External Flow

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Mechanical Engineering 390

Fluid Mechanics

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California State University Northridge

Outline

- Review head loss in internal flows
- Definition of lift and drag forces
  - Result of pressure and shear stress forces
- External flows over a flat plate
- Laminar and turbulent regions
- Flows over cylinders and spheres
- Pressure distributions and lift
- Lift coefficients for an airfoil

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Review Energy Equation

- Energy equation between inlet (1) and outlet (2)

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

- Previous applications allowed us to compute the head loss from other data in this equation
  - Call this the ***measured*** head loss
    - We can compute it, but we have no way of knowing its cause

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Review Head Loss

- During the last two weeks we learned to compute the head loss

$$h_L = \left(f\frac{\ell}{D} + \sum K_L \right)\frac{V^2}{2g}$$

- Call this the design head loss
  - Allows us to determine the head loss from empirical relations among Re, f, and ε/D
    - Also have various K_L for minor losses
    - Once we know the head loss, we can find other terms in the energy equation

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Review Energy/Head Loss

- Combine energy equation with f and K_L

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - \left(f\frac{\ell}{D} + \sum K_L \right)\frac{V^2}{2g}$$

- Top equation: compute h_L if you know all other terms in the equation
- Bottom equation: find flow properties accounting for calculated head loss

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Review Energy/Head Loss II

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - \left(f\frac{\ell}{D} + \sum K_L \right)\frac{V^2}{2g}$$

- Simplest case: $z_1 - z_2 = V_1 - V_2 = h_s = 0$

$$p_1 - p_2 = \gamma\left(f\frac{\ell}{D} + \sum K_L \right)\frac{V^2}{2g} = \left(f\frac{\ell}{D} + \sum K_L \right)\frac{\rho V^2}{2}$$

- Often have $V_2 = V_1$ so head loss is

$$p_1 + \gamma z_1 - \left(p_2 + \gamma z_2\right) = \left(f\frac{\ell}{D} + \sum K_L \right)\frac{\rho V^2}{2}$$

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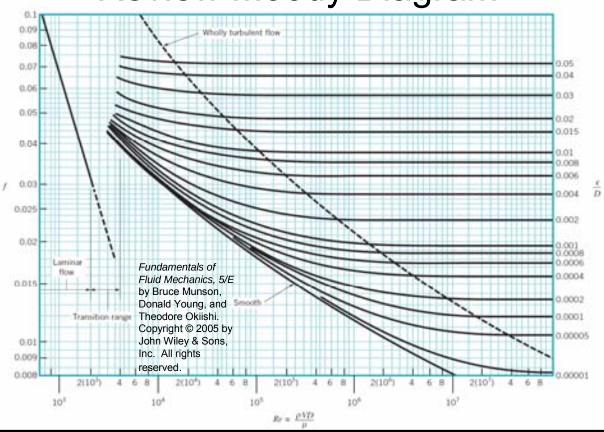
Review Moody Diagram



Review Moody Equations

- Colebrook equation (turbulent) $\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$
- Haaland equation (turbulent) $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left(\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$
- Laminar

$$f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{128\mu\ell Q}{\pi D^4}}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{256\mu}{\pi D^3}V\frac{\pi}{4}D^2}{\rho V^2} = \frac{64}{\frac{\rho V D}{\mu}} = \frac{64}{\text{Re}}$$

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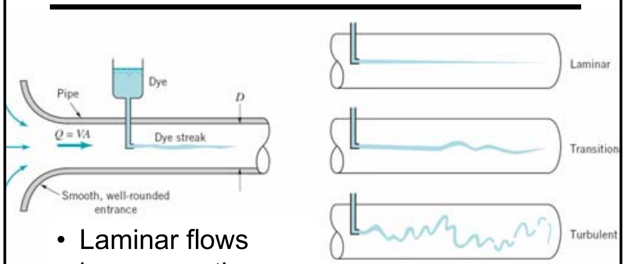
Review Laminar *vs*. Turbulent



- Laminar flows have smooth layers of fluid
- Turbulent flows have fluctuations

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Other Review Items

- Not for partially filled pipes
- Find minor loss coefficients from various tables and charts
- Parabolic laminar velocity profile
  - Adjustment coefficients for momentum and kinetic energy flows terms
- Flat turbulent velocity profile
  - No adjustment terms required
- Non-circular pipes use $D_h = 4A/P$

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Other Review Items II

- Wall shear stress related to pressure drop: $\Delta p = 2\tau_w\ell/R = 4\tau_w\ell/D$ (for horizontal pipe)
- Entry lengths for developing flows
  - laminar $\frac{\ell_e}{D} = 0.06\,\text{Re}$ turbulent $\frac{\ell_e}{D} = 4.4\,\text{Re}^{1/6}$
  - $\ell_e \approx 10D$ for turbulent flows
- h_L varies as D^{-4} for laminar flows and D^{-5} for fully turbulent flows with fixed Q

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Review Head Loss Problems

- Find the pressure drop given fluid data, pipe dimensions, ε, and flow (volume flow, mass flow, or velocity)
  - Get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Find ρ and μ for fluid at given T and P
  - Compute $\text{Re} = \rho V D/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar $f = 64/\text{Re}$; Colebrook for turbulent
  - Compute $h_L = f\,(\ell/D)\,V^2/2g$

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Review Lead Loss Problems II

- Find the diameter for a given head loss given fluid data, ε, and flow (volume flow, mass flow, or velocity)
  - Find ρ and μ for fluid at given T and P
  - Guess D; get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Compute $Re = \rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar $f = 64/Re$; Colebrook for turbulent
  - Compute $h_{L,calculated} = f\,(\ell/D)\,V^2/2g$
  - Iterate on D until $h_{L,calculated} = h_{L,required}$

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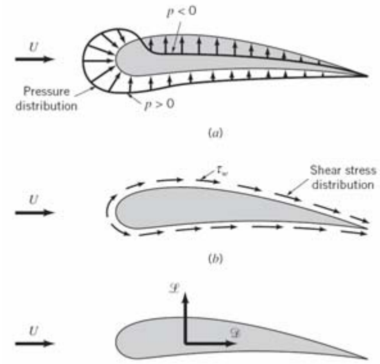
External Flow Forces

- Forces due to pressure distribution and shear stress
- Net result is a lift force, L, and a drag force, D



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External Flow Characteristics

- Flow over a flat plate is the simplest example
  - $Re = U_\infty \ell/\nu$
    - U_∞ = velocity upstream and far from plate
    - ℓ = length of plate in direction of flow
- Low Reynolds number flow has viscous effects important over wide region
- As Re increases region of significant viscous effects narrows

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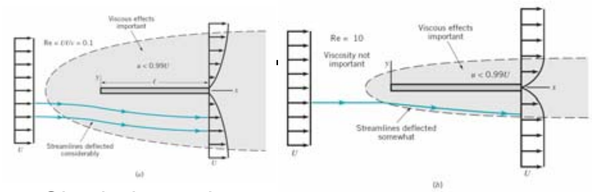
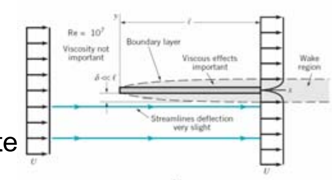


- Shaded area is where viscous effects are important for (a) small, (b) moderate and (c) large Re



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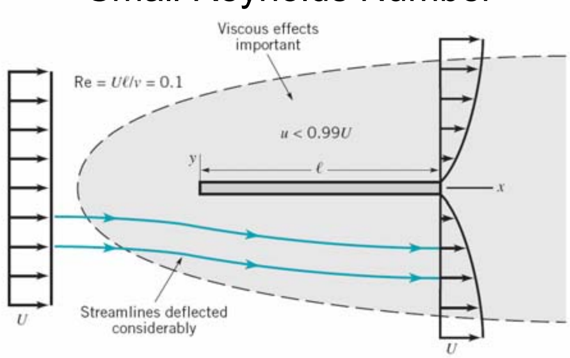
Small Reynolds Number



(a)

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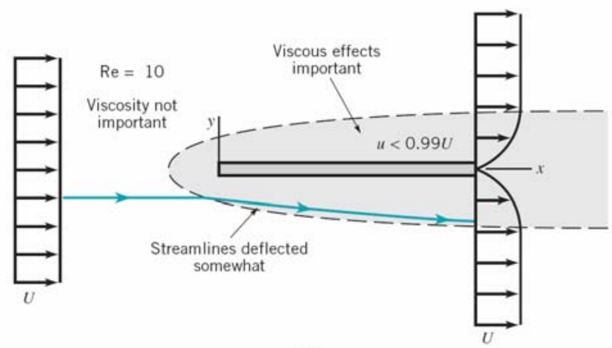
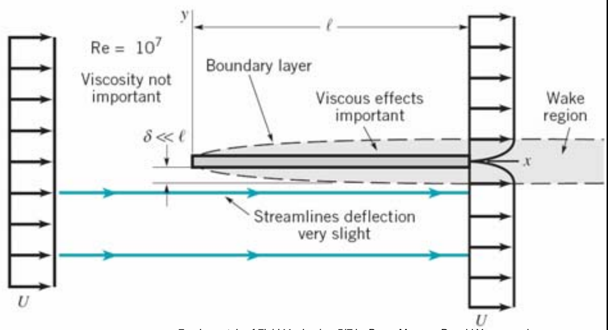
Moderate Reynolds Number



(b)

Large Reynolds Number



Flow Over a Sphere

- Similar effects of different flow regimes as Reynolds number increases
- Drag force due to both pressure drag and friction drag
  - Flat plate only has friction drag
- No simple relation between drag coefficient and Reynolds number
  - Nature of flow changes as Reynolds number increases

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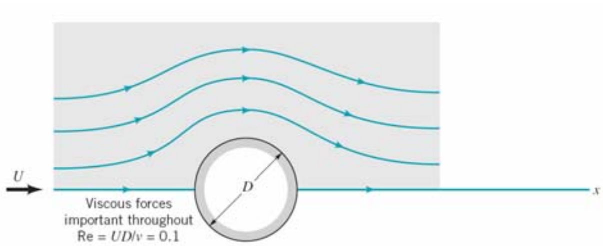
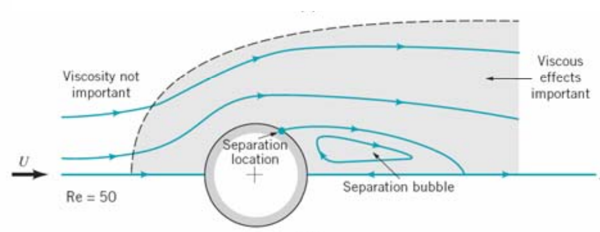
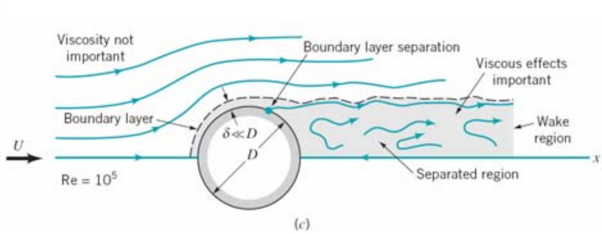
Low Reynolds Number



Moderate Reynolds Number



Large Reynolds Number



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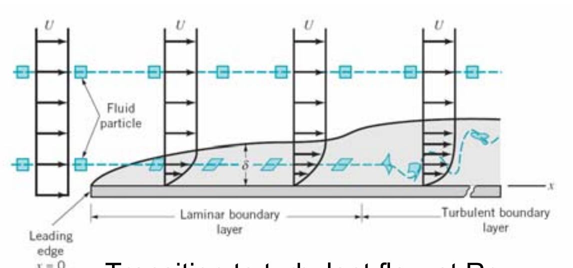
Flat Plate Boundary Layer



- Transition to turbulent flow at $Re_x = U_\infty x/\nu = 2x10^5$ to $5x10^5$

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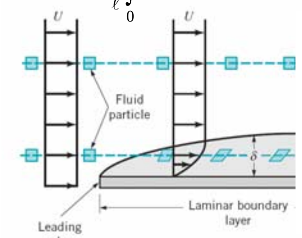
Friction and Drag Coefficients

Laminar: $c_f(x) = \dfrac{\tau_w(x)}{\dfrac{\rho U_\infty^2}{2}} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$ $\quad C_{Df} = \dfrac{\mathrm{D}}{\dfrac{\rho U_\infty^2}{2} b\ell} = \dfrac{1.328}{\sqrt{\mathrm{Re}_\ell}}$

$\mathrm{D} = \ell b \dfrac{1}{\ell}\displaystyle\int_0^\ell \tau_w(x)dx \quad \ell = length \quad b = width \quad \mathrm{Re}_z = \dfrac{U_\infty z}{\nu}$

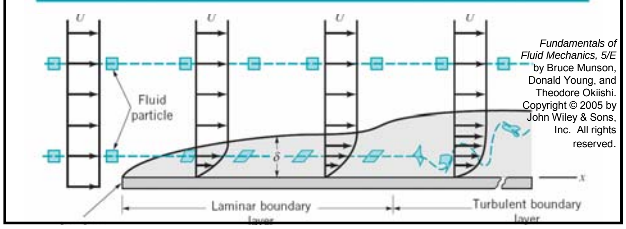


Length of plate is small enough that entire flow is laminar

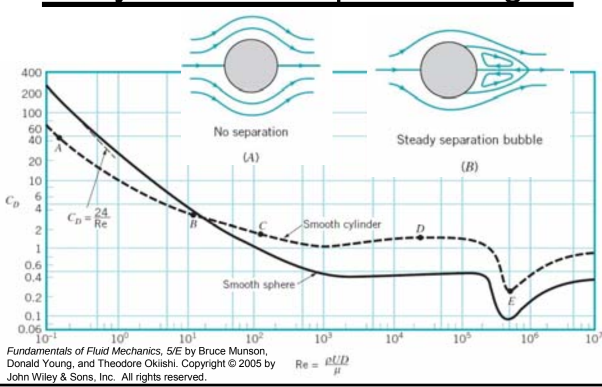
$$\mathrm{Re}_\ell < \mathrm{Re}_{crit} = 5x10^5$$

Flat Plate Drag Coefficients

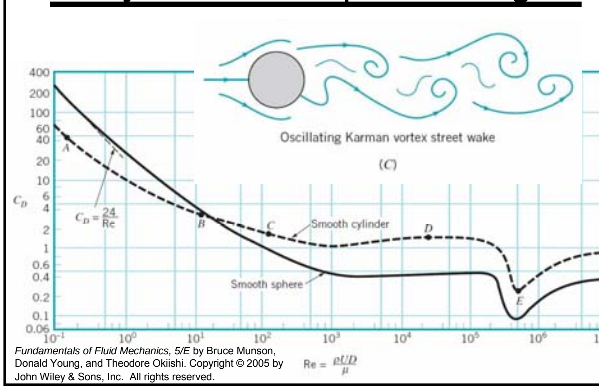
Equation	Flow Conditions
$C_{Df} = 1.328/(\mathrm{Re}_\ell)^{0.5}$	Laminar flow
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58} - 1700/\mathrm{Re}_\ell$	Transitional with $\mathrm{Re}_{xcr} = 5 \times 10^5$
$C_{Df} = 0.455/(\log \mathrm{Re}_\ell)^{2.58}$	Turbulent, smooth plate
$C_{Df} = [1.89 - 1.62 \log(\varepsilon/\ell)]^{-2.5}$	Completely turbulent



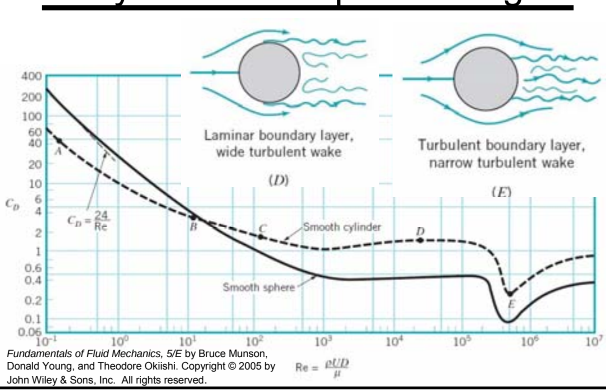
Cylinder and Sphere Drag



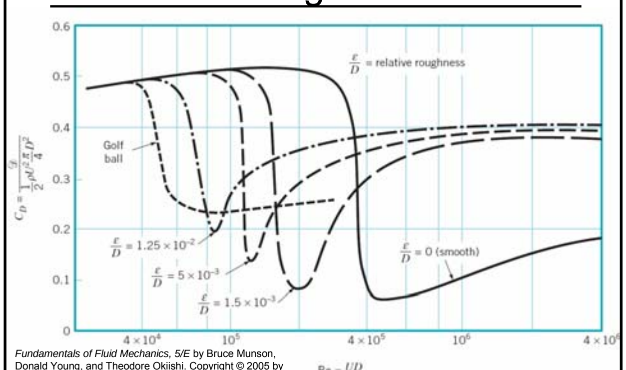
Cylinder and Sphere Drag



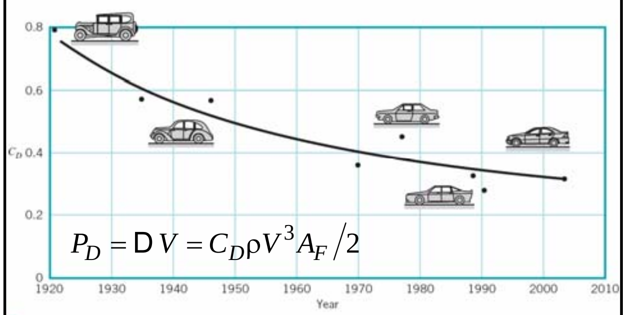
Cylinder and Sphere Drag



Surface Roughness Effect



Automobile Drag Coefficients



$P_D = \mathrm{D}\,V = C_D \rho V^3 A_F / 2$

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Shape	Reference area	Drag coefficient C_D
Parachute	Frontal area $A = \frac{\pi}{4}D^2$	1.4
Porous parabolic dish	Frontal area $A = \frac{\pi}{4}D^2$	Porosity: 0 / 0.2 / 0.5; → 1.42 / 1.20 / 0.82; ← 0.95 / 0.90 / 0.80; Porosity = open area/total area
Average person	Standing Sitting Crouching	$C_DA = 9\ \text{ft}^2$ $C_DA = 6\ \text{ft}^2$ $C_DA = 2.5\ \text{ft}^2$
Fluttering flag	$A = \ell D$	ℓ/D: 1, 2, 3; C_D: 0.07, 0.12, 0.15
Empire State Building	Frontal area	1.4

Figures 9.28 to 9.30 (pp 536-538 give drag coefficients for "objects of interest"

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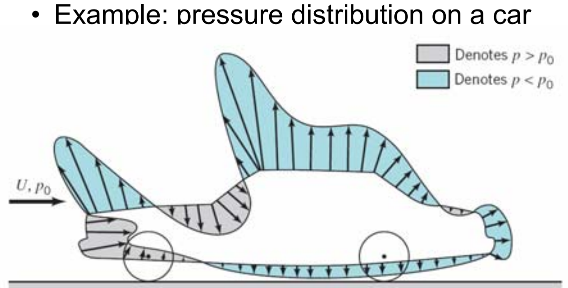
Lift

- Example: pressure distribution on a car



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Lift Coefficient, C_L

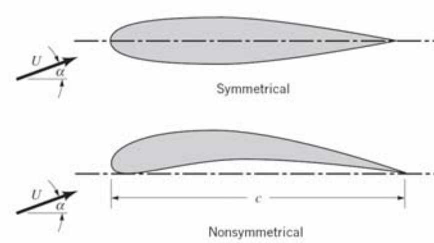
- Defined similarly to drag coefficient

$$C_L = \frac{\mathrm{L}}{A\frac{\rho U_\infty^2}{2}}$$

b is width

c is chord length

α is angle of attack



Area A ≈ bc = (width)(cord length)

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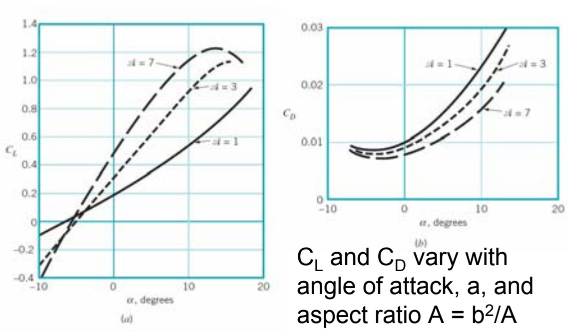
Lift and Drag Coefficients



C_L and C_D vary with angle of attack, a, and aspect ratio A = b^2/A

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