)
7. Solve the initial value problem  $y'' + 4y = u(t - 3)$ ,  $y(0) = 1$ ,  $y'(0) = 0$ , using Laplace transforms. Show the details of your work.
8. Obtain the solution to the coupled spring-mass system governed by the following equations

$$\frac{d^2 y_1}{dt^2} = -\frac{k_1 + k_2}{m_1} y_1 + \frac{k_2}{m_1} y_2 \qquad \frac{d^2 y_2}{dt^2} = \frac{k_2}{m_2} y_1 - \frac{k_3 + k_2}{m_2} y_2$$

Use the following properties and initial conditions:  $m_1 = m_2 = 10$  kg;  $k_1 = k_3 = 20$  kg/s<sup>2</sup>;  $k_2 = 40$  kg/s<sup>2</sup>,  $y_1(0) = y_2(0) = 0$ ;  $y_1'(0) = 1$  m/s, and  $y_2'(0) = -1$  m/s. Solve the problem using Laplace transforms; show your work.