Compressible Flows
Open-Channel Flows

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Mechanical Engineering 390

Fluid Mechanics

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California State University Northridge

This Week

- Cover two topics and students choose which one they want to follow
  - Open channel flows for CE students
  - Compressible flow for ME students
- Brief overview of both subjects today
- Group work on Thursday will use homework problems in place of additional exercises
  - Quiz May 5 will have choice of problems

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Some Similarities

- Compressible
  - Wave speed for ideal gases $c = (kRT)^{1/2}$
  - Mach number = V/c
  - Flow is subsonic, sonic and supersonic for Ma < 1, Ma = 1, and Ma > 1
  - Can have shock wave from Ma > 1 to Ma < 1 with increase in pressure
- Open channel
  - Wave speed for small y, $c = (gy)^{1/2}$
  - Froude number = V/c
  - Flow is subcritical, critical, and supercritical for Fr < 1, Fr = 1, and Fr > 1
  - Can have hydraulic jump from Fr > 1 to Fr < 1 with increase in height

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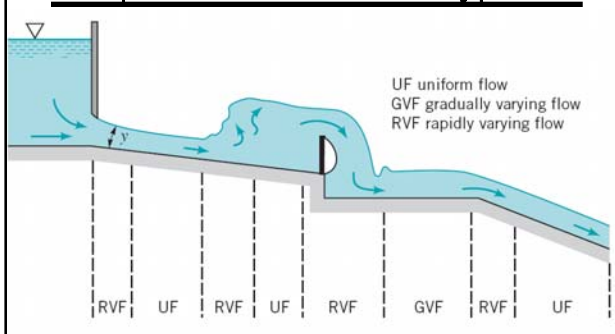
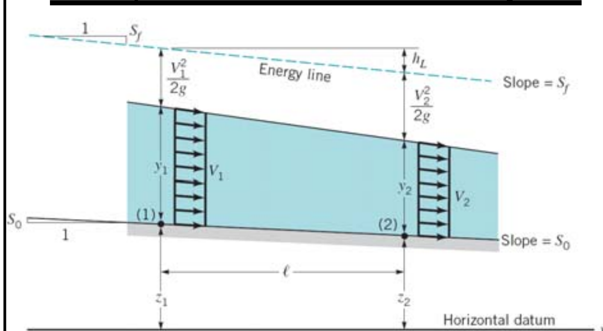
Open Channel Flow Types



California State University Northridge *Fundamentals of Fluid Mechanics, 5/E* by Bruce Munson, Donald Young, and Theodore Okiishi, 4

Open Channel Geometry



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Energy Equation

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

- $z_1 - z_2 = S_0\ell$ $h_L = S_f\ell$
- $p_1/\gamma = y_1$ and $p_2/\gamma = y_2$

$$y_2 + \frac{V_2^2}{2g} = E_2 = y_1 + \frac{V_1^2}{2g} + S_0\ell - S_f\ell = E_1 + (S_0 - S_f)\ell$$

- q = flow per unit width = Q/b = Vyb/b = Vy

$$E = y + \frac{V^2}{2g} = y + \frac{(q/y)^2}{2g} = y + \frac{q^2}{2gy^2}$$

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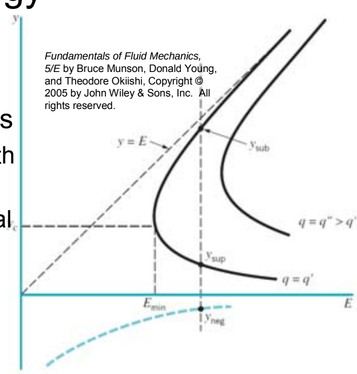
Specific Energy

- Depth versus energy
- Three solutions
  - Negative depth not possible
  - One subcritical solution
  - Other is supercritical



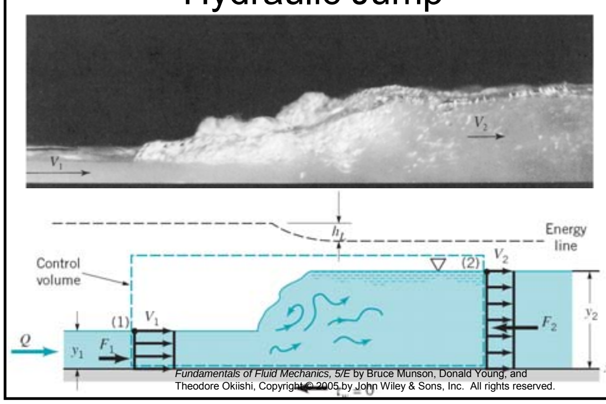
Chezy-Manning

- Equation for flow velocity considering head loss $V = \frac{R_h^{2/3} S_0^{1/2}}{n}$
- R_h = A/P is hydraulic radius
  - A is cross sectional area of flow
  - Wetted perimeter P does not include area of open surface
  - Coefficient n has units of $s \cdot m^{-1/3}$
    - Multiply result for V by 1.4859 $ft^{1/3}/m^{1/3}$ to get V in ft/s when R_h is in ft

TABLE 10.1
Values of the Manning Coefficient, *n* (Ref. 6) **Units: $s \cdot m^{-1/3}$**

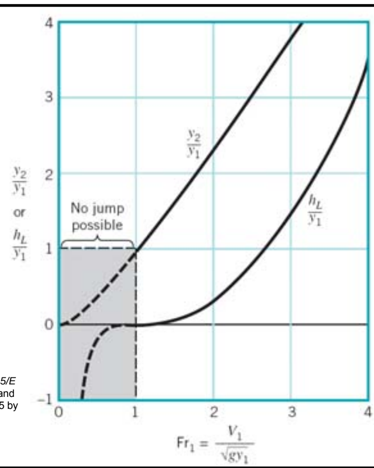
Wetted Perimeter	*n*	Wetted Perimeter	*n*
A. Natural channels		*D. Artificially lined channels*	
Clean and straight	0.030	Glass	0.010
Sluggish with deep pools	0.040	Brass	0.011
Major rivers	0.035	Steel, smooth	0.012
B. Floodplains		Steel, painted	0.014
Pasture, farmland	0.035	Steel, riveted	0.015
Light brush	0.050	Cast iron	0.013
Heavy brush	0.075	Concrete, finished	0.012
Trees	0.15	Concrete, unfinished	0.014
C. Excavated earth channels		Planed wood	0.012
Clean	0.022	Clay tile	0.014
Gravelly	0.025	Brickwork	0.015
Weedy	0.030	Asphalt	0.016
		Corrugated metal	0.022
Stony, cobbles	0.035	Rubble masonry	0.025

Hydraulic Jump



Jump

- Hydraulic jump *vs.* upstream Froude number, Fr_1 (eqns 10.24 and 10.25)



Open Channel Flow Problems

- Determine allowable depths from energy equation for given flow rate
  - q = Q/b substituted for V = Q/(by) = q/y
    - Plot $E = y + (q/y)^2/(2g)$ for given Q and b
    - Find allowed y values for given E
    - Relate y and $E_{min} = (3/2)y_c = (3/2)(q^2/g)^{1/3}$
- Use Chezy-Manning equation to find velocities and flow rates $V = \frac{R_h^{2/3} S_0^{1/2}}{n}$

Compressible Flow Topics

- Frictionless flow with no heat transfer (isentropic flow)
  - Area must change
- Constant area with heat transfer but no friction (Fanno flow)
- Constant area with friction but no heat transfer (Rayleigh flow)
- Normal shock

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Compressible Equations

- Based on ideal gas with sound speed, c^2 = kRT and Mach number Ma = V/c
- Specific heat ratio k = c_p/c_v (= 1.4 for air)
  - Equations apply to any substance
  - Charts for air with k = 1.4
- Reference point is stagnation point with V = 0
- T_0 = stagnation temperature = T + $V^2/2c_p$

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More Compressible Flow

- For low pressure ratios flow cannot have velocity greater than sonic velocity without a converging-diverging nozzle
- Pressure ratios less than PR_{crit} will not accelerate flow beyond sonic velocity

$$PR_{crit} = \left(\frac{2}{k+1}\right)^{k/(k+1)} \qquad PR_{crit} = 0.528 \ \ for\ k = 1.4$$

- See diagrams for isentropic flow and normal shock waves

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Isentropic Flow Equations

- Stagnation point ("0") in large tank where V = 0
  - Variables change with flow through duct based on duct area

$$\rho = \rho_0\left(1+\frac{k-1}{2}Ma^2\right)^{1/(1-k)} \qquad T = T_0\left(1+\frac{k-1}{2}Ma^2\right)^{-1}$$

$$p = p_0\left(1+\frac{k-1}{2}Ma^2\right)^{k/(1-k)} \qquad p^* = p_0\left(\frac{k+1}{2}\right)^{k/(1-k)} \ \ for\ Ma = 1$$

Different from incompressible flow definition $p_o = p + \rho V^2/2$

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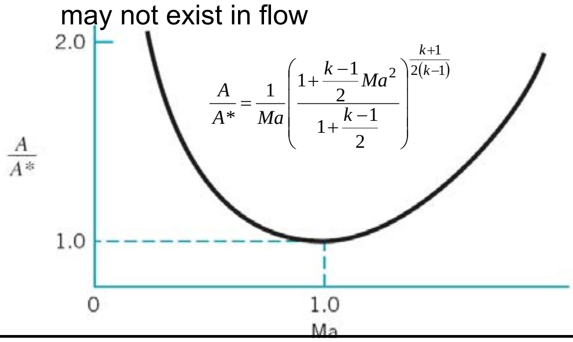
Area Ratio A/A*

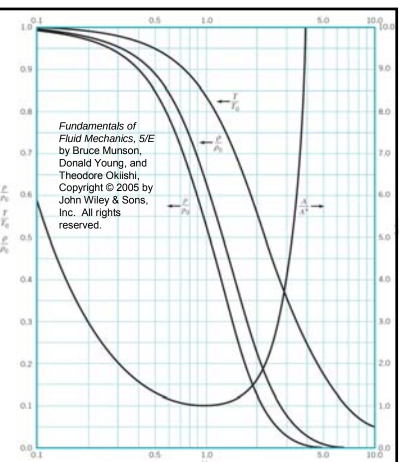
- Flow is sonic at A/A* = 1, but area, A*, may not exist in flow



$$\frac{A}{A^*} = \frac{1}{Ma}\left(\frac{1+\frac{k-1}{2}Ma^2}{1+\frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

Isentropic

- No heat transfer or friction
- Area must vary
- Sonic flow at throat (minimum area)



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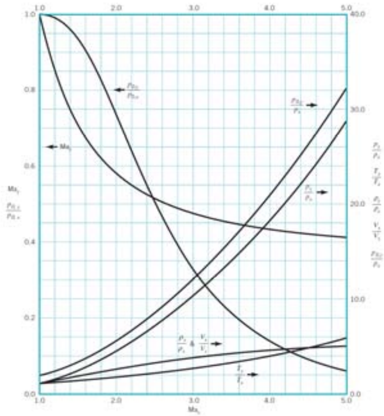
Shock

- Normal shock
  - x is upstream (into shock)
  - y is downstream (leaving shock)



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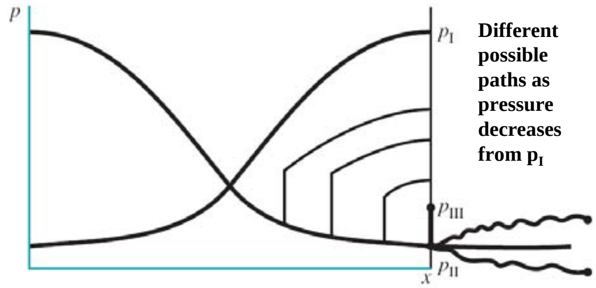
19

Nozzle with Shock



Different possible paths as pressure decreases from p_I

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Mass Flow at Exit Pressure

- Problem using isentropic flow equations
  - Flow is sonic only at throat in a converging-diverging duct and only if $p_{out} < p^*$
  - For converging duct, use normal flow relations if $p_{out} > p^*$; $Ma_{out} = 1$ for $p_{out} < p^*$

$$Ma_{out} = \sqrt{\left(\frac{2}{k-1}\right)\left[\left(\frac{p_0}{p_{out}}\right)^{(k-1)/k} - 1\right]} \qquad T_{out} = T_0\left(1 + \frac{k-1}{2}Ma_{out}^2\right)^{-1}$$

$$\dot{m} = \rho AV = \rho AcMa = \frac{p}{RT}AMa\sqrt{kRT} = \sqrt{\frac{k}{RT}}\,pAMa$$

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21