

# Engineering Economics Fundamentals

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Mechanical Engineering 483

## ***Alternative Energy Engineering II***

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### **Reading**

Tonight and next Monday:

Notes on engineering economics

Next two classes: No assigned reading

### **Homework**

Third homework assignment due next Wednesday

## Today's Class

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- Review last class
- Modeling general fuels
- Air/fuel ratio and  $\lambda$  = relative ratio
- Products for complete combustion
- Combustion efficiency
- Time value of money
  - Present and future worth
  - Series of payments

The material in the lectures for tonight will be a review for students who have completed a course in engineering economic analysis such as the MSE 304, Engineering Economy, course required of all engineering majors at CSUN.

The material covered in this lecture will summarize the important points of this material and provide equations that can be used for simple computations.

The key concept is the time value of money. Money that could be used to purchase advanced energy technology to provide future savings on fuel costs could also be invested at some interest rate. A purely economic decision to purchase the new energy technology is based on a comparison of the savings from that technology with the return from some alternative investment for the purchase price.

## Basic Combustion Analysis

- General fuel formula:  $C_xH_yS_zO_wN_v$
- $x$ ,  $y$ ,  $z$ ,  $w$ , and  $v$  from ultimate analysis or analysis of gas mixtures
- Mineral matter included in ultimate analysis represented as %Ash or %MM
- $M_{\text{fuel}} = 12.0107x + 1.00794y + 32.065z + 15.9994w + 14.0067v$
- $m_{\text{fuel}} = M_{\text{fuel}} / (1 - \%MM)$

For ultimate analysis  $x = \text{wt}\%C/12.0107$ ,  $y = \text{wt}\%H/1.00794$ ,  $z = \text{t}\%S/32.065$ ,  $w = \text{wt}\%O/16.0004$ ,  $v = \text{wt}\%N/14.0067$

For mixture of gases where species  $k$  has mole fraction  $\omega_k$ ,  $x_k$  C atoms,  $y_k$  H atoms, etc. compute  $x$  and  $y$  for fuel formula as follows. (Similar formulas apply for other atoms in fuel "molecule".)

$$x = \sum_{\text{species}} \omega_k x_k \qquad y = \sum_{\text{species}} \omega_k y_k$$

When an ultimate analysis is used to compute  $x$ ,  $y$ ,  $z$ ,  $w$ , and  $v$ , the value of  $M_{\text{fuel}}$  is 100 mass units. When a gas analysis is used to compute  $x$ ,  $y$ ,  $z$ ,  $w$ , and  $v$  for the mixture,  $M_{\text{fuel}}$  will be the mean molar mass of the mixture.

$M_{\text{fuel}}$  represents the combustible part of the fuel. The actual fuel mass is given by  $m_{\text{fuel}}$ .

## Combustion Air

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- $A = x + y/4 + z - w/2 =$  stoichiometric moles  $O_2$ /mole fuel
- Actual  $O_2$ /Stoichiometric  $O_2 = \lambda$
- Air/fuel ratio =  $m_{air}/m_{fuel} = 138.28\lambda A/m_{fuel}$
- $C_xH_yS_zO_wN_v + \lambda A(O_2 + 3.77 N_2) \rightarrow xCO_2 + (y/2)H_2O + zSO_2 + (\lambda - 1)AO_2 + 3.77\lambda A + v/2)N_2$
- Can relate  $\lambda$  to fraction of  $O_2$  in dry exhaust (see notes page)

The stoichiometric  $O_2$  requirement is the minimum amount of  $O_2$  for complete combustion:  $x$  moles of  $O_2$  are required to convert  $C_x$  to  $xCO_2$ ;  $y/4$  moles of  $O_2$  are required to convert  $C_y$  to  $(y/2)H_2O$ ;  $z$  moles of  $O_2$  are required to convert  $S_z$  to  $zSO_2$ ; the  $O_w$  in the fuel supplies  $w/2$  moles of  $O_2$  needed for combustion.

Combustion processes usually have more oxygen than the stoichiometric requirement. The ratio,  $\lambda$ , is known as the relative air/fuel ratio.

Exhaust gas measurements remove the water in the exhaust to avoid contamination in the sampling system and analyzers. The resulting exhaust is called dry exhaust. Measurements are typically made in terms of this dry exhaust stream. The equations that relate  $\lambda$  to the (dry) exhaust oxygen concentration are copied below:

$$\frac{\% O_2|_{dry}}{100} = \frac{(\lambda - 1)A}{x + 4.77\lambda A - A + z + \frac{v}{2}}$$

$$\lambda = \frac{A + \frac{\% O_2|_{dry}}{100} \left[ x - A + z + \frac{v}{2} \right]}{A \left( 1 - 4.77 \frac{\% O_2|_{dry}}{100} \right)}$$

## Emission Rates

- Often stated as pollutant mass per unit heat input from fuel

- Equation used:  $E_i = \rho_{i,d} F_d \frac{20.9}{20.9 - \%O_{2,d}}$

- Compute  $\rho_{i,d} = y_{i,d} M_i P_{std} / R_u T_{std}$

- $F_d$  is dry exhaust volume/heat input
  - Use default values of compute by equation

$$F_d = \frac{K(K_C \%C + K_H \%H + K_O \%O + K_S \%S + K_N \%N)}{Q_c}$$

Some values of  $F_d$  from <http://www.epa.gov/ttn/emc/promgate/m-19.pdf> (accessed February 6, 2007) are shown below

Fuel	dscm/J	dscf/MMBtu
Bituminous Coal	$2.63 \times 10^{-7}$	9,780
Lignite	$2.65 \times 10^{-7}$	9,860
Oil	$3.247 \times 10^{-7}$	9,190
Natural Gas	$2.34 \times 10^{-7}$	8,710

Constants for computing  $F_d$  from ultimate analysis (note that 84% C is entered as 84 **not** .84).

The units for the higher heating value ( $Q_c$ ) are Btu/lb<sub>m</sub> or kJ/kg.

$K$  = Conversion factor,  $10^{-3}$  (kJ/J)/(%) [ $10^6$  Btu/million Btu].

$K_C$  = (9.57 scm/kg)/% [(1.53 scf/lb)/%].

$K_H$  = (22.7 scm/kg)/% [(3.64 scf/lb)/%].

$K_N$  = (0.86 scm/kg)/% [(0.14 scf/lb)/%].

$K_O$  = (2.85 scm/kg)/% [(0.46 scf/lb)/%].

$K_S$  = (3.54 scm/kg)/% [(0.57 scf/lb)/%].

$P_{std} = 101.325 \text{ kPa} = 14.696 \text{ psia}$ ,  $R_u = 8.31447 \text{ kPa}\cdot\text{m}^3/\text{kgmol}\cdot\text{K} = 10.7316 \text{ psia}\cdot\text{ft}^3/\text{lbmol}\cdot\text{R}$ ,  $T_{std} = 288.15 \text{ K} = 518.67 \text{ R}$ ,

## Other Equations

- Pollutant mass per unit heat input

$$\frac{m_{CO_2}}{Q_{fuel}} = \frac{3.6642 \text{ wt\% } C}{Q_c \cdot 100} \quad \frac{m_{SO_2}}{Q_{fuel}} = \frac{1.9979 \text{ wt\% } S}{Q_c \cdot 100}$$

- Combustion Efficiency

$$\eta_{comb} = \frac{|q|}{|q|_{max}} = 1 - \frac{\left[1 + \frac{Air}{Fuel}\right]}{Q_c} \int_{T_{in}}^{T_{out}} c_{p,Air} dT' - \frac{xf\Delta h_{CO}}{M_{fuel}Q_c}$$

In the equation for combustion efficiency,

Air/fuel is the air to fuel (mass) ratio

$C_{p,air} = 0.24 \text{ Btu/lb}_m \cdot R = 1.005 \text{ kJ/kg} \cdot K$

f = molar exhaust ratio  $CO/(CO + CO_2)$

x = carbon atoms in fuel formula,  $C_xH_y\dots$

$Q_c$  = heat of combustion (Btu/lb<sub>m</sub> or kJ/kg)

$\Delta h_{CO} = 282,990 \text{ kJ/kgmol} = 121,665 \text{ Btu/lbmol}$

$M_{fuel}$  is combustible fuel molar mass lb<sub>m</sub>/lbmol or kg/kmol

The heat of combustion can be taken as either the higher or lower heating values. Use of the higher heating value will result in a lower combustion efficiency.

If  $c_{p,air}$  is assumed constant the integral can be simply replaced by  $c_{p,air}(T_{out} - T_{in})$ . Note that if  $T_{out} = T_{in}$  and  $x = 0$  the combustion efficiency is 100% by this formula.

## Energy Economics

- How much should one pay now for a more efficient product compared to future energy savings?
  - Payoff period analysis payoff period = (initial cost) / (future savings)
  - Better analysis considers time value of money
    - Easy comparison is increased home mortgage payment for energy efficient construction versus energy savings over same time period

The straightforward way to analyze economic decision on energy technology is called the payoff period. For example, assume you are comparing two similar refrigerators, one of which costs \$400 more than the other and the label says that the more efficient refrigerator will use 500 kWh per year less than the less efficient. If your electricity cost is 10 cents per kWh, this will be a savings of \$50/year. So an initial cost increment of \$400 would be recouped in  $(\$400) / (\$50/\text{year}) = 8$  years. You could make a decision on which refrigerator to buy depending on whether or not you would be happy with this time period.

This payback analysis does not account for the time value of money. If you were to borrow money for the purchase you could compare the difference in your monthly payments for each refrigerator to the expected cost savings of  $(\$50/\text{year}) / (12 \text{ months}/\text{year}) = \$4.17 / \text{month}$ . If difference in the monthly payment for the more efficient (and more expensive) refrigerator was more than this, it would not pay to buy it. Here you are accounting for the time value of money since you are comparing two payments at the same point in time.

In analyzing the time value of money one has to also consider the effects of inflation. Although your electricity now costs 10 cents per kWh, future inflation may increase the cost. If you were considering purchasing a hybrid car, what would you assume for the future cost of gasoline that you would use to determine your fuel savings.

## Time Value of Money

- Money can be invested to earn more
- Simplest idea is interest on a loan with a single payment at the end of the loan
  - $F$  = future value = amount to repaid
  - $P$  = present value = amount of loa
  - $t$  = time period for the loan
  - $i$  = annual interest rate on the loan
  - $F = P + itP = (1 + it)P$ 
    - Typically assume that  $t = 1$  time period and  $i$  is interest rate per period giving  $F = P(1 + i)$

The key component of engineering economic analysis is recognizing the time value of money. Most of use are familiar with this through the interest charged on loans or credit cards. In investment, companies look for a desired rate of return, which is similar to an interest rate.

Interest rates are usually expressed as a percentage per unit time period. For example, many credit cards have an interest rate of 1.5% per month. Home loans are usually expressed as a percent per year.

Typically formulas for interest are expressed If the time period is one month, then the interest rate must be in units of 1/month. If  $t = 1$  period the time is not explicitly shown in the equation and we write  $F = P(1 + i)$ . This is the form used in almost all economic analyses. However, it depends on having the correct time units for interest rate that correspond to one time period.

If the money is used for more than one time period of the loan, and during the second time period additional interest is paid not only on the original amount but also on the original interest payment, the loan is said to be compounded. Most analysis of the time value of money consider the effects of compounding which we will consider next.



## Compounding

- Loan is made for several years
- Interest  $iP$  on initial principal at end of year increases principal  $P + iP = (1 + i)P$ 
  - In second year interest payment is  $i(1+i)P$  increasing principal to  $(1 + i)P + i(1 + i)P = (1 + i)(1 + i)P = (1 + i)^2P$
  - We can infer the formula for the future amount,  $F$ , after the last interest payment at the end of year  $n$
- $F = (1 + i)^n P$

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–  $F$  = future value = amount to repaid

In engineering economic analysis one recognizes the time value of money and compounding. The basic argument is that a company can invest its money in the company to produce future profits or it could make other investments that might return more income. For investments, companies look for a desired rate of return, which is similar to an interest rate.

A key component of interest is compounding. Even if there is only a single payment at the end of the loan, the interest is quoted as an annual interest. In addition, the interest is applied each year (or more frequently). Future interest is based on the original loan amount plus the already incurred interest. For example, a loan of \$10,000 with a 10% per year interest rate would have the following interest accumulations each year:

Year 0: \$10,000

Year 1:  $\$10,000 + 10\%(\$10,000) = \$11,000$

Year 2:  $\$11,000 + 10\%(\$11,000) = \$12,100$

Year 3:  $\$12,100 + 10\%(\$12,100) = \$13,310$

Without compounding the interest of 10% per year for three years would be \$3,000; with compounding, the total interest is \$3,310. Note that the formula shown above predicts this result:  $\$10,000(1 + 0.1)^3 = \$10,000(1.331) = \$13,310$ .

Note also that interest rates in percentages must be converted to decimal fractions before being used in interest formulas.

## Interest Rate

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- Can be compounded at greater frequency than annual
  - However, interest rate is still quoted as an annual rate
    - If interest is compounded  $k$  times a year the interest rate for each period is  $i/k$
    - Value at the end of the year  $(1 + i/k)^k$  times the value at the start of the year
    - As  $k \rightarrow \infty$ ,  $(1 + i/k)^k \rightarrow e^{i(1)}$
    - For multiple (or partial) years,  $t$ ,  $F = Pe^{it}$

Loans can be compounded at a frequency greater than an annual frequency. Home mortgages and credit card debt are compounded monthly. The ultimate compounding rate is instantaneous. In the limit as the number of compounding periods approaches infinity, the formula for the future worth becomes exponential:  $F = Pe^{it}$ .

Consider an interest rate of 6% per year. If there is no compounding during the year the value at the end of the year is 1.06 times the value at the start. If there is semi annual compounding, the value at the end of the year is  $(1 + .06/2)^2 = 1.0609$  times the value at the start of the year. Continuing the progression we have the following results for the end/start ratio as a function of compounding periods:

3	1.06121	10	1.06146
4	1.06136	20	1.061741
6	1.06152	100	1.061817

We see that even with 10 compounding periods we are quickly approaching the instantaneous value of  $e^{i(1)} = e^{0.06} = 1.061837$

When compounding is used we speak of the nominal annual rate (here 6%) and the effective annual rate, which depends on the number of compounding periods. The table above shows, for example, that quarterly compounding gives an effective annual rate of 6.136%.

## Annualized Cost

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- This is like the payments on a loan over the lifetime of the object purchased
  - Typical engineering economic analysis looks at cash flows at the end of a year
  - Equivalent annual cash flow for present worth,  $P$ , at interest rate  $i$ , lifetime  $n$

$$\frac{A}{P} = \frac{i}{1 - (1 + i)^{-n}}$$

- E.g.:  $i = 10\%$ ,  $n = 30$ ,  $A/P = .1/[1 - (1 + .1)^{-30}]$

The ratio  $A/P$  can be found in tables in textbooks on engineering economics and is present on many calculators and in the Excel function `pmt`(interest rate, number of periods, present value). To compute the  $A/P$  ratio a value of -1 must be entered for the present value; the `pmt` function has additional arguments that can be ignored for the purposes of the  $A/P$  formula.

This formula allows one to compute the annual costs of an operation such as an electric power plant, including the annual operating and maintenance costs and fuel costs, plus the annualized cost of the initial plant construction.

This formula is sometimes called the capital recovery factor. In applications of the inverse of this formula, finding the present worth of a series of future payments, the interest rate is sometimes called the discount rate. This meaning is that an individual or company would “discount” future cash flows to compute their present value.

The analysis of projects with an initial cost and recurring costs must consider the time value of money. This can be done by computing the present worth,  $P$ , of the annual recurring costs,  $A$ , by the formula:  $P = A[1 - (1 + i)^{-n}]/i$ . This value is then added to the initial cost to obtain the total cost at present value.

An alternative approach is to convert the initial cost to an equivalent set of annual costs and add the annual cost to the recurring cost to get the total cost for each year over the life of the project.

## A/P Formula Use

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- Can be used for any time period as long as  $i$  is the interest rate per period
  - Used for home mortgage calculations with monthly interest rate
  - Easy to compute  $A$  and  $P$ 
    - $n = -\ln(1 - iP/A) / \ln(1 + i)$
    - No exact formula for  $i$
  - Some calculators have financial functions
  - Excel spreadsheet formulas shown below

Computations with the A/P formula can be done in Excel using the following functions. Note that all these formulas assume that  $A$  and  $P$  have opposite signs. If  $P$  represents money you receive it is positive and the value of  $A$  will be negative. If  $P$  represents money you loan, it will be negative, and the value of  $A$  will be positive.

The function **PMT( $i$ ,  $N$ ,  $P$ )** computes the value of  $A$ . To obtain a positive value for the result  $A$  of the PMT function, a minus sign is placed before the present value  $P$  in the function.

The function **NPER( $i$ ,  $A$ ,  $P$ )** computes the value of  $N$  (Remember to use opposite signs for  $A$  and  $P$ .)

The function **RATE( $N$ ,  $A$ ,  $P$ )** computes the periodic interest rate,  $i$ , (Remember to use opposite signs for  $A$  and  $P$ .)

For a non-uniform series of payments at uniform time intervals the Excel function **NPV( $i$ , values)** computes the present value of the payments. In this formula  $i$  is the interest rate and **values** is a range of cells that contains the set of different payments. The present value is computed at one time period before the first payment in the values cells.

See Excel documentation for using the following functions: **IRR** computes the internal rate of return on a series of payments at uniform time intervals, but different amounts; **XIRR** and **XNPV** are similar to the IRR and NPV functions except that they consider non-uniform time intervals.

## Example Problem

- You have been offered to purchase a photovoltaic solar collector for your house. The collector costs \$35,730, but LADWP will give you a credit of \$14,573 and you can get a Federal tax credit of \$10,719. The expected electricity savings is \$720 per year over a lifetime of 25 years. What is the effective interest rate on this solar collector?

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We can use the formula for the A/P ratio, where P is the initial cost and A is the annual savings. For n = 25 years and i as the annual interest rate the formula gives:

$$\frac{A}{P} = \frac{i}{1 - (1 + i)^{-n}} \quad \$720/\text{yr} / \$10498 = \frac{0.068584}{\text{yr}} = \frac{i}{1 - (1 + i)^{-25}}$$

Solve by trial and error or calculator/spreadsheet that computes interest rate. Find "interest rate" is 4.66%.

What if there were no rebates or tax credits, what would the effective interest rate be?

$$\frac{A}{P} = \frac{i}{1 - (1 + i)^{-n}} \quad \$720/\text{yr} / \$35730 = \frac{0.020151}{\text{yr}} = \frac{i}{1 - (1 + i)^{-25}}$$

Again using a trial-and-error solution or having a formula in a calculator/spreadsheet gives an interest rate of -4.7015%.

## Considering Inflation

- What is  $i$  in equation  $F = P(1 + i)^n$ ?
  - Real interest rate accounts for inflation
  - Market interest rate,  $i_m$ , paid by banks or charged by lenders, includes inflation
  - Inflation rate,  $f$
- If  $i$  is real interest rate,  $F$  is in terms of constant dollars at time of investment
- If we use  $i_m$ , the market rate,  $F$  is in terms of then-current dollars

We are all aware that inflation reduces the purchasing power of money over time. One thousand dollars in year zero has a purchasing power of  $\$1000/(1.03) = \$970.87$  after one year; after two years it is  $= \$970.87/1.03 = \$1000/(1.03)^2 = \$942.60$ , etc. In general the purchasing power of a year zero dollar is  $1/(1 + f)^n$  of the original dollar.

The actual price that we pay includes the effect of inflation; this approach is said to use current dollars. When we use this accounting we want to consider the market interest rate that includes the effects of inflation.

The real interest rate, which does not consider the effect of inflation is used for internal economic analyses. In this case we say that we are using constant dollars (as opposed to currency that loses its value due to inflation.). It is useful for comparing options, but it will not give results that match future marketplace prices that change due to inflation.

The decision of which analysis to use (a) market interest rate and current dollars or (b) real interest rate and constant dollars, is a matter of choice, convention, or specifications determined by some government agency or standard-setting body.

## Considering Inflation II

- For the market rate,  $i_m$ , the future worth in then-current dollars,  $F = P(1 + i_m)^n$
- The future worth in constant dollars,  $F'$ , will be less than  $F$  due to inflation:
- $F' = F/(1 + f)^n = P(1 + i_m)^n/(1 + f)^n$
- The future worth in constant dollars is found by using the real interest rate,  $i$ 
  - $F' = P(1 + i)^n = P(1 + i_m)^n/(1 + f)^n$
  - $i_m = i + f + if$

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The formulas on this chart are based on using discrete formulas. Starting with an initial present worth,  $P$ , the market interest rate,  $i_m$  (the actual rate you would get from a bank or pay to a lender), would produce a future worth,  $F = P(1 + i_m)^n$ , in then current dollars. This is how many dollars it would produce, but what is the purchasing power of these future dollars compared to the purchasing power they would have if they existed at the time the process started?

The future  $F$  dollars would only be worth  $F'$  dollars using a constant dollar measure. The relation between  $F$  and  $F'$  is given by the inflation rate,  $f$ . If we had  $F'$  dollars at the start of the investment (when we really had  $P$  dollars) the inflation rate would produce  $F = F'(1 + f)^n$  dollars (called constant or non-inflated dollars).

Combining these results gives  $F' = F/(1 + f)^n = P(1 + i_m)^n/(1 + f)^n$ . The real interest rate,  $i$ , produces these same  $F'$  dollars by the usual formula for future worth:  $F' = P(1 + i)^n$ . Comparing the two equations for  $F'$  gives:

$$F' = P(1 + i)^n = P(1 + i_m)^n/(1 + f)^n$$

$$P(1 + i)^n (1 + f)^n = P[(1 + i)(1 + f)]^n = P[1 + i + f + if]^n = P(1 + i_m)^n$$

Comparing the last two equations above shows that the market interest rate is the following combination of the actual interest rate and the inflation rate.

$$i_m = i + f + if$$

This is often approximated by ignoring the last term, which is a reasonable approximation if both  $i$  and  $f$  are small..

## Other Factors

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- Choice of index to account for inflation
- Taxes and depreciation may be considered
- Rate depends on source of funds
  - Borrowed money is tax deductible
  - Equity funding depends on desired rate of return on capital (not tax deductible)
- “Year zero” initial construction costs actually spread over several years

Current year dollars account for inflation, typically using the consumer price index. See <http://www.bls.gov/cpi/> (last accessed February 7, 2007) for the consumer price index. This is a general index. Other possible inflation indices are available for specific items. For example, the inflation rate for fuel costs may be different from that for the general consumer price index.

The application of the previous formula for A/P and adding in a constant annual cost for maintenance, operations, and supplies is equivalent to doing the analysis in constant dollars. (Assuming a constant annual cost for these items ignores inflation.) In this case the correct interest rate to use in the real interest rate.

Advanced considerations include sources of cash and the tax implications of those sources. Analyses that consider tax also consider the impact of depreciation on the project accounting.

The example on pages 216-217 (with details on page 245 and 246) shows the application of inflation to the cost analysis of a power plant. (In that analysis the factor of 100 converts dollars to cents and the factor of 8766 is the average hours per year for a four year period. The last two homework problems ask you to apply this analysis to a combined-cycle gas turbine power plant and a solar photovoltaic collector. You can compare your results to those in the text for a nuclear power plant (light-water reactor).



## Payback Period

- Defined as initial cost in dollars divided by annual savings (or income) in dollars per year
- Simplified analysis of fixed cost versus annual savings
- Engineering economics texts recommend **not** using this measure
- But – it is easier to understand than the discount rate

If you are choosing between two possibilities in the purchase of a new appliance, with a cost difference  $D$ , between the two and an estimated annual savings  $S$  for the more expensive appliance. It is easy to compute the payback period  $D/S$  and then make a decision if the savings to you are worth it.

In a formal engineering economic analysis you would have to decide what a reasonable discount (or interest) rate,  $i$ , you would want to earn on the extra initial cost,  $D$ . You would also have to estimate how long you would own the appliance. You could then compute the present worth of the savings,  $P$ , from the reciprocal of the  $A/P$  formula:

$$\frac{P}{A} = \frac{1 - (1 + i)^{-n}}{i}$$

For example if you were to keep the appliance for ten years and you wanted a discount rate of 3% the value of  $P/A$  would be 8.53 so the initial cost difference,  $D$ , would have to be less than 8.53 times the annual savings  $S$ . If the initial cost difference were exactly this figure ( $D = 8.53S$ ) then the payback period,  $D/S$ , would be 8.53 years. If you wanted a 6% discount rate, your cost difference,  $D$ , would have to be less than 6.14 times the savings. If you kept the appliance for only three years you would want  $D < 2.8S$  for a 3% discount rate and  $D < 2.58S$  for a 10% rate.