Review for Final Exam

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Mechanical Engineering 390
Fluid Mechanics

May 6, 2008

Outline

- Properties, statics and manometers
- Bernoulli and continuity equation
- Momentum and energy equations
- Dimensional analysis and similitude
- Pipe flow
- External flow
- Choice of open-channel flow or compressible flow

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The Final

- Tuesday, May 13, 5:30 to 7:30 pm
- Open textbook only
  - No class notes, homework solutions, classmate conversations, text messaging, etc.
- Will be problems similar to the midterm and quizzes
- Think of it as the midterm times 120/75 or four back-to-back quizzes

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But, First a Word About Units

Quantity	SI units	EE units	BG units
Density	kg/m^3	lb$_m$/ft^3	slug/ft^3
Pressure & shear stress	kPa = kN/m^2	1 psi = 1 lb$_f$/in^2 = 144 psf = 144 lb$_f$/ft^2	
Velocity	m/s	ft/s	
Viscosity	N·s/m^2 = kg/m·s	lb$_f$·s/ft^2 = 32.2 lb$_m$/ft·s	lb$_f$·s/ft^2 = slug/ft·s
Specific weight = ρg	N/m^3	lb$_f$/ft^3	
	Tabulated values at standard gravity		

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Density and Related Summary

- **Density:** ρ = mass per unit volume with units of kg/m^3 or slugs/ft^3
- **Specific weight:** $\gamma = \rho g$ with units of N/m^3 or lb$_f$/ft^3 (varies with local g)
- **Specific gravity:** SG = $\rho/\rho_{ref} = \gamma/\gamma_{ref}$
  - Liquid ρ_{ref}: water at 4°C with ρ = 1000 kg/m^3 and γ = 9806.65 N/m^3 or water at 60°F with ρ = 1.94 slugs/ft^3 and γ = 62.4 lb$_f$/ft^3
  - Gas ρ_{ref}: air at 15°C (59°F) with ρ = 1.23 kg/m^3 = 0.00238 slugs/ft^3 and γ = 62.4 lb$_f$/ft^3

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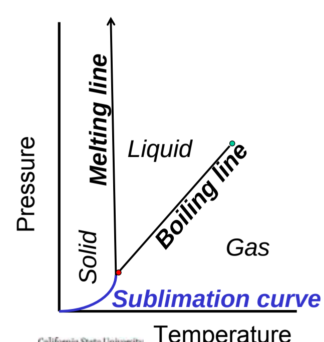
Transitions Between Phases



- For **phase transitions** pressure and temperature are related
- **Vapor pressure** is the pressure at which liquid-vapor transition occurs

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Ideal Gases

- From chemistry: P∀ = nℛT (∀ is volume)
  - n = m / M is the number of moles
    - for mass in kg n is in kilogram moles (kmol); for mass in lb_m, n is in pound moles (lbmol)
  - ℛ = 8.31447 kJ/kmol·K = 10.7316 psia·ft³ / lbmol·R is universal gas constant
  - R = ℛ/M is engineering gas constant that is different for each gas
  - Real gases like ideal gases at low pressures
  - P = nℛT /∀ = (m/M)ℛT /∀ = (m/∀)(ℛ/M)T

P = ρRT

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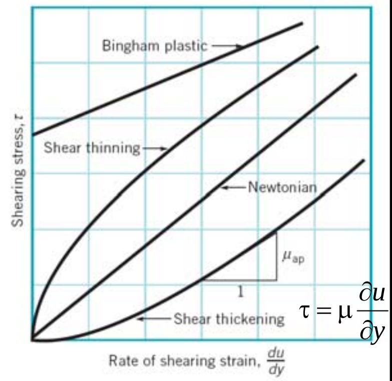
Viscosity

Newtonian and non-Newtonian
Variation of shear stress with rate of shearing strain for several types of fluids, including common non-Newtonian fluids.



$$\tau = \mu \frac{\partial u}{\partial y}$$

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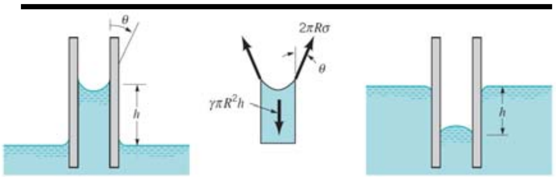
Surface Tension Rise



- Vertical force balance: γπR²h = 2πRσcosθ
  - Surface tension depends on fluid and temperature, wetting angle, θ, depends on fluid and surface

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

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Pressure Relations

- Pressure is a scalar
- The force exerted by a pressure is the same in all directions
- Want to see how pressure changes in a static (nonmoving fluid)
- Look at balance of pressure force and fluid weight over a differential volume element, δxδyδz

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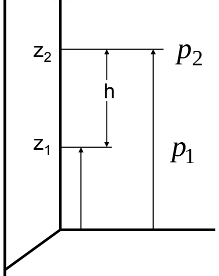
Incompressible Fluid

$$p_2 + \gamma z_2 = p_1 + \gamma z_1$$



- Which pressure is higher?

$$p_1 = p_2 + \gamma(z_2 + z_1)$$

$$p_1 = p_2 + \gamma h > p_2$$

- Pressure increases with depth

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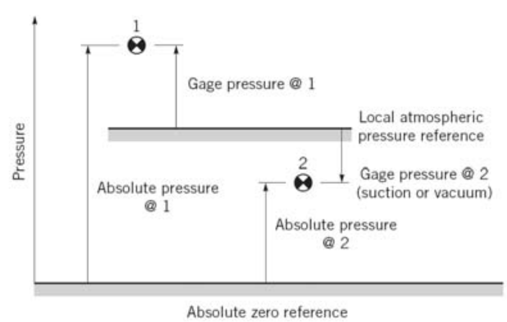
Gage and Absolute Pressure



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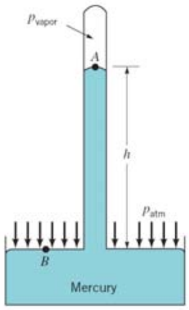
Barometric Pressure



- Mercury barometer used to measure atmospheric pressure
  - Top is evacuated and fills with mercury vapor
  - $P_{atm} = \gamma h + p_{vapor}$
  - p_{vapor} = 0.000023 psia = 0.1586 Pa at 68°F (20°C)
  - h = 760 mm = 29.92 in for standard atmosphere

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Figure 2.8, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

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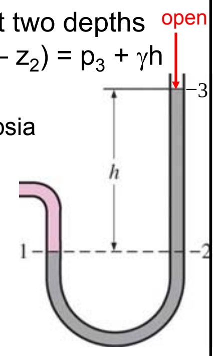
Solving Manometer Problems

- Basic equation: pressures at two depths in same fluid: $p_2 = p_3 + \gamma(z_3 - z_2) = p_3 + \gamma h$
- "Open" means $p = p_{atm}$
  - p_{atm} = 101.325 kPa = 14.696 psia
  - For gage pressure, $p_{atm} = 0$
- Same pressures at same level on two sides of a manometer with same fluid
  - $p_1 = p_2$



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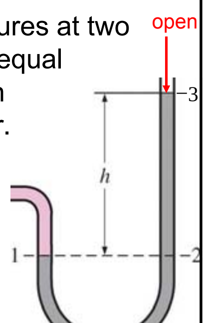
Solving Manometer Problems II

- Write equations for (1) pressures at two depths in same fluid and (2) equal pressures at same level (with at all branches in manometer.
- Eliminate intermediate pressures from equations to get desired ΔP
- Watch units for length, psi or psf, N or kN
  - For gases $\gamma \Delta z \approx 0$



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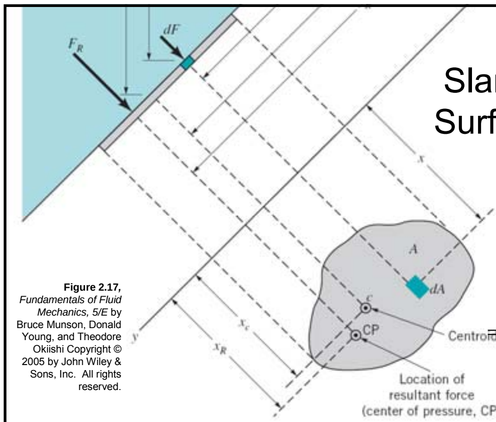
Slanted Surface



$$F_R = \int_A p\,dA = \int_A \gamma h\,dA = \int_A \gamma y \sin\theta\,dA$$

Figure 2.17, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

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Resultant Force

$$F_R = \gamma \sin\theta \int_A y\,dA = \gamma y_c A \sin\theta$$

- Center of pressure, **not y_c**, is ***location*** of resultant force (in diagram $y_c = y_{start} + a/2$)

$$y_{CP} = \frac{I_x}{A y_c} = \frac{I_{xc}}{A y_c} + y_c$$

$I_x = I_{xc} + A y_C^2$ (in diagram $I_{xc} = ba^3/12$ and $A = ab$

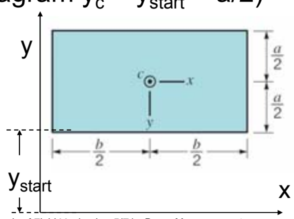

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Figure 2.18(a), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

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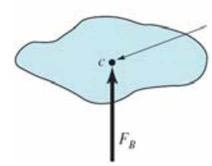
Buoyancy



- Buoyant force, F_B, due to difference in pressure between top and bottom
- $F_B = \gamma_{fluid} V_b$
- Passes through the centroid of the submerged body

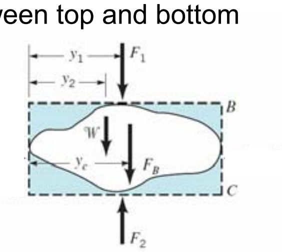


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Figure 2.24(b,c), *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

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Streamlines

- A line everywhere tangent to a the velocity vector is a streamline (s, n) = distance (along, normal to) streamline

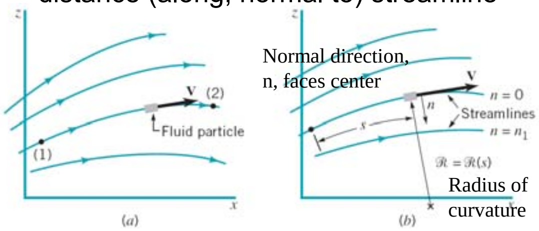

Figure 3.1, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Bernoulli Equation

- Limited to steady, inviscid, streamline flow
- Restriction to steady flow comes from assumption that velocity changes in space, but not with time ($\partial V/\partial t = 0$)
- Have to know ρ-p equation to integrate
  - Simplest relation is constant density

For constant density
$$g(z_2-z_1)+\frac{p_2-p_1}{\rho}+\frac{(V_2^2-V_1^2)}{2}=0$$

Bernoulli Equation

Bernoulli Forms and Dimensions

- Energy – dimensions of L^2/T^2
$$g(z_2-z_1)+\frac{p_2-p_1}{\rho}+\frac{(V_2^2-V_1^2)}{2}=0$$
- Head – dimensions of L
$$z_2-z_1+\frac{p_2-p_1}{\gamma}+\frac{(V_2^2-V_1^2)}{2g}=0$$
- Pressure – dimensions of $F/L^2 = M/LT^2$
$$\gamma(z_2-z_1)+p_2-p_1+\frac{\rho(V_2^2-V_1^2)}{2}=0$$

Forces Normal to Streamline

- Similar force balance involving pressure and weight as forces
- Result is $\gamma\frac{dz}{dn}+\frac{\partial p}{\partial n}+\frac{\rho V^2}{\Re}=0$ ℜ is radius of curvature
- For negligible density $\frac{\partial p}{\partial n}=-\frac{\rho V^2}{\Re}$
- For swirling flows, pressure **decreases** towards the center

Dynamic and Total Pressure

- Bernoulli equation analysis of stagnation point flow shows stagnation pressure, $p_2 = p_1 + \rho V_1^2/2$
- Call $\rho V^2/2$ the dynamic pressure
- Call $p + \rho V^2/2$ the stagnation pressure
- Call pressure, p, static pressure to distinguish this from other pressures
- Total pressure is $p + \gamma z + \rho V^2/2$

Static and Stagnation Pressure

- Parallel streamlines
- h is static pressure at (1)
- H is stagnation pressure at (2)

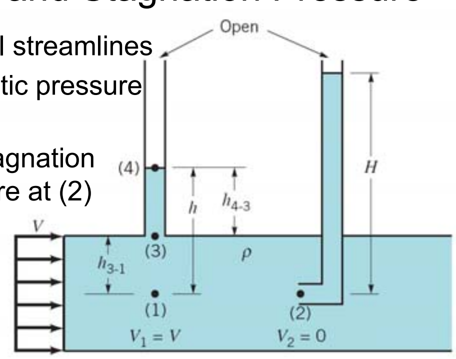

Figure 3.4, *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Incompressible?

- Accurate results for gas flows can be found from incompressible flow equations provided that the Mach number is less than 0.3
  - Ma = 0.3 gives relative error of 0.13% for computation of stagnation pressure
- Results less accurate for Ma > 0.3
- Catastrophic errors for Ma > 1
  - 1145% relative error for Ma = 4
  - Shock wave for Ma > 1 changes equation

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Continuity Equation

$$\left.\frac{dm}{dt}\right|_{\substack{control\\volume}} = \sum \dot{m}_{in} - \sum \dot{m}_{out} = \sum \rho_{in} A_{in} V_{in} - \sum \rho_{out} A_{out} V_{out}$$

- For steady flow dm/dt = 0

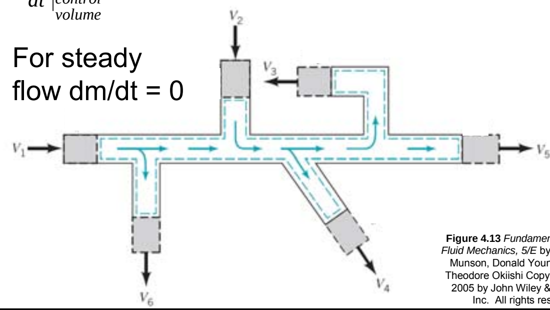

Figure 4.13 *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Cavitation

- Change from small to large velocity ($V_1 << V_2$ will cause large pressure decrease: $p_2 = p_1 + \rho(V_1^2 - V_2^2)$
- Could reduce pressure to vapor pressure of liquid
- This causes vapor bubbles to form and burst exerting pressure on surfaces
- Cavitation can, over time, damage surfaces and must be avoided

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Flowrate Measurement

- Flow meters use basic idea of Bernoulli equation and continuity
  - Continuity: area differences give related velocity differences: $V_2 = V_1A_1/A_2$
  - Bernoulli: velocity differences give pressure differences: $V_2^2 - V_1^2 = 2(p_1 - p_2)/\rho$
  - Combine these equations: $V_2^2 - V_1^2 = (V_1A_1/A_2)^2 - V_1^2 = V_1^2[1 - (A_1/A_2)^2] = 2(p_1 - p_2)/\rho$

$$V_1 = \sqrt{\frac{2(p_1 - p_2)}{\rho\left[1 - (A_2/A_1)^2\right]}}$$

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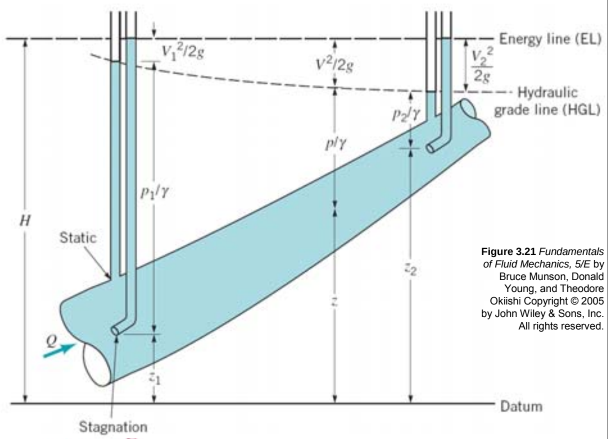

Figure 3.21 *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi

Momentum Conservation

- Apply general conservation equation to each component of momentum
  - $D(mV_k)_{system}/Dt = \Sigma F_k$ (k = x, y, z)

$$\frac{DB_{system}}{Dt} = \frac{\partial B_{cv}}{\partial t} + \sum_{o=1}^{N_{outlets}} \rho_o V_o A_o b_o - \sum_{i=1}^{N_{inlets}} \rho_i V_i A_i b_i$$

$$\frac{\partial (mV_k)_{cv}}{\partial t} + \sum_{o=1}^{N_{outlets}} \dot{m}_o V_{k,o} - \sum_{i=1}^{N_{inlets}} \dot{m} V_{k,i} = \sum F_k$$

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Energy Equations

$$E = \frac{mV^2}{2} + mgz + m\breve{u} \quad \Rightarrow \quad e = \frac{E}{m} = \frac{V^2}{2} + gz + \breve{u}$$

$$\frac{DE_{system}}{Dt} = \frac{\partial E_{cv}}{\partial t} + \sum_{o=1}^{N_{outlets}} \dot{m}_o e_o - \sum_{i=1}^{N_{inlets}} \dot{m}_i e_i = \dot{Q}_{\substack{net \\ in}} + \dot{W}_{\substack{net \\ in}}$$

$$\dot{W}_{\substack{shaft \\ net\ in}} - \sum_{o=1}^{N_{outlets}} \dot{m}_o \frac{P_o}{\rho_o} + \sum_{i=1}^{N_{inlets}} \dot{m}_i \frac{P_i}{\rho_i} = \dot{W}_{\substack{net \\ in}} \qquad \breve{h} = \breve{u} + P/\rho$$

$$\frac{\partial E_{cv}}{\partial t} + \sum_{o=1}^{N_{outlets}} \dot{m}_o \left(\frac{V_o^2}{2} + gz_o + \breve{h}_o \right) - \sum_{i=1}^{N_{inlets}} \dot{m}_i \left(\frac{V_i^2}{2} + gz_i + \breve{h}_i \right) = \dot{Q}_{\substack{net \\ in}} + \dot{W}_{\substack{shaft \\ net\ in}}$$

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Energy and Bernoulli

- Steady flows, one inlet, one outlet

$$\frac{V_o^2}{2g} + \frac{P_o}{\rho g} + z_o = \frac{V_i^2}{2g} + \frac{P_i}{\rho g} + z_i + h_s - h_L$$

- h_L = head loss due to viscous forces, always positive
- h_s = shaft work head, positive for shaft work input negative for output

$$h_s = \frac{\dot{W}_{\substack{shaft \\ net\ in}}}{\dot{m}g} = \frac{\dot{W}_{\substack{shaft \\ net\ in}}}{\rho V A g} = \frac{\dot{W}_{\substack{shaft \\ net\ in}}}{\gamma Q}$$

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Dimensionless Analysis

- Buckingham Pi Theorem (not covered)
  - Can resolve a problem with N variables and D dimensions in to a set of N – D dimensionless variables
- Discussed important dimensionless variables
  - Reynolds number
  - Froude number
  - Loss coefficients ($\Delta p/(\rho V^2/2)$)

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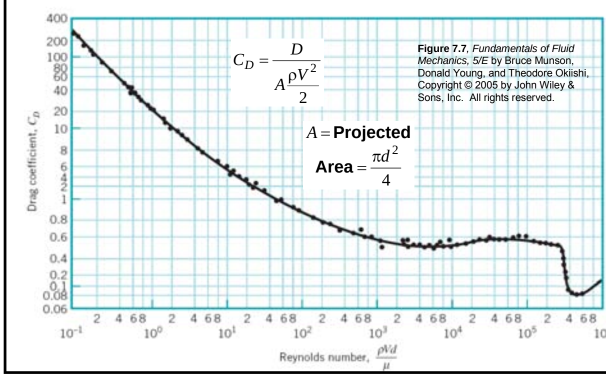
Example: Sphere Drag



Models and Similitude

- In testing models in flow want important dimensionless parameters to be the same in model and prototype
  - Model is tested, prototype is full-size
  - May not be possible to satisfy all
- Model size will be smaller than prototype with length scale that applies to all dimensions

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Renolds Number Similarity

- If Re and C_D are important parameters we want $Re_m = Re_P$

$$\frac{\rho_m V_m L_{c,m}}{\mu_m} = \frac{\rho_p V_p L_{c,p}}{\mu_p} \quad \Rightarrow \quad \frac{\rho_m V_m \mu_p}{\rho_p V_p \mu_m} = \frac{L_{c,p}}{L_{c,m}}$$

- If model test uses the same fluid as the prototype will use, we must have

$$\frac{V_m}{V_p} = \frac{L_{c,p}}{L_{c,m}}$$

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Similarity in Re and C_D

- Re similarity (with same fluid properties)

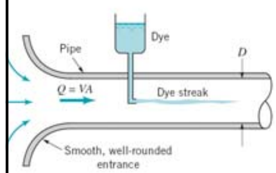
$$\frac{V_m}{V_p} = \frac{L_{c,p}}{L_{c,m}}$$

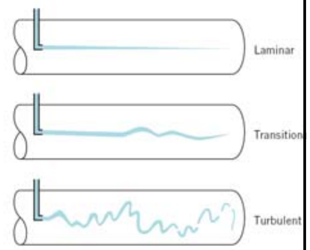
- Drag coefficient similarity (same fluid properties)

$$\frac{F_{D,m}}{\rho_m V_m^2 A_m/2} = \frac{F_{D,p}}{\rho_p V_p^2 A_p/2} \Rightarrow \frac{F_{D,p}}{F_{D,m}} = \frac{\rho_p V_p^2 A_p}{\rho_m V_m^2 A_m} = \frac{V_p^2 L_{c,p}^2}{V_m^2 L_{c,m}^2}$$

- For both Re and C_D similarity drag force is the same in model and prototype

Laminar *vs.* Turbulent Flows





- Laminar flows have smooth layers of fluid
- Turbulent flows have fluctuations

Laminar-Turbulent Transition

- Criterion for transition is based on Reynolds number
- Critical Reynolds number based on flow
- No sharp distinction
  - For pipe flow Re < 2100 is laminar and Re > 4100 to 10,000 is turbulent
  - For flat plate flow transition is at about Re = 500,000

Energy Equation Head Loss

- Energy equation between inlet (1) and outlet (2)

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

- Previous applications allowed us to compute the head loss from other data in this equation
  - Call this the ***measured*** head loss
    - We can compute it, but we have no way of knowing its cause

Head Loss

- Computed head loss in pipe flows from equation like the following

$$h_L = \left(f\frac{\ell}{D} + \sum K_L \right) \frac{V^2}{2g}$$

- Call this the design head loss
  - Allows us to determine the major head loss from empirical relations among Re, f, and ε/D, and minor head loss from emprical K_L
    - Multiple straight pipes have multiple f values

Energy/Head Loss

- Combine energy equation with f and K_L

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - \left(f\frac{\ell}{D} + \sum K_L \right) \frac{V^2}{2g}$$

- Top equation: compute h_L if you know all other terms in the equation
- Bottom equation: find flow properties accounting for calculated head loss

Energy/Head Loss II

$$z_2+\frac{p_2}{\gamma}+\frac{V_2^2}{2g}=z_1+\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+h_s-\left(f\frac{\ell}{D}+\sum K_L\right)\frac{V^2}{2g}$$

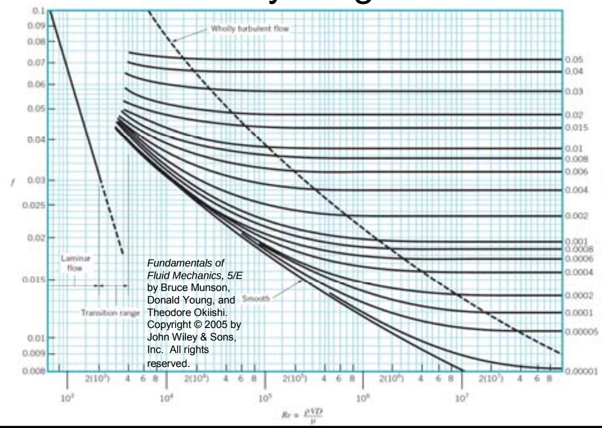
- Simplest case: $z_1 - z_2 = V_1 - V_2 = h_s = 0$

$$p_1-p_2=\gamma\left(f\frac{\ell}{D}+\sum K_L\right)\frac{V^2}{2g}=\left(f\frac{\ell}{D}+\sum K_L\right)\frac{\rho V^2}{2}$$

- Often have $V_2 = V_1$ so head loss is

$$p_1+\gamma z_1-(p_2+\gamma z_2)=\left(f\frac{\ell}{D}+\sum K_L\right)\frac{\rho V^2}{2}$$

Moody Diagram



Moody Equations

- Colebrook equation (turbulent) $\frac{1}{\sqrt{f}}=-2.0\log_{10}\left(\frac{\varepsilon/D}{3.7}+\frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$
- Haaland equation (turbulent) $\frac{1}{\sqrt{f}}\approx-1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}}+\left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$
- Laminar

$$f=\frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2}=\frac{\frac{128\mu\ell Q}{\pi D^4}}{\frac{1}{2}\frac{\ell}{D}\rho V^2}=\frac{\frac{256\mu}{\pi D^3}V\frac{\pi}{4}D^2}{\rho V^2}=\frac{64}{\frac{\rho V D}{\mu}}=\frac{64}{\mathrm{Re}}$$

Pipe Flow Calculations

- Not for partially filled pipes
- Find minor loss coefficients from various tables and charts
- Parabolic laminar velocity profile
  - Adjustment coefficients for momentum and kinetic energy flows terms
- Flat turbulent velocity profile
  - No adjustment terms required
- Non-circular pipes use $D_h = 4A/P$

More Pipe Flow Calculations

- Wall shear stress related to pressure drop: $\Delta p = 2\tau_w\ell/R = 4\tau_w\ell/D$ (for horizontal pipe)
- Entry lengths for developing flows
  - laminar $\frac{\ell_e}{D}=0.06\,\mathrm{Re}$ turbulent $\frac{\ell_e}{D}=4.4\,\mathrm{Re}^{1/6}$
  - $\ell_e \approx 10D$ for turbulent flows
- h_L varies as D^{-4} for laminar flows and D^{-5} for fully turbulent flows with fixed Q

Head Loss Problems

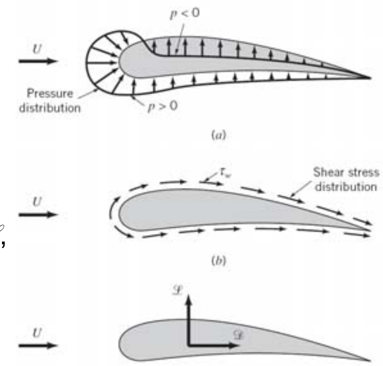
- Find the pressure drop given fluid data, pipe dimensions, ε, and flow (volume flow, mass flow, or velocity)
  - Get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Find ρ and μ for fluid at given T and P
  - Compute $\mathrm{Re} = \rho V D/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar $f = 64/\mathrm{Re}$; Colebrook for turbulent
  - Compute $h_L = f(\ell/D)V^2/2g$

Head Loss Problems II

- Find the diameter for a given head loss given fluid data, ε, and flow (volume flow, mass flow, or velocity)
  - Find ρ and μ for fluid at given T and P
  - Guess D; get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Compute $Re = \rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar $f = 64/Re$; Colebrook for turbulent
  - Compute $h_{L,calculated} = f\,(\ell/D)\,V^2/2g$
  - Iterate on D until $h_{L,calculated} = h_{L,required}$

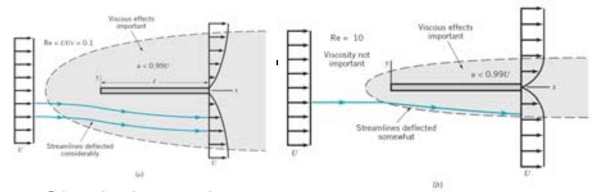
External Flow Forces

- Forces due to pressure distribution and shear stress
- Net result is a lift force, $\mathscr{L}$, and a drag force, $\mathscr{D}$

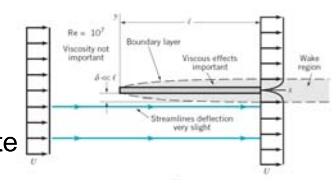


External Flow Characteristics

- Flow over a flat plate is the simplest example
  - $Re = U_\infty \ell/\nu$
    - U_∞ = velocity upstream and far from plate
    - ℓ = length of plate in direction of flow
- Low Reynolds number flow has viscous effects important over wide region
- As Re increases region of significant viscous effects narrows



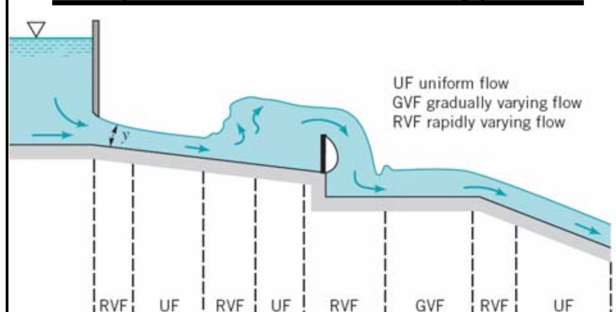
- Shaded area is where viscous effects are important for (a) small, (b) moderate and (c) large Re



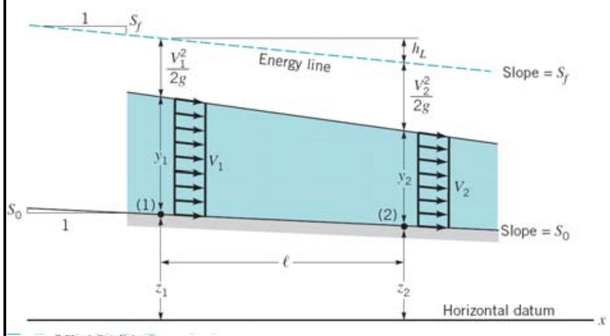
Some Similarities

- Compressible
  - Wave speed for ideal gases $c = (kRT)^{1/2}$
  - Mach number = V/c
  - Flow is subsonic, sonic and supersonic for Ma < 1, Ma = 1, and Ma > 1
  - Can have shock wave from Ma > 1 to Ma < 1 with increase in pressure
- Open channel
  - Wave speed for small y, $c = (gy)^{1/2}$
  - Froude number = V/c
  - Flow is subcritical, critical, and supercritical for Fr < 1, Fr = 1, and Fr > 1
  - Can have hydraulic jump from Fr > 1 to Fr < 1 with increase in height

Open Channel Flow Types



Open Channel Geometry



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Energy Equation

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

- $z_1 - z_2 = S_0\ell$ $h_L = S_f\ell$
- $p_1/\gamma = y_1$ and $p_2/\gamma = y_2$

$$y_2 + \frac{V_2^2}{2g} = E_2 = y_1 + \frac{V_1^2}{2g} + S_0\ell - S_f\ell = E_1 + (S_0 - S_f)\ell$$

- q = flow per unit width = Q/b =Vyb/b = Vy

$$E = y + \frac{V^2}{2g} = y + \frac{(q/y)^2}{2g} = y + \frac{q^2}{2gy^2}$$

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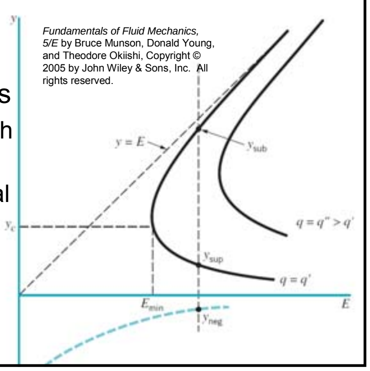
Specific Energy

- Depth versus energy
- Three solutions
  - Negative depth not possible
  - One subcritical solution
  - Other is supercritical



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Chezy-Manning

- Equation for flow velocity considering head loss $V = \frac{R_h^{2/3} S_0^{1/2}}{n}$
- R_h = A/P is hydraulic radius
  - A is cross sectional area of flow
  - Wetted perimeter P does not include area of open surface
  - Coefficient n has units of s·m$^{-1/3}$
    - Multiply result for V by 1.4859 ft$^{1/3}$/m$^{1/3}$ to get V in ft/s when R_h is in ft

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TABLE 10.1
Values of the Manning Coefficient, n (Ref. 6)

Units: s·m$^{-1/3}$

Wetted Perimeter	n	Wetted Perimeter	n
A. Natural channels		*D. Artificially lined channels*	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. Floodplains		Steel, painted	0.014
Pasture, farmland	0.035	Steel, riveted	0.015
Light brush	0.050	Cast iron	0.013
Heavy brush	0.075	Concrete, finished	0.012
Trees	0.15	Concrete, unfinished	0.014
C. Excavated earth channels		Planed wood	0.012
Clean	0.022	Clay tile	0.014
Gravelly	0.025	Brickwork	0.015
Weedy	0.030	Asphalt	0.016
Stony, cobbles	0.035	Corrugated metal	0.022
		Rubble masonry	0.025

Compressible Flow Problems

- Frictionless flow with no heat transfer (isentropic flow)
  - Area must change
- Constant area with heat transfer but no friction (Fanno flow)
- Constant area with friction but no heat transfer (Rayleigh flow)
- Normal shock

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Compressible Equations

- Based on ideal gas
- Use specific heat ratio k (k = 1.4 for air)
  - Equations apply to any substance
  - Charts for air with k = 1.4
- Reference: stagnation point with V = 0
- T_0 = stagnation temperature = T + $V^2/2c_p$
- Stagnation Pressure $p/p_0 = (T/T_0)^{k/(k-1)}$

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More Compressible Flow

- For low pressure ratios flow cannot have velocity greater than sonic velocity without a converging-diverging nozzle
- Pressure ratios less than PR_{crit} will not accelerate flow beyond sonic velocity

$$PR_{crit} = \left(\frac{2}{k+1}\right)^{k/(k+1)} \qquad PR_{crit} = 0.528 \ for\ k = 1.4$$

- See diagrams for isentropic flow and normal shock waves

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Isentropic Flow Equations

- Stagnation point ("0") in large tank where V = 0
  - Variables change with flow through duct based on duct area

$$\rho = \rho_0\left(1+\frac{k-1}{2}Ma^2\right)^{1/(1-k)} \qquad T = T_0\left(1+\frac{k-1}{2}Ma^2\right)^{-1}$$

$$p = p_0\left(1+\frac{k-1}{2}Ma^2\right)^{k/(1-k)} \qquad p^* = p_0\left(\frac{k+1}{2}\right)^{k/(1-k)} \ for\ Ma = 1$$

Different from incompressible flow definition $p_o = p + \rho V^2/2$

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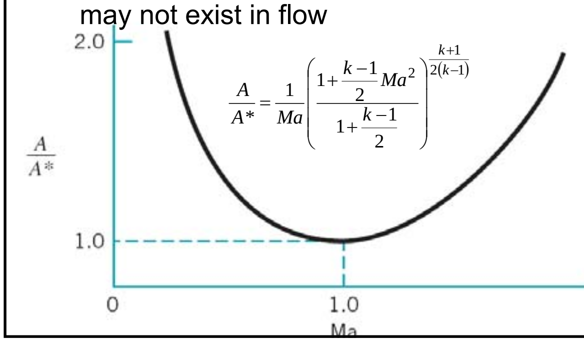
Area Ratio A/A*

- Flow is sonic at A/A* = 1, but area, A*, may not exist in flow



What is A*?

- For an isentropic flow, A* is the area at which the flow will have Ma = 1
- Converging-diverging nozzle with sonic flow at the throat: A* is throat area
- Following a normal shock in the diverging portion of the duct, A/A* can be used to determine Mach number as a function of physical area

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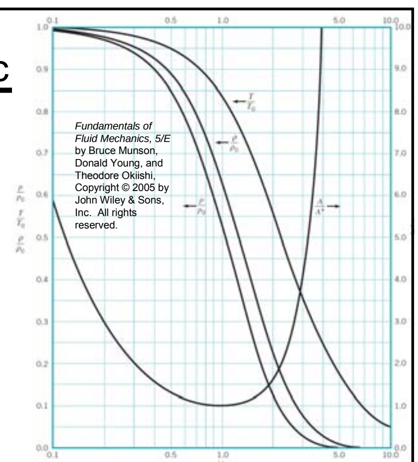
Isentropic

- No heat transfer or friction
- Area must vary
- Sonic flow at throat (minimum area)



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Mass Flow at Exit Pressure

- Problem using isentropic flow equations
  - Flow is sonic only at throat in a converging-diverging duct and only if $p_{out} < p^*$
  - For converging duct, use normal flow relations if $p_{out} > p^*$; $Ma_{out} = 1$ for $p_{out} < p^*$

$$Ma_{out} = \sqrt{\left(\frac{2}{k-1}\right)\left[\left(\frac{p_0}{p_{out}}\right)^{(k-1)/k} - 1\right]} \qquad T_{out} = T_0\left(1 + \frac{k-1}{2}Ma_{out}^2\right)^{-1}$$

$$\dot{m} = \rho AV = \rho AcMa = \frac{p}{RT}AMa\sqrt{kRT} = Ma_{out}\sqrt{\frac{k}{RT_{out}}}pA_{out}$$

California State University Northridge $p = p_{out}$ for $p_{out} > p^*$ or $p_{Ma=1}$ for $p_{out} < p^*$ 67