College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 483
Alternative Energy Engineering II

Spring 2010 Number: 17724 Instructor: Larry Caretto

Notes on Engineering Economic Analysis

Introduction

The economic analysis of alternative energy sources typically involves the comparison of an initial cost with a future savings. For example the decision to pay more money for a vehicle with a hybrid drive train is based on a comparison of the higher initial price for the hybrid drivetrain withs the future savings in fuel costs. Similarly the decision to install a photovoltaic solar collector on your home balances the initial cost of the collector against the future savings in electricity bills.

A simple way to make such comparisons is to divide the initial cost to the expected savings rate, a calculation that results in a **payback period**. For example, if an initial cost of $2,000 resulted in a savings of $500 per year, the payback period would be computed as ($2,000) / ($500/year) = 4 years. An individual could then judge if the payback period was short enough to justify the initial investment.

One problem with the payback period analysis is that it does not account for the **time value of money**. The money spent on the initial alternative energy technology could be invested in some other area that would pay a return on the initial investment. If the return from this investment were greater than the savings from the alternative energy technology, it would make more economic sense to make this investment instead of purchasing the alternative energy technology.

The purpose of these notes is to summarize the basic ideas of applying the concept of the time value of money to the economic analysis of engineering decision making. In this course, we will apply these ides to the economic analysis of different energy technologies.

The time value of money

The time value of money is specified in terms of an interest rate, i. If an initial amount of money, P, called the present worth or present value (hence the symbol P), is invested at an interest rate, i, for a time period, t, the investment will earn an interest payment at the end of the time period of itP. The sum of the initial investment and the interest payment is called the future worth or future value, F. We this have the following equation.

$$F = P + iPt = P(1 + it) \qquad [1]$$

Note that the dimensions of interest rate are 1/time. Typically the interest rate is expressed as a percentage and one has to be careful to convert the percentage to a decimal fraction before using it in an equation. The typical unit for interest rate is 1/year. If the interest rate is applied for t = 1 year, the t term in equation [1] is usually omitted and one write the future value equation as follows.

$$F = P(1 + i) \qquad [2]$$

This equation assumes that the time period is one unit; this is typically one year when the interest rate has units of 1/year. However other periods, such as monthly may be used. When using equation [2] in place of equation [1] it is important to ensure that the 1/time units for the interest

Jacaranda (Engineering) 3333 | Mail Code 8348 | Phone: 818.677.6448
E-mail: lcaretto@csun.edu | Fax: 818.677.7062

rate are the same as the time units for the period. If the period is one month, then the units for the interest rate must be 1/month.

The calculation of the interest rate for a different time unit is simply done by using the unit conversion factor for the time units. For example, when applying equation [2] to a period of one quarter (1/4 of a year) an annual interest rate of 6% would be converted to a quarterly interest rate as (6%/year)(1 year/4 quarters) = 1.5%/quarter. Interest rates converted in this fashion are called nominal or base interest rates to distinguish them from the effective interest rates discussed below.

If the value of P(1 + i) from equation [2] is invested for a second year it will earn additional interest of P(1 + i)i. At the end of the second year the total amount form the initial principal value, P, and the two interest payments can be found as follows.

$$F = P(1 + i) + P(1 + i)i = P(1 + i)(1 + i) = P(1 + i)^2 \qquad [3]$$

If we continue to reinvest the total amount (initial investment plus accumulated interest) each year we will continue to earn interest on the original amount plus the accumulated interest. Extending the analysis that led to equation [3] gives the following result for the value at the end of N years.

$$F = P(1 + i)^N \qquad [4]$$

If we solve this equation for P we can answer the following question: how much do we have to invest at an interest rate i to have a future value of F?

$$P = F(1 + i)^{-N} \qquad [5]$$

The fact that this multiyear investment earns interest not only on the initial investment, but also on the interest earned in earlier years is called **compounding**.

In some cases the compounding period can be different from the time units used for the interest rate. For example, some bank savings accounts provide a quarterly interest payment. In this example we can use equation [1] to determine the future value at the end of one quarter by setting t = ¼ year.

$$F = P[1 + i(1/4)] \qquad [6]$$

(Note that this result is the same as using equation [2] with a quarterly interest rate computed as the annual interest rate divided by 4 quarters per year.) We can apply equation [4] to determine the amount that we would have at the end of one year (N = 4 quarters).

$$F = P[1 + i(1/4)]^4 = P(1 + i/4)^4 \qquad [7]$$

What interest rate would be required to have the same future value at the end of one year without the quarterly compounding? We can find this by rewriting equation [2] in terms of an effective interest rate, i_{eff}, such that $F = P(1 + i_{eff})$; setting this expression for F equal to the expression for F in equation [7].gives.

$$P(1 + i_{eff}) = P(1 + i/4)^4 \Rightarrow i_{eff} = (1 + i/4)^4 - 1 \qquad [8]$$

We can generalize this equation for the case of n_c compounding periods in one year.

$$i_{eff} = \left(1 + \frac{i}{n_c}\right)^{n_c} - 1 \qquad [9]$$

Table 1 shows the value of the effective interest rate for different nominal (base) interest rates and different compounding periods. This shows that the effect of compounding is greater for higher interest rates and diminishes as the number of compounding periods becomes large.[1]

Table 1 – Effective Interest Rate as a Function of Base Rates and Compounding Periods

Compounding Periods	Effective interest rates for base Interest rates in next row						
	0.25%	0.50%	1%	2%	4%	10%	20%
1	0.25000%	0.50000%	1.00000%	2.00000%	4.00000%	10.0000%	20.0000%
2	0.25016%	0.50062%	1.00250%	2.01000%	4.04000%	10.2500%	21.0000%
3	0.25021%	0.50083%	1.00334%	2.01336%	4.05357%	10.3370%	21.3630%
4	0.25023%	0.50094%	1.00376%	2.01505%	4.06040%	10.3813%	21.5506%
5	0.25025%	0.50100%	1.00401%	2.01606%	4.06451%	10.4081%	21.6653%
7	0.25027%	0.50107%	1.00430%	2.01722%	4.06923%	10.4389%	21.7983%
10	0.25028%	0.50113%	1.00451%	2.01810%	4.07277%	10.4622%	21.8994%
15	0.25029%	0.50117%	1.00468%	2.01877%	4.07554%	10.4804%	21.9790%
20	0.25030%	0.50119%	1.00476%	2.01911%	4.07692%	10.4896%	22.0190%
∞	0.25031%	0.50125%	1.00502%	2.02013%	4.08108%	10.5171%	22.1403%

Sample Problem: Interest on a credit card is typically stated as 1.5% per month. What is the nominal annual interest rate? What is the effective annual interest rate if the interest is compounded monthly?

Solution: Since there are 12 months per year, the nominal annual interest rate is simply the product (12 months/year)(1.5%/month) = 18%/year.

The effective annual interest rate is found from equation [9] with n_c = 12 compounding periods and i = 18%, the nominal annual interest rate just found.

$$i_{eff} = \left(1 + \frac{i}{n_c}\right)^{n_c} - 1 = \left(1 + \frac{0.18}{12}\right)^{12} - 1 = 0.1956 = 19.56\%$$

Remember that interest rates are converted to decimal fractions before being used in equations.

Present worth of a series of uniform payments

In the introduction we discussed the tradeoff between an initial one-time investment and a continuous series of future savings. To analyze this examine an initial investment with a present worth, P, and a set of N equal time increments with a fixed payment, A, at the end of each time increment.[2] We want to find the equivalence, considering the time value of money, between the initial investment, P, and the series of payments, A. Typically the schedule for the payments will be monthly or annual payments. If the interest rate for one time period of the payments is i, we can determine the present worth of each future payment, A, from equation [5]. The present worth of the first payment will be $A(1 + i)^{-1}$; for the second payment, at the end of two time periods, the present worth will be $A(1 + i)^{-2}$. In general the present worth of the payment at the end of k time

[1] The result for an infinite number of compounding periods, called instantaneous compounding can be found by taking the limit as n_c approaches infinity. This leads to the following equation for instantaneous compounding: $(i_{eff})_{instantaneous} = e^i – 1$.

[2] The notation A for each payment in this series comes from the use of this formula for determining a set of annual payments, sometimes called an annuity. However, any time interval can be used for the payments.

periods will be $A(1 + i)^{-k}$. To get the present worth of all payments, we have to sum the present worth of all N individual payments. This gives.

$$P = \sum_{k=1}^{N} A(1+i)^{-k} \tag{10}$$

Appendix A shows that this summation can be expressed as the ratio, P/A, in either of two equivalent forms shown below.

$$\frac{P}{A} = \frac{1-(1+i)^{-N}}{i} = \frac{(1+i)^{N}-1}{i(1+i)^{N}} \tag{11}$$

This equation answers the question of what present worth is required to provide a series of a uniform payment, A, at the end of N periods when the interest rate per period is i. The answer to the opposite question, what is the uniform payment that we would get from a present worth P for N time periods with an interest rate per period of i is simply the reciprocal of equation [11].

$$\frac{A}{P} = \frac{i}{1-(1+i)^{-N}} = \frac{i(1+i)^{N}}{(1+i)^{N}-1} \tag{12}$$

A key point to remember for use of equations [11] and [12] is that the interest rate must be the interest rate per period. If the payments are monthly and the interest rate is given as an annual interest rate, the monthly interest rate must be found to use these equations.

Sample problem: An individual is offered a loan of $10,000 at an annual interest rate (nominal) of 9%. The loan is to be paid off in a series of uniform monthly payments over six years. What is the amount of the monthly payment?

Solution: The period between payments is one month. The interest rate per monthly period is (9%/year)(12 months/year) = 0.75%/month. The total number of payments, N = 6 x 12 = 72. We can use equation [12], multiplied by P, to determine the payment, A. [3]

$$A = \frac{iP}{1-(1+i)^{-N}} = \frac{\frac{0.0075}{month}(\$10{,}000)}{1-(1+0.0075)^{-72}} = \boxed{\frac{\$180.26}{month}}$$

Sample problem continued: The amount of a loan is called the principal. The monthly payment in the previous problem is used two ways: (1) the first part is the payment of interest (computed as the monthly rate times the remaining balance of the loan principal). The remainder of the monthly payment is used to reduce the principal. Determine how much the loan principal is reduced during the first year of payments.

Solution: For the first payment, the monthly interest is the monthly interest rate times the original principal of $10,000. This is (0.0075)($10,000) = $75. The remainder of the monthly payment, $180.26 - $75 = $105.26 is used to reduce the principal. After the first loan payment the

[3] Note that the units of 1/month are not used for interest rate in the (1 + i) term in the equation for A. This is because of the shorthand that we used in going from equation [1] to from equation [2]. The calculation of A should really have the term as (1 + it), where t = 1 month. Since we have dropped this term, we have not shown the units for i in this sample calculation.

remaining principal is $10,000 - $105.26 = $9895.74. For the second month, the interest payment is (0.0075)($9,895.74) = $74.21. Continuing the calculations in this fashion gives the results for the entire loan shown in Table 2. After the first year (12 payments) the loan balance is $8,683.45. (See the N = 13 line in Table 2.) The amount paid towards reducing the principal in the first year is $10, 000 - $8,683.45 = $1,316.55.

Table 2 – Amounts Paid to Interest and Principal for Sample Problem

N	Principal	Amount to Interest	Amount to Principal	N	Principal	Amount to Interest	Amount to Principal	N	Principal	Amount to Interest	Amount to Principal
1	$10,000.00	$75.00	$105.26	25	$7,243.39	$54.33	$125.93	49	$3,945.38	$29.59	$150.67
2	$9,894.74	$74.21	$106.05	26	$7,117.46	$53.38	$126.88	50	$3,794.71	$28.46	$151.80
3	$9,788.69	$73.42	$106.84	27	$6,990.58	$52.43	$127.83	51	$3,642.91	$27.32	$152.94
4	$9,681.85	$72.61	$107.65	28	$6,862.75	$51.47	$128.79	52	$3,489.97	$26.17	$154.09
5	$9,574.20	$71.81	$108.45	29	$6,733.96	$50.50	$129.76	53	$3,335.88	$25.02	$155.24
6	$9,465.75	$70.99	$109.27	30	$6,604.20	$49.53	$130.73	54	$3,180.64	$23.85	$156.41
7	$9,356.48	$70.17	$110.09	31	$6,473.47	$48.55	$131.71	55	$3,024.23	$22.68	$157.58
8	$9,246.39	$69.35	$110.91	32	$6,341.76	$47.56	$132.70	56	$2,866.65	$21.50	$158.76
9	$9,135.48	$68.52	$111.74	33	$6,209.06	$46.57	$133.69	57	$2,707.89	$20.31	$159.95
10	$9,023.74	$67.68	$112.58	34	$6,075.37	$45.57	$134.69	58	$2,547.94	$19.11	$161.15
11	$8,911.16	$66.83	$113.43	35	$5,940.68	$44.56	$135.70	59	$2,386.79	$17.90	$162.36
12	$8,797.73	$65.98	$114.28	36	$5,804.98	$43.54	$136.72	60	$2,224.43	$16.68	$163.58
13	$8,683.45	$65.13	$115.13	37	$5,668.26	$42.51	$137.75	61	$2,060.85	$15.46	$164.80
14	$8,568.32	$64.26	$116.00	38	$5,530.51	$41.48	$138.78	62	$1,896.05	$14.22	$166.04
15	$8,452.32	$63.39	$116.87	39	$5,391.73	$40.44	$139.82	63	$1,730.01	$12.98	$167.28
16	$8,335.45	$62.52	$117.74	40	$5,251.91	$39.39	$140.87	64	$1,562.73	$11.72	$168.54
17	$8,217.71	$61.63	$118.63	41	$5,111.04	$38.33	$141.93	65	$1,394.19	$10.46	$169.80
18	$8,099.08	$60.74	$119.52	42	$4,969.11	$37.27	$142.99	66	$1,224.39	$9.18	$171.08
19	$7,979.56	$59.85	$120.41	43	$4,826.12	$36.20	$144.06	67	$1,053.31	$7.90	$172.36
20	$7,859.15	$58.94	$121.32	44	$4,682.06	$35.12	$145.14	68	$880.95	$6.61	$173.65
21	$7,737.83	$58.03	$122.23	45	$4,536.92	$34.03	$146.23	69	$707.30	$5.30	$174.96
22	$7,615.60	$57.12	$123.14	46	$4,390.69	$32.93	$147.33	70	$532.34	$3.99	$176.27
23	$7,492.46	$56.19	$124.07	47	$4,243.36	$31.83	$148.43	71	$356.07	$2.67	$177.59
24	$7,368.39	$55.26	$125.00	48	$4,094.93	$30.71	$149.55	72	$178.48	$1.34	$178.92

In this table N is the number of the payment; the Principal column shows the loan balance prior to the payment. The amount to interest is the periodic interest rate times the principal. The payment of this loan is $180.26; the amount to principal is the difference between the loan payment and the amount to interest. The new principal is the old principal minus the amount to principal. The loan interest is rounded to two decimal places. Because of this the final calculation is not correct; the final amount to principal is less than the amount of the principal. In practice, the final payment would be reduced by $0.44 (from $180.26 to $179.82) to pay off the principal exactly.

The information in Table 2 becomes important when economic analyses are done considering taxes. Interest paid on a loan is a deductable business expense. The amount paid for principal is not. When considering the annual income statement of a company for preparing tax returns, it is important to know how much of a loan payment goes to interest.

Sample problem concluded: What is the total amount of the payments on this loan. How much goes to interest?

Solution: There are 72 payments of $180.25 for a total of $12,978.72. Subtracting the initial loan amount of $10,000 from this total shows that the total interest paid is $2,978.72.[4]

Practical calculations

Equations [11] and [12] can be readily solved for A or P. Both of these equations can be solved for N to give

$$N = -\frac{\ln\left(1 - \frac{iA}{P}\right)}{\ln(1+i)} \qquad [13]$$

However, there is no explicit solution for i. Traditionally engineering economics textbooks have included tables of quantities such as A/P as a function of i and N. Such tables allow i or N to be determined by interpolation. Modern calculators, especially financial calculators have equations for the various functions considered here. Spreadsheets, such as Excel, also have formulas for computing the various terms in these equations. Figure 1 is an example of an Excel spreadsheet with formulas for financial calculations. This spreadsheet uses names for cell locations[5] and displays the formulas instead of the numerical results.

Figure 1: Excel Spreadsheet with Financial Formulas

Annual interest rate	5%
Number of periods per year	12
Interest rate per period	=Annual_interest_rate/Number_of_periods_per_year
Total number of periods	360
Present Value	100000
Periodic payment	=PMT(Interest_rate_per_period,Total_number_of_periods,-Present_Value)
Calculate number of periods	=NPER(Interest_rate_per_period,Periodic_payment,-Present_Value)
Calculate interest rate	=RATE(Total_number_of_periods,Periodic_payment,-Present_Value)

In this spreadsheet the names Annual_interest_rate, Number_of_periods_per_year, Total_number_of_periods and Present_value have been defined to represent the cells that contain the values 5%, 12, 360, and 100000, respectively. The names Interest_rate_per_period represents the cell that contains the formula for defining the periodic interest rate, which is 0.041667 (= 5%/12) in this example.[6]

[4] It is possible to obtain an interest-only loan in which the monthly payments would be only (.0075)($10,000) = $75. However a lump-sum payment of $10,000 would be required at the end of the loan. Here the total interest would be $5,400. Another possibility is a simple interest loan in which the total interest would be ($10,000)(0.09/year)(6 years) = $5400. The total amount of principal plus interest would be paid according to some payment schedule. For example, the total principal plus interest of $15,400could be paid in twelve semi-annual payments of $1,283.33.

[5] A cell name in Excel provides an alternative way for referring to a cell location. The usual cell reference is a (column)(row) combination such as C14. Cell names provide an alternative way to refer to cell locations that gives someone reviewing the spreadsheet a better idea of what the variables in an equation represent.

[6] Note that percentages in Excel are a formatting option. The underlying value is not changed. Thus, the explicit conversion of percentages to decimal fractions is ***not*** required in Excel.

The function PMT computes the value of A from equation [12]. Excel spreadsheet financial formulas include the **direction of the payment**. Thus the PMT function assumes that the present value and the payment result will have opposite signs.[7] To obtain a positive value for the payment result, a minus sign is placed before the present value in the PMT formula. The cell that contains the formula is given the name Periodic_payment.

The function NPER computes the value of N, the number of periods, which could be found from equation [13]. Again, this Excel formula uses opposite signs for A and P; to accommodate this, a minus sign is placed in front of the present value. The function RATE computes the periodic interest rate, i, which satisfies both equations [11] and [12].

In the actual spreadsheet for this example the computed payment is $536.82 per month and the computed values of N and i match the input values of 360 and 5%/12.

What is the interest rate?

The interest rate used previously in these notes is easily understood as the interest that you have to pay on a loan or the interest that you would receive on a bank savings account. In some cases the term **discount rate** is used. This is commonly used for the rate that the Federal Reserve charges for loans to banks. It is also used in bond sales where the initial price for a bond with a certain face value (say $1000) is "discounted" so that the initial price for the bond is less than its face value. A company comparing alternative project proposals, each with different cash flows, will often examine the **internal rate of return**. This is the equivalent interest rate that the cash flows for the project would pay if the cash outflows were costs and the cash inflows were profits. When an individual or company is considering investments on different projects with expected cash flows they will generally seek a **minimally acceptable rate of retur (MARR)** on the investment. Regardless of which term is used – interest rate, discount rate, internal rate of return, MARR – the formulas derived above will apply.

Non-uniform payment series and comparison of alternatives

The formulas used so far relate a present value to a single future value and an initial present value to a uniform series of payments. Texts on engineering economics also present formulas for a series of increasing (or decreasing payments) where the rate of increase (or decrease) is the same and the payment interval is constant. These notes do not consider such formulas.

In the most general economic analysis of engineering projects, the amount of the cash flows will not be uniform and the time intervals may not be constant. A general comparison of alternatives can be done by a method known as **discounted cash flows**. In this approach each cash input or output is converted to an equivalent value at a fixed time using equations [4] or [5]. Typically this fixed time is the start or end of the project so the cost is expressed as a present value at the start of the project or a final value at the end of the project.

The calculation of the net present value of a series of cash flows that occur at regular intervals (yearly, monthly, weekly, *etc*.) essentially applies equation [5] to this series of evenly spaced cash flows (with positive and negative signs) to determine the net present value. If the cash flow at the end of period k is CF_k, the net present value, NPV, is found from the following equation when the time interval is constant and the interest rate refers to the annual period.

[7] If P is negative, indicating a cash outflow (a loan that you make) then A is positive indicating a cash inflow (the payments that the borrower returns to you). If P is positive, indicating a cash inflow (a loan that you receive), then A is negative indicating a cash outflow as you make payments on the loan.

$$NPV = \sum_{k=0}^{N} CF_k (1+i)^{-k} \quad \text{Equal time intervals} \qquad [14]$$

The more general case of unequal cash flows with uneven time increments can be expressed by the following equation where the interest rate is the daily interest rate (the annual rate divided by 365.25 days/year), d_0 is the date of the initial cash flow, expressed as a serial date, and d_k is the date of the k^{th} cash flow.

$$NPV = \sum_{k=0}^{N} CF_k \left(1+i_{daily}\right)^{d_0-d_k} \quad \text{Unequal time intervals} \qquad [15]$$

As usual, i is the interest rate for the period. The summations are started at k = 0 to consider a cash flow at the start of the first period. The Excel function NPV(rate, values) has the interest rate as its first argument and the series of payments from k = 1 to k = N as the remaining arguments. This function ***does not*** consider any cash flow at the start of the first interval (the k = 0 term in equation [14]). If an Excel spreadsheet has the interest rate in cell A1, the initial (k = 0) cash flow in cell A2, and the remaining cash flows in cells A3:A12, the net present value, including the k = 0 term, would be computed by the spreadsheet formula = A2 + NPV(A1, A3:A12). If the initial cash flow in cell A2 were included in the arguments to the NPV function, it would be considered as the cash flow at the end of the first time increment.

If the net future value is required, it can be computed from the net present value using equation [4].

The Excel function XNPV computes the net present value for a series of uneven cash flows at different dates, where the date intervals need not be the same. In this function, the initial cash flow is accounted for. If an Excel spreadsheet has the interest rate in cell A1, the initial (k = 0) cash flow in cell A2, the remaining cash flows in cells A3:A12, and the dates for each cash flow in cells B2:B12 the net present value, including the k = 0 term, would be computed by the spreadsheet formula =XNPV(A1, A2:A12, B2:B12). Note the difference between XNPV and NPV: XNPV includes the initial (k = 0) cash flow; NPV does not.

The internal rate of return (IRR) is the interest rate that makes the net present value equal to zero for a series of positive and negative cash flows. (Typically these start with a negative cash flow to purchase an item or make some other investment, followed by positive cash flows representing income from the investment.) If we use the net present value from equation [14], where the cash flows can vary, but the time interval is the same, the IRR is the value of i that makes NPV = 0 for an arbitrary series of cash flows. The Excel function IRR computes the internal rate of return for a series of cash flows at equally-spaced time intervals. The IRR function includes the k = 0 cash flow. For the spreadsheet with the initial (k = 0) cash flow in cell A2 and the remaining cash flows in cells A3:A12, the internal rate of return would be computed by the function call =IRR(A2:A12).

If we want to compute the IRR for a series of cash flows at different times we can use the Excel function XIRR. If an Excel spreadsheet has the initial (k = 0) cash flow in cell A2, the remaining cash flows in cells A3:A12, and the dates for each cash flow in cells B2:B12 the net present value, including the k = 0 term, would be computed by the spreadsheet formula =XNPV(A2:A12, B2:B12).

Both the IRR and XIRR functions have a final, optional argument, which is an initial guess for the interest rate. This argument can be used if the results from using either of these functions without this optional argument do not give correct results. You can check the results of the IRR or XIRR functions by using, respectively, the NPV or XNPV functions. If the IRR or XIRR function has the correct result the NPV or XNPV functions should give zero to within roundoff error when used with the same data set and an interest rate equal to that found by IRR or XIRR.

Effect of inflation

Inflation (deflation) in a currency is said to occur when the purchasing power of that currency decreases (increases) over time. If the purchase price of an identical item is d_1 at some initial time and increases to d_2, N time periods later, the inflation rate for each time period, f, assumed constant over N periods, is defined as follows:

$$d_2 = d_1 (1 + f)^N \qquad [16]$$

Although the equation has the same form as in interest rate equation, inflation does not represent an increase in the amount of money available. Rather it is a decrease in the purchasing power of the currency.

Measures of inflation in the US are maintained by the bureau of labor statistics. The most commons indices are the consumer price index (CPI) and producer price index (PPI). See the web site http://www.bls.gov/CPI/ for information on the CPI; the site http://www.bls.gov/ is the general site for various statistics including the CPI, PPI, and the employment cost index. Table 2 shows the average inflation rate in the CPI for all US cities. The inflation rate shown for a given year is the inflation rate from June of the previous year to June of the year shown. The cumulative inflation rate is the rate from June 1996 to June of the year shown.[8]

Table 3 - Inflation in US Average Urban Consumer Price Index Ending in June of Year Shown

Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Annual	2.3%	1.7%	2.0%	3.7%	3.2%	1.1%	2.1%	3.3%	2.5%	4.3%	2.7%	5.0%	-1.4%
Cumulative	2.3%	4.0%	6.1%	10.0%	13.6%	14.8%	17.2%	21.1%	24.1%	29.5%	33.0%	39.6%	37.6%

The inflation rate from June 1996 to June 1997 was 2.3%; the inflation rate from June 1996 to June 2009 was 37.6%. The negative inflation rate from June 2008 to June 2009 indicates a deflation in this period.

In equation [16] we assumed that the inflation rate was constant. In order to compute the average (constant) inflation rate for a period of N years from the total inflation over the period we need to understand how an overall inflation rate is related to a series of individual rates. Assume that a price P increases by 10% then increases by 12% again. What is the total price increase? After the first increase the new price is P(1 + 10%). After the second increase the price becomes P(1 + 10%)(1 + 12%). The total percent increase, x, from the two price increases is found by setting P(1 + x) = P(1 + 10%)(1 + 12%) so that x = 10% + 12% + (10%)(12%) = 23.2%. In general, the total fractional increase in a quantity g_{total} due to a series of individual increases g_i is given by the following equation: $(1 + g_{total}) = (1 + g_1)(1 + g_2) \ldots (1 + g_N)$ so that

$$g_{total} = \prod_{i=1}^{N} (1 + g_i) - 1 \qquad [17]$$

The product operator, Π, in this equation is the multiplication analog of the summation operator, Σ.

To get the average gain per period for a total gain g_{total} over N periods we set each g_i in equation [17] equal to an average growth rate for each period, $\overline{g}$,assumed constant over the N time periods. This gives

[8] Note that the cumulative inflation rate is not simply the sum of the individual inflation rates. Instead it is computed as shown in the text following the table.

$$g_{total} = \prod_{i=1}^{N}\left(1+\overline{g}\right)-1 = \left(1+\overline{g}\right)^N - 1 \quad \Rightarrow \quad \overline{g} = \left(1+g_{total}\right)^{1/N} - 1 \qquad [18]$$

Sample Problem: What is the average inflation rate for the years 1996 to 2009 from Table 2?

Solution: From Table 2 the total inflation over this 13-year period is 37.6%. Using equation [18] (with the symbol f for inflation rate in place of g) we find the average rate as follows.

$$\overline{f} = \left(1+f_{total}\right)^{1/N} - 1 = \left(1+0.376\right)^{1/13} - 1 = 2.488\%$$

How is inflation considered in engineering cost analyses? There are two approaches for doing this. The first approach assumes that the interest rate includes the effects of inflation. In this case we have the so-called **market interest rate** and the dollar amounts are expressed in so-called current dollars (or current Euros or whatever currency is used). The current currency amount is the amount of currency in the actual transaction. An alternative is to use the concept of constant dollars that corrects for the effects of inflation. Both of these approaches will be outlined below.

Let's start with the example of an investment of $1000 in June 1996 that pays a constant interest rate of 5% compounded annually. From equation [4] we can compute the value in June 2009 (thirteen years later) as follows: F = ($1,000)(1 + 0.05)^{13} = $1,885.66. What can we buy with this amount, compared to the purchasing power of our original investment? According to the data in Table 2, it will cost us 37.6% more to buy the item in June 2006 than it would have cost us in June 1996. Thus our new purchasing power is only $1,885.55/(1 + 0.376) = $1369.92 in terms of the value of the dollar in June 1996 when the initial investment was made. We could make the same computation using the average inflation rate of 2.488% just found. In this case we would compute $1,885.55/(1 + 0.02488)^{13} = $1369.92

The interest rate of 5% used in the above example is called the **market interest rate**. This is the interest rate that one usually sees offered for investments or charged for loans. This market interest rate includes the effect of inflation. A person offering this rate is willing to accept the return of the 5% investment with the realization that the effect of inflation will reduce the purchasing power of the eventual return on the investment. How do we distinguish between the market interest rate and the **true interest rate** or **actual interest rate** which ignores the effects of inflation?

Because the true interest rate, i_{true}, ignores the effects of inflation, we say that the calculations are in hypothetical "constant dollars"; this is a currency measure that ignores the effects of inflation. In the previous example we saw that a $1,000 investment in June 1996 yielded a constant dollar return (*i.e*. a return of equivalent purchasing power) of $1369.92 in June 2009. Using equation [4] with these constant dollar amounts we can compute the true interest rate as follows.

$$\$1369.92 = (\$1,000)(1 + i_{true})^{13} \quad \Rightarrow \quad i_{true} = [(\$1369.92) / (\$1,000)]^{1/13} - 1 = 2.451\%$$

What is the relationship between the true interest rate, the market interest rate and the inflation rate? We previously used the usual formula that relates present and future values using the market interest rate $F = P (1 + i_{market})^N$ to show that the future value of a $1,000 investment in June 1996 was $1,885.55 in June 2009. We then used the inflation formula to determine the future value in constant dollars, $F_{const} = F(1 + f)^{-N}$, Finally we defined the true interest rate by the equation $F_{const} = P(1 + i_{true})^N$. We can combine these three equations as follows.

$$F_{const} = P(1+i_{true})^N = \frac{F}{(1+f)^N} = \frac{P(1+i_{market})^N}{(1+f)^N} \quad [19]$$

Considering only the two (equal) terms multiplied by P gives

$$P(1+i_{true})^N = \frac{P(1+i_{market})^N}{(1+f)^N} \quad \Rightarrow \quad (1+i_{true})^N = \left(\frac{1+i_{market}}{1+f}\right)^N \quad [20]$$

Taking the 1/N root of both sides of this equation give

$$1+i_{true} = \frac{1+i_{market}}{1+f} \quad \Rightarrow \quad (1+i_{true})(1+f) = 1+i_{true}+f+i_{true}f = 1+i_{market} \quad [21]$$

This gives the following relationship between the three rates considered here: the market interest rate, the true interest rate, and the inflation rate.

$$i_{market} = i_{true} + f + i_{true}f \quad [22]$$

Note that i_{market} is not equal to the sum of the true interest rate and the inflation rate. However, if both of these are small, the market rate will be approximately the same as the sum of the true interest rate and the inflation rate.

Example problem: What is the true interest rate if the market rate is 5% and the inflation rate is 2.488%? What would the error be by using the approximation that $i_{market} = i_{true} + f$?

Solution: We can solve equation [22] for the true interest rate and substitute the data given with the following result.

$$i_{true} = \frac{i_{market} - f}{1+f} = \frac{0.05 - 0.02488}{1 + 0.02488} = 0.02451 = 2.451\%$$

Using the approximation that $i_{market} = i_{true} + f$ gives $i_{true} = i_{market} - f$ = 5% - 2.488% = 2.512%. This is an error of 0.061 percentage points.

Comment: We see that the input data for this problem are the same as the data that have been used in the examples of this section. Thus the true interest rate computed from equation [22] has the same value found previously. Because both i_{true} and f are relatively small the approximation gives only a small error.

Which approach – true interest rate and constant dollars or market interest rate and current dollars – should we use in calculations of alternative energy economics? The choice may be made by a regulatory agency. For example, public utilities, whose profit is regulated by governmental utility agencies, must use standard accounting determined by the agency. For simple problem solving you should notice that the same equations are used for market interest rates, which do not explicitly account for inflation and present results in current dollars, and for true interest rates, which present results in constant dollars. Here is a set of guidelines for problem solving:

- If you have only the true interest rate, you must do calculations in constant dollars.
- If you have only the market interest rate, you must do calculations in current dollars.

- If you have two of the three quantities (1) market interest rate, (2) true interest rate, and (3) inflation rate, you can find the third from equation [22]. You can then choose to do the calculations in either constant or current dollars.

In comparing options for alternative energy projects, it is often useful to explicitly account for the inflation rate. This is particularly true when comparing a conventional fossil fuel option in which the fuel inflation can be a significant factor with an alternative such as solar and wind in which the ongoing costs, such as maintenance, will not be as significant as the ongoing costs for the fossil fuel option. However, it is simpler to use the market interest rate because this gives all figures in current dollars.

Death and taxes

Well, only taxes will be considered here, and only briefly. All previous sections have made no mention of taxes; such analyses are called before-tax analyses. This section will outline the considerations that are required for an after-tax analysis.

We are all familiar with the "taxman" who visits our homes every April 15 with the requirement of filling out forms that show what percentage of our income we owe to the government. These are personal income taxes. In addition to such taxes, business entities pay a corporate income tax. The marginal tax rate (the tax rate on the highest level of taxable income) ranges from 34% to 39% for taxable incomes over $75,000.

The computation of taxable income allows deductions for interest payments and depreciation. Depreciation is an accounting tool where income is saved on a regular schedule to replace purchased equipment. The rate at which companies may charge depreciation is governed by regulations of the Internal Revenue Service (IRS). Although there are several methods of depreciation, the commonly used method is called the modified accelerated cost recovery system (MACRS), which allows a high rate of depreciation in early years. This is intended to encourage companies to purchase new equipment since the accelerated depreciation schedule allows then to reduce their taxable income.

Interest payments on loans are a particularly important item in after-tax analyses. Companies can finance new projects by a combination of debt and equity financing. Equity financing is the use of existing cash reserves (or the sale of stock by a corporation); debt financing is the borrowing of money, either as a loan or bond financing for large corporations. Most investments involve a combination of these two.[9] When considering the return that a company makes on its investments, only the equity financing is considered. The debt financing is accounted as a future expense of interest payments on the debt. As noted above, such interest payments are tax deductable.

Energy Accounting

Our main concern here will be with the tradeoffs between the investment in a new energy technology with higher first costs, but lower operating costs than conventional energy technology. We will be comparing the increased purchase price, an initial investment, against the ongoing future savings, that will be a time series of payments. Equation [12] will be our main tool for making this comparison. We can use this equation to determine the present worth of a series of cost savings and compare it to the initial purchase price.

For example, the purchase of a photovoltaic solar collector represents an initial cost. The annual electricity savings represents a series of costs that can be reduced to a present worth using equation [12] for a specified interest rate (what the purchaser would obtain from investing the

[9] A company's debt-to-equity ratio, the ratio of long term debt divided by common shareholder equity is a measure of the risk of investing in the company.

money instead of buying the solar system) and expected lifetime for the collector. If the present worth of the savings is less than the purchase cost, the other investment would make more economic sense.

Such a calculation might better be done with a real interest rate, since the cost of the electricity saved would be expected to increase over time. Because, the A/P formula in equation [12] assumes equal future payments, you could not account for any future increased savings due to inflation.

Appendix A – Derivation of Equations Used in Notes

Derivation of equation [11] for a series of uniform payments

We start with equation [10] and divide both sides of the equation by uniform payment, A.

$$\frac{P}{A} = \sum_{k=1}^{N}(1+i)^{-k} \qquad [A-1]$$

We start by writing the last term in the summation as a separate term (and decreasing the upper limit of the summation by one to account for this.)[10]

$$\frac{P}{A} = \sum_{k=1}^{N-1}(1+i)^{-k} + (1+i)^{-N} \qquad [A-2]$$

Next we extract a common factor of (1 + i) from the summation.

$$\frac{P}{A} = (1+i)\sum_{k=1}^{N-1}(1+i)^{-k-1} + (1+i)^{-N} \qquad [A-3]$$

Now we define a new summation index, m = k + 1. When k = 1, m = 2; when k = N – 1, m = N. This gives the following result.

$$\frac{P}{A} = (1+i)\sum_{m=2}^{N}(1+i)^{-m} + (1+i)^{-N} \qquad [A-4]$$

Finally we add and subtract the same term, $(1 + i)^{-1}$ from the summation term. Because $(1 + i)^{-1}$ has the same form as the general term in the summation for m = 1, the addition of this term simply adds the m = 1 term to the summation. This gives.

$$\begin{aligned}\frac{P}{A} &= (1+i)\left[\sum_{m=2}^{N}(1+i)^{-m} + (1+i)^{-1} - (1+i)^{-1}\right] + (1+i)^{-N} \\ &= (1+i)\left[\sum_{m=1}^{N}(1+i)^{-m} - (1+i)^{-1}\right] + (1+i)^{-N}\end{aligned} \qquad [A-5]$$

We see that the final summation term in this equation is the same as the summation term in equation [A-1] with the dummy index k replaced by the dummy index m. Thus

[10] Students may not have seen the typical approach for working with equations for summations. Getting a closed form equation for the summation is approached by writing specific terms in the sum outside the sum then getting a common factor that changes the expression inside the summation. A revised summation index is defined and the summation is manipulated so that the right side of the equation contains the original sum plus some other terms. The original left side (which equals the original sum) is then substituted for the original sum on the right side and the resulting equation is then solved for the initial sum. If this sounds confusing, just regard the derivation as a demonstration of this general approach to the specific problem considered here.

we can replace the second summation term by its equivalent term P/A from equation [A-1].[11] Doing this and solving for P/A gives.

$$\frac{P}{A} = (1+i)\left[\frac{P}{A} - (1+i)^{-1}\right] + (1+i)^{-N} = (1+i)\frac{P}{A} - 1 + (1+i)^{-N}$$
$$[1-(1+i)]\frac{P}{A} = -i\frac{P}{A} = -1 + (1+i)^{-N} \qquad \text{[A-6]}$$

The first equation below is found by solving equation [A-6] for P/A; the second equation is found by multiplying the first by $(1 + i)^N$.

$$\frac{P}{A} = \frac{1-(1+i)^{-N}}{i} = \frac{(1+i)^N - 1}{i(1+i)^N} \qquad \text{[A-7]}$$

Either of these equations may be used to solve for P/A.

[11] The concept of a dummy index means that we can use any letter for the summation index and the result will be the same. For example, $\sum_{k=1}^{2} x^k = \sum_{m=1}^{2} x^m = x + x^2$,