Engineering Economics Fundamentals

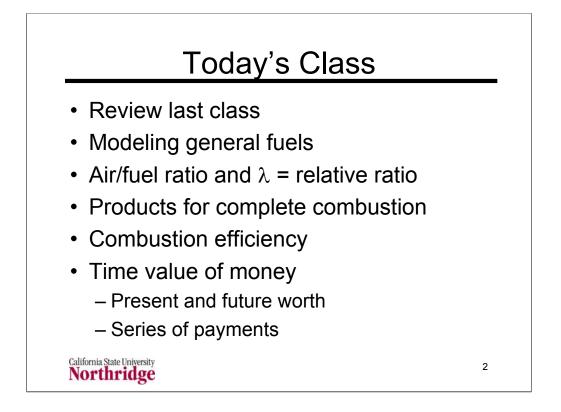
Larry Caretto Mechanical Engineering 483 *Alternative Energy Engineering II*

February 3, 2010

California State University Northridge

Reading

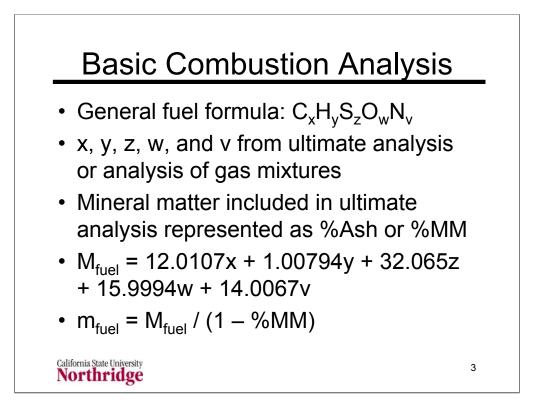
<u>Tonight and next Monday:</u> Notes on engineering economics <u>Next two classes</u>: No assigned reading **Homework** Third homework assignment due next Wednesday



The material in the lectures for tonight will be a review for students who have completed a course in engineering economic analysis such as the MSE 304, Engineering Economy, course required of all engineering majors at CSUN.

The material covered in this lecture will summarize the important points of this material and provide equations that can be used for simple computations.

The key concept is the time value of money. Money that could be used to purchase advanced energy technology to provide future savings on fuel costs could also be invested at some interest rate. A purely economic decision to purchase the new energy technology is based on a comparison of the savings from that technology with the return from some alternative investment for the purchase price.



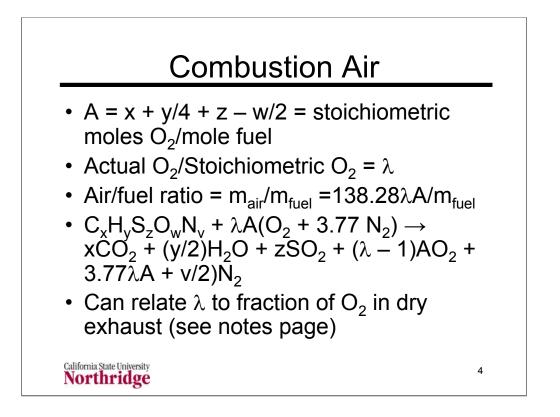
For ultimate analysis x = wt%C/12.0107, y = wt%H/1.00794, z = t%S/32.065, w = wt%O/16.0004, v = wt%N/14.0067

For mixture of gases where species k has mole fraction ω_k , x_k , C atoms, y_k H atoms, etc. compute x and y for fuel formula as follows. (Similar formulas apply for other atoms in fuel "molecule".)

$$x = \sum_{species} \omega_k x_k \qquad \qquad y = \sum_{species} \omega_k y_k$$

When an ultimate analysis is used to compute x, y, z, w, and v, the value of M_{fuel} is 100 mass units. When a gas analysis is used to compute x, y, z, w, and v for the mixture, M_{fuel} , will be the mean molar mass of the mixture.

 $\rm M_{fuel}$ represents the combustible part of the fuel. The actual fuel mass is given by $\rm m_{fuel}.$



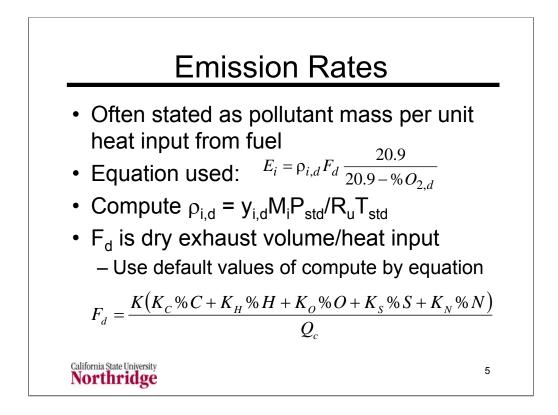
The stoichiometric O₂ requirement is the minimum amount of O₂ for complete combustion: x moles of O₂ are required to convert C_x to xCO₂; y/4 moles of O₂ are required to convert C_y to (y/2)H₂O; z moles of O₂ are required to convert S_z to zSO₂; the O_w in the fuel supplies w/2 moles of O₂ needed for combustion.

Combustion processes usually have more oxygen than the stoichiometric requirement. The ratio, λ , is known as the relative air/fuel ratio.

Exhaust gas measurements remove the water in the exhaust to avoid contamination in the sampling system and analyzers. The resulting exhaust is called dry exhaust. Measurements are typically made in terms of this dry exhaust stream. The equations that relate λ to the (dry) exhaust oxygen concentration are copied below:

$$\frac{\% O_2|_{dry}}{100} = \frac{(\lambda - 1)A}{x + 4.77\lambda A - A + z + \frac{\nu}{2}}$$

$$\lambda = \frac{A + \frac{\% O_2|_{dry}}{100} \left[x - A + z + \frac{v}{2} \right]}{A \left(1 - 4.77 \frac{\% O_2|_{dry}}{100} \right)}$$



Some values of F_d from http://www.epa.gov/ttn/emc/promgate/m-19.pdf (accessed February 6, 2007) are shown below

Fuel	dscm/J	dscf/MMBtu
Bituminous Coal	2.63x10 ⁻⁷	9,780
Lignite	2.65x10 ⁻⁷	9,860
Oil	3 2.47x10 ⁻⁷	9,190
Natural Gas	2.34x10 ⁻⁷	8,710

Constants for computing F_d from ultimate analysis (note that 84%C is entered as 84 **not** .84.

The units for the higher heating value (Q_c) are Btu/lb_m or kJ/kg.

K = Conversion factor, 10⁻³ (kJ/J)/(%) [10⁶ Btu/million Btu].

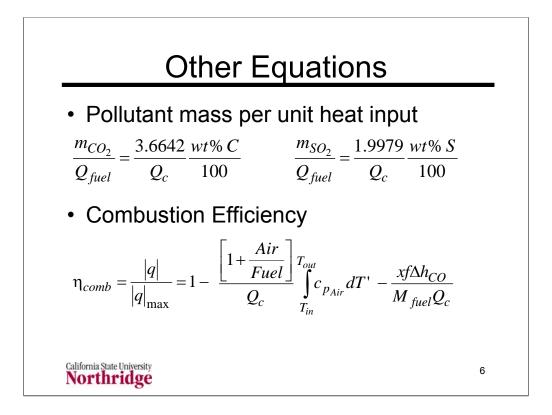
K_H = (22.7 scm/kg)/% [(3.64 scf/lb)/%].

 $K_{N} = (0.86 \text{ scm/kg})/\% [(0.14 \text{ scf/lb})/\%].$

 $K_{O} = (2.85 \text{ scm/kg})/\% [(0.46 \text{ scf/lb})/\%].$

 $K_{\rm S} = (3.54 \text{ scm/kg})/\% [(0.57 \text{ scf/lb})/\%].$

 P_{std} = 101.325 kPa = 14.696 psia, R_u = 8.31447 kPa•m3/kgmol•K = 10.7316 psia•ft3/lbmol•R, T_{std} = 288.15 K = 518.67 R,



In the equation for combustion efficiency,

Air/fuel is the air to fuel (mass) ratio

 $C_{p,air} = 0.24 \text{ Btu/lb}_m \cdot R = 1.005 \text{ kJ/kg} \cdot K$

f = molar exhaust ratio $CO/(CO + CO_2)$

x = carbon atoms in fuel formula, $C_x H_y$...

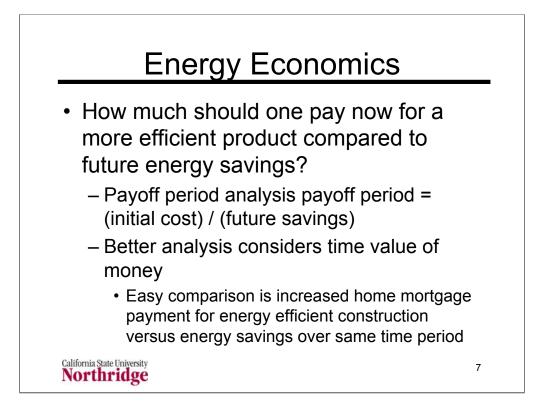
 Q_c = heat of combustion (Btu/lb_m or kJ/kg)

∆h_{CO} 282,990 kJ/kgmol = 121,665 Btu/lbmol

M_{fuel} is combustible fuel molar mass Ib_m/Ibmol or kg/kmol

The heat of combustion can be taken as either the higher or lower heating values. Use of the higher heating value will result in a lower combustion efficiency.

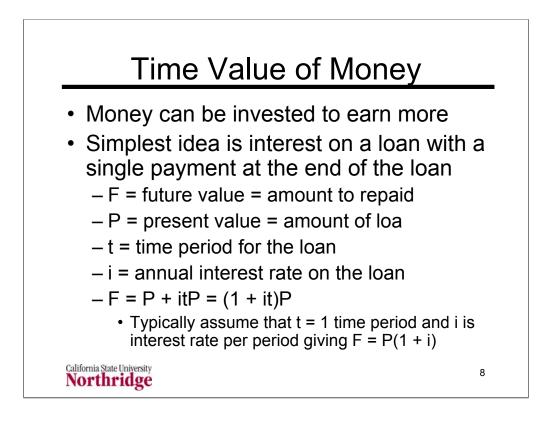
If $c_{p,air}$ is assumed constant the integral can be simply replaced by $c_{p,air}(T_{out} - T_{in})$. Note that if $T_{out} = T_{in}$ and x = 0 the combustion efficiency is 100% by this formula.



The straightforward way to analyze economic decision on energy technology is called the payoff period. For example, assume you are comparing two similar refrigerators, one of which costs \$400 more than the other and the label says that the more efficient refrigerator will use 500 kWh per year less than the less efficient. If your electricity cost is 10 cents per kWh, this will be a savings of \$50/year. So an initial cost increment of \$400 would be recouped in (\$400) / (\$50/year) = 8 years. You could make a decision on which refrigerator to by depending on whether or not you would be happy with this time period.

This payback analysis does not account for the time value of money. If you were to borrow money for the purchase you could compare the difference in your monthly payments for each refrigerator to the expected cost savings of (\$50/year) / (12 months/year) = \$4.17 / month. If difference in the monthly payment for the more efficient (and more expensive) refrigerator was more than this, it would not pay to buy it. Here you are accounting for the time value of money since you are comparing two payments at the same point in time.

In analyzing the time value of money one has to also consider the effects of inflation. Although your electricity now costs 10 cents per kWh, future inflation may increase the cost. If you were considering purchasing a hybrid car, what would you assume for the future cost of gasoline that you would use to determine your fuel savings.

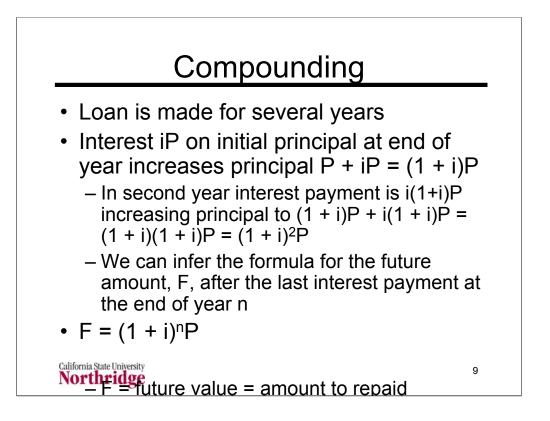


The key component of engineering economic analysis is recognizing the time value of money. Most of use are familiar with this through the interest charged on loans or credit cards. In investment, companies look for a desired rate of return, which is similar to an interest rate.

Interest rates are usually expressed as a percentage per unit time period. For example, many credit cards have an interest rate of 1.5% per month. Home loans are usually expressed as a percent per year.

Typically formulas for interest are expressed If the time period is one month, then the interest rate must be in units of 1/month. If t = 1 period the time is not explicitly shown in the equation and we write F = P(1 + i). This is the form used in almost all economic analyses. However, it depends on having the correct time units for interest rate that correspond to one time period.

If the money is used for more than one time period of the loan, and during the second time period additional interest is paid not only on the original amount but also on the original interest payment, the loan is said to be compounded. Most analysis of the time value of money consider the effects of compounding which we will consider next.



In engineering economic analysis one recognizes the time value of money and compounding. The basic argument is that a company can invest its money in the company to produce future profits or it could make other investments that might return more income. For investments, companies look for a desired rate of return, which is similar to an interest rate.

A key component of interest is compounding. Even if there is only a single payment at the end of the loan, the interest is quoted as an annual interest. In addition, the interest is applied each year (or more frequently). Future interest is based on the original loan amount plus the already incurred interest. For example, a loan of \$10,000 with a 10% per year interest rate would have the following interest accumulations each year:

Year 0: \$10,000

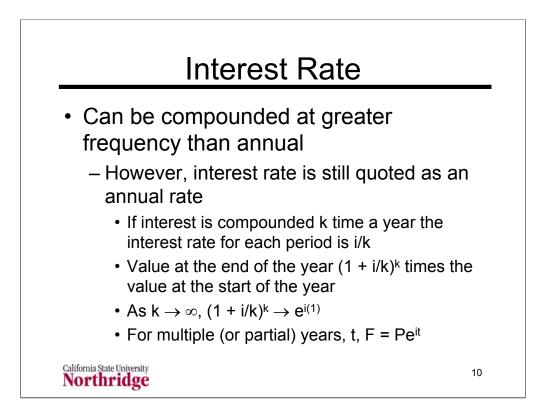
Year 1: \$10,000 + 10%(\$10,000) = \$11,000

Year 2: \$11,000 + 10%(\$11,000) = \$12,100

Year 3: \$12,100 + 10%(\$12,100) = \$13,310

Without compounding the interest of 10% per year for three years would be \$3,000; with compounding, the total interest is \$3,310. Note that the formula shown above predicts this result: $10,000(1 + 0.1)^3 = 10,000(1.331) = 13,310$.

Note also that interest rates in percentages must be converted to decimal fractions before being used in interest formulas.



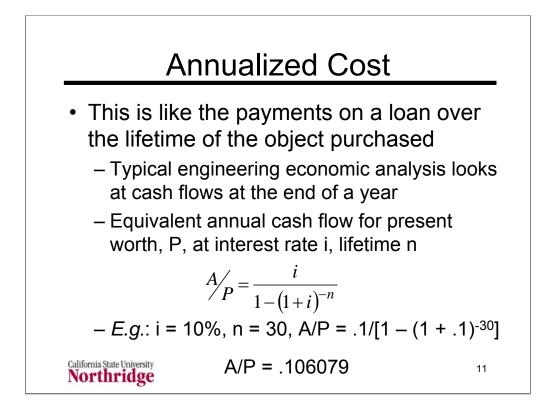
Loans can be compounded at a frequency greater than an annual frequency. Home mortgages and credit card debt are compounded monthly. The ultimate compounding rate is instantaneous. In the limit as the number of compounding periods approaches infinity, the formula for the future worth becomes exponential: $F = Pe^{it}$.

Consider an interest rate of 6% per year. If there is no compounding during the year the value at the end of the year is 1.06 times the value at the start. If there is semi annual compounding, the value at the end of the year is $(1 + .06/2)^2 = 1.0609$ times the value at the start of the year. Continuing the progression we have the following results for the end/start ratio as a function of compounding periods:

3	1.06121	10	1.06146
4	1.06136	20	1.061741
6	1.06152	100	1.061817

We see that even with 10 compounding periods we are quickly approaching the instantaneous value of $e^{i(1)} = e^{0.06} = 1.061837$

When compounding is used we speak of the nominal annual rate (here 6%) and the effective annual rate, which depends on the number of compounding periods. The table above shows, for example, that quarterly compounding gives an effective annual rate of 6.136%.



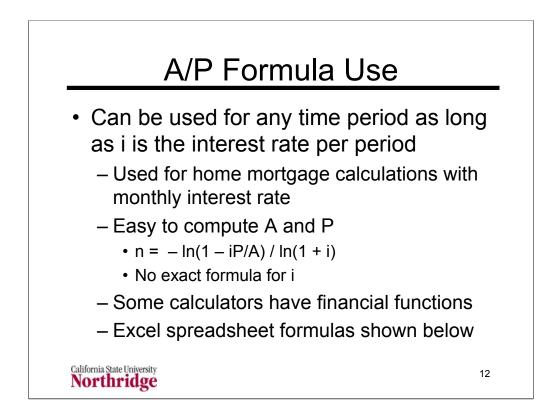
The ratio A/P can be found in tables in textbooks on engineering economics and is present on many calculators and in the Excel function pmt(interest rate, number of periods, present value). To compute the A/P ratio a value of -1 must be entered for the present value; the pmt function has additional arguments that can be ignored for the purposes of the A/P formula.

This formula allows one to compute the annual costs of an operation such as an electric power plant, including the annual operating and maintenance costs and fuel costs, plus the annualized cost of the initial plant construction.

This formula is sometimes called the capital recovery factor. In applications of the inverse of this formula, finding the present worth of a series of future payments, the interest rate is sometimes called the discount rate. This meaning is that an individual or company would "discount" future cash flows to compute their present value.

The analysis of projects with an initial cost and recurring costs must consider the time value of money. This can be done by computing the present worth, P, of the annual recurring costs, A, by the formula: $P = A[1 - (1 + i)^{-n}]/i$. This value is then added to the initial cost to obtain the total cost at present value.

An alternative approach is to convert the initial cost to an equivalent set of annual costs and add the annual cost to the recurring cost to get the total cost for each year over the life of the project.



Computations with the A/P formula can be done in Excel using the following functions. Note that all these formulas assume that A and P have opposite signs. If P represents money you receive it is positive and the value of A will be negative. If P represents money you loan, it will be negative, and the value of A will be positive.

The function **PMT(i, N, P)** computes the value of A. To obtain a positive value for the result A of the PMT function, a minus sign is placed before the present value P in the function.

The function **NPER(i, A, P)** computes the value of N (Remember to use opposite signs for A and P.)

The function **RATE(N, A, P)** computes the periodic interest rate, i, (Remember to use opposite signs for A and P.)

For a non-uniform series of payments at uniform time intervals the Excel function **NPV(i, values)** computes the present value of the payments. In this formula **i** is the interest rate and **values** is a range of cells that contains the set of different payments. The present value is computed at one time period before the first payment in the values cells.

See Excel documentation for using the following functions: **IRR** computes the internal rate of return on a series of payments at uniform time intervals, but different amounts; **XIRR** and **XNPV** are similar to the IRR and NPV functions except that they consider non-uniform time intervals.

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Example Problem

 You have been offered to purchase a photovoltaic solar collector for your house. The collector costs \$35,730, but LADWP will give you a credit of \$14,573 and you can get a Federal tax credit of \$10,719. The expected electricity savings is \$720 per year over a lifetime of 25 years. What is the effective interest rate on this solar collector?

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We can use the formula for the A/P ratio, where P is the initial cost and A is the annual savings. For n = 25 years and i as the annual interest rate the formula gives:

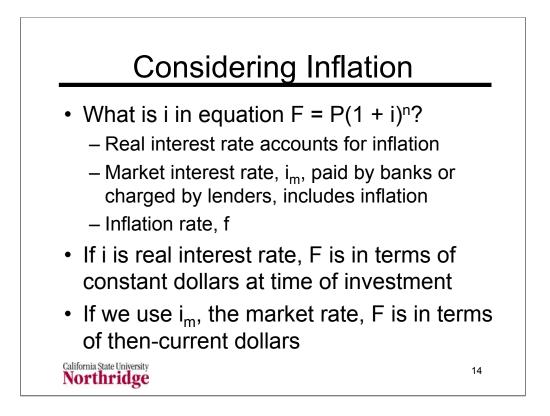
$$A_{P} = \frac{i}{1 - (1 + i)^{-n}} \qquad \$720/yr / \$10498 = \frac{0.068584}{yr} = \frac{i}{1 - (1 + i)^{-25}}$$

Solve by trial and error or calculator/spreadsheet that computes interest rate. Find "interest rate" is 4.66%.

What if there were no rebates or tax credits, what would the effective interest rate be?

$$A_{P} = \frac{i}{1 - (1 + i)^{-n}} \qquad \$720/yr / \$35730 = \frac{0.020151}{yr} = \frac{i}{1 - (1 + i)^{-25}}$$

Again using a trial-and-error solution or having a formula in a calculator/spreadsheet gives an interest rate of -4.7015%.

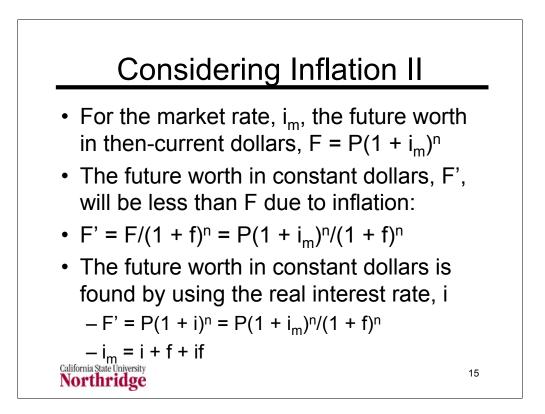


We are all aware that inflation reduces the purchasing power of money over time. One thousand dollars in year zero has a purchasing power of 1000/(1.03) = 970.87 after one year; after two years it is = $970.87/1.03 = 1000/(1.03)^2 = 942.60$, *etc.* In general the purchasing power of a year zero dollar is $1/(1 + f)^n$ of the original dollar.

The actual price that we pay includes the effect of inflation; this approach is said to use current dollars. When we use this accounting we want to consider the market interest rate that includes the effects of inflation.

The real interest rate, which does not consider the effect of inflation is used for internal economic analyses. In this case we say that we are using constant dollars (as opposed to currency that loses its value due to inflation.).. It is useful for comparing options, but it will not give results that match future marketplace prices that change due to inflation.

The decision of which analysis to use (a) market interest rate and current dollars or (b) real interest rate and constant dollars, is a matter of choice, convention, or specifications determined by some government agency or standard-setting body.



The formulas on this chart are based on using discrete formulas. Starting with an initial present worth, P, the market interest rate, i_m (the actual rate you would get from a bank or pay to a lender), would product a future worth, $F = P(1 + i_m)^n$, in then current dollars. This is how many dollars it would produce, but what is the purchasing power of these future dollars compared to the purchasing power they would have if they existed at the time the process started?

The future F dollars would only be worth F' dollars using a constant dollar measure. The relation between F and F' is given by the inflation rate, f. If we had F' dollars at the start of the investment (when we really had P dollars) the inflation rate would produce $F = F'(1 + f)^n$ dollars (called constant or non-inflated dollars.

Combining these results gives $F' = F/(1 + f)^n = P(1 + i_m)^n/(1 + f)^n$. The real interest rate, i, produces these same F' dollars by the usual formula for future worth: $F' = P(1 + i)^n$. Comparing the two equations for F' gives: $F' = P(1 + i)^n = P(1 + i_m)^n/(1 + f)^n$

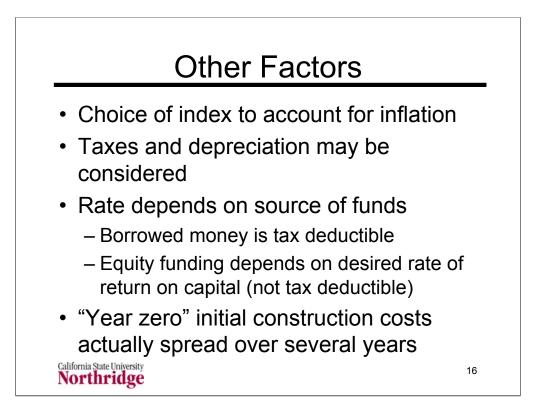
$$P(4 + 1) P(4 + 0) P(4 + 1) (4 + 0) P(4 + 1)$$

 $P(1 + i)^{n} (1 + f)^{n} = P[(1 + i) (1 + f)]^{n} = P[1 + i + f + if]^{n} = P(1 + i_{m})^{n}$

Comparing the last two equations above shows that the market interest rate is the following combination of the actual interest rate and the inflation rate.

 $i_m = i + f + if$

This is often approximated by ignoring the last term, which is a reasonable approximation if both i and f are small.

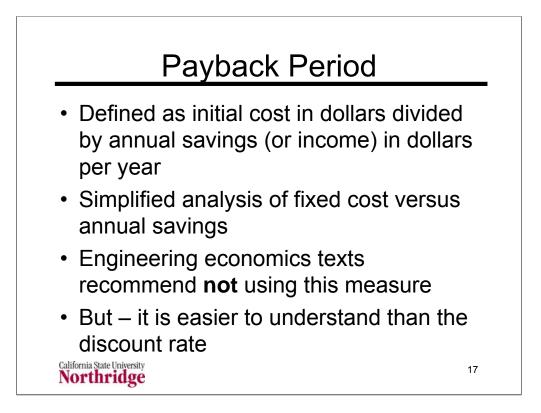


Current year dollars account for inflation, typically using the consumer price index. See http://www.bls.gov/cpi/ (last accessed February 7, 2007) for the consumer price index. This is a general index. Other possible inflation indices are available for specific items. For example, the inflation rate for fuel costs may be different from that for the general consumer price index.

The application of the previous formula for A/P and adding in a constant annual cost for maintenance, operations, and supplies is equivalent to doing the analysis in constant dollars. (Assuming a constant annual cost for these items ignores inflation.) In this case the correct interest rate to use in the real interest rate.

Advanced considerations include sources of cash and the tax implications of those sources. Analyses that consider tax also consider the impact of depreciation on the project accounting.

The example on pages 216-217 (with details on page 245 and 246) shows the application of inflation to the cost analysis of a power plant. (In that analysis the factor of 100 converts dollars to cents and the factor of 8766 is the average hours per year for a four year period. The last two homework problems ask you to apply this analysis to a combined-cycle gas turbine power plant and a solar photovoltaic collector. You can compare your results to those in the text for a nuclear power plant (light-water reactor).



If you are choosing between two possibilities in the purchase of a new appliance, with a cost difference D, between the two and an estimated annual savings S for the more expensive appliance. It is easy to compute the payback period D/S and then make a decision if the savings to you are worth it.

In a formal engineering economic analysis you would have to decide what a reasonable discount (or interest) rate, i, you would want to earn on the extra initial cost, D. You would also have to estimate how long you would own the appliance. You could then compute the present worth of the savings, P, from the reciprocal of the A/P formula:

$$P_A = \frac{1 - (1 + i)^{-n}}{i}$$

For example if you were to keep the appliance for ten years and you wanted a discount rate of 3% the value of P/A would be 8.53 so the initial cost difference, D, would have to be less than 8.53 times the annual savings S. If the initial cost difference were exactly this figure (D = 8.53S) then the payback period, D/S, would be 8.53 years. If you wanted a 6% discount rate, your cost difference, D, would have to be less than 6.14 times the savings. If you kept the appliance for only three years your would want D < 2.8S for a 3% discount rate and D < 2.58S for a 10% rate.