College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 501A
Seminar in Engineering Analysis

Fall 2017 Number 15966 Instructor: Larry Caretto

September 20 Homework Problems

1. Find the spectrum (all the eigenvalues) and eigenvectors for the matrix, **A**, at the right.

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Find the principal axes and the corresponding expansion (or compression) factors for the elastic deformation matrix, **A**, shown at the right.

$$\mathbf{A} = \begin{bmatrix} 3/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1 \end{bmatrix}$$

3. Is the matrix at the right symmetric, skew-symmetric or orthogonal? Find the eigenvalues of the matrix and show how this illustrates Theorems 1 or Theorem 5 in section 8.3.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

4. Give a geometrical interpretation of the coordinate transformation **y** = **Ax**, with the matrix **A** as shown on the right.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

5. Find a basis for eigenvectors and diagonalize the matrix, **A**, shown at the right.

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$$

6. Find out what kind of conic section (or pair of straight lines) is represented by the equation $9x_1^2 - 6x_1x_2 + x_2^2 = 40$. Transform it to principal axes. Express $\mathbf{x}^T = [x_1 \;\; x_2]$ in terms of the new coordinate vector $\mathbf{y}^T = [y_1 \;\; y_2]$,

7. Determine and sketch disks that contain the eigenvalues of the matrix A at the right.

$$\mathbf{A} = \begin{bmatrix} 5 & -2 & 2 \\ 2 & 0 & 4 \\ 4 & 2 & 7 \end{bmatrix}$$

8. Find the similarity transformation, $\mathbf{T}^{-1}\mathbf{AT}$ such that the Gerschgorin disk with a center at 5 for the matrix **A** at the right is reduced to 1/100 of its original value.

$$\mathbf{A} = \begin{bmatrix} 5 & 10^{-2} & 10^{-2} \\ 10^{-2} & 8 & 10^{-2} \\ 10^{-2} & 10^{-2} & 9 \end{bmatrix}$$