Pipe Flow

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Mechanical Engineering 390

Fluid Mechanics

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California State University Northridge

Quiz Seven Results

- 32 students
- 25 maximum possible
- Average (mean) = 17.3
- Standard deviation = 3.58
- Median = 17
- Grade distribution

8 12 13 13 14 14 15 15 15 15 15
15 15 16 16 17 17 18 19 19 20
20 20 20 20 20 22 22 22 22 22 22

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Quiz Eight Solution

- Find prototype velocity for V_m = 160 mph in 70°F air for various scales and prototype pressures such that Ma < 0.3
- Reynolds number similarity with ideal gas law, $\rho = P/RT$ and μ = function of T only

$$\mathrm{Re}_m = \frac{\rho_m V_m \ell_m}{\mu_m} = \mathrm{Re} = \frac{\rho V \ell}{\mu} \Rightarrow \frac{V_m}{V} = \frac{\rho \ell \mu_m}{\rho_m \ell_m \mu} \Rightarrow V = V_m \frac{\rho_m \ell_m \mu}{\rho \ell \mu_m}$$

$$\frac{V}{V_m} = \frac{\rho_m \ell_m \mu}{\rho \ell \mu_m} = \frac{P_m}{R_m T_m}\frac{RT}{P}\frac{\ell_m \mu}{\ell \mu_m} = \frac{\ell_m}{\ell}\frac{P_m}{P} \Rightarrow V = V_m \frac{\ell_m}{\ell}\frac{P_m}{P}$$

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Quiz Eight Solution II

- Have ℓ_m/ℓ = 1/3 and 1/8 with p_m/p = 1 and 5

$$V = V_m \frac{\ell_m}{\ell}\frac{P_m}{P}$$

- For V_m = 160 mph, we have V = 53.3 mph, 20 mph, 267 mph, and 100 mph
- For T = 70°F, c = 1128 ft/s = 769 mph (Table B.3, p. 762, 30 mph = 44 ft/s
- For Ma = 0.3, V = (0.3)(769 mph) = 231mph
- All speeds except 267 mph speed okay

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Quiz Eight Solution III

- Need equal drag coefficients

$$C_{D,m} = \frac{F_{D,m}}{A_m \frac{\rho_m V_m^2}{2}} = C_D = \frac{F_D}{A\frac{\rho V^2}{2}} \Rightarrow F_D = F_{D,m}\frac{A\rho V^2}{A_m \rho_m V_m^2}$$

$$= F_{D,m}\frac{\ell^2}{\ell_m^2}\frac{\rho}{\rho_m}\frac{V^2}{V_m^2} = F_{D,m}\frac{\ell^2}{\ell_m^2}\frac{P}{RT}\frac{R_m T_m}{P_m}\left(\frac{\ell_m}{\ell}\frac{P_m}{P}\right)^2 = F_{D,m}\frac{\ell^2}{\ell_m^2}\frac{P}{P_m}\left(\frac{\ell_m}{\ell}\frac{P_m}{P}\right)^2 = \frac{P_m}{P}F_{D,m}$$

- When p_m = p, drag forces are the same
- When p_m = 5p prototype drag force is 5 times model drag force

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Quiz Seven Comments

- Energy equation has specific terms for inlet (i) and outlet (o) in various forms

$$\frac{p_o}{\gamma} + \frac{V_o^2}{2g} + z_o = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i + h_s - h_L \qquad \textit{head}$$

$$\frac{p_o}{\rho} + \frac{V_o^2}{2} + gz_o = \frac{p_i}{\rho} + \frac{V_i^2}{2} + gz_i + w_{\substack{shaft\\net\ in}} - loss \qquad \textit{energy}$$

$$\dot{m}\left(\frac{p_o}{\rho} + \frac{V_o^2}{2} + gz_o\right) = \dot{m}\left(\frac{p_i}{\rho} + \frac{V_i^2}{2} + gz_i\right) + \dot{W}_{\substack{shaft\\net\ in}} - P_{Loss} \qquad \textit{power}$$

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Quiz Seven Comments II

- Work outputs are negative
- Vacuum pressures are negative
- Still have problems with units
  - Always convert pressures to lb_f/ft^2
  - Conversion factor: 70.726 psf/in Hg found as γ_{Hg} = 847 lb_f/ft^3 divided by 12 in/ft
- Note conversions for head

$$\dot{W}_{\substack{shaft\\net\ in}} = \dot{m}w_{\substack{shaft\\net\ in}} = \dot{m}gh_s = \gamma Q h_s$$

$$P_{Loss} = \dot{m}(loss) = \dot{m}gh_L = \gamma Q h_L$$

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Outline

- Laminar and turbulent flows
- Developing and fully-developed flows
- Laminar and turbulent velocity profiles: effects on momentum and energy
- Calculating head losses in pipes
  - Major losses from pipe only
  - Minor losses from fittings, valves, *etc*.
- Noncircular ducts

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Piping System

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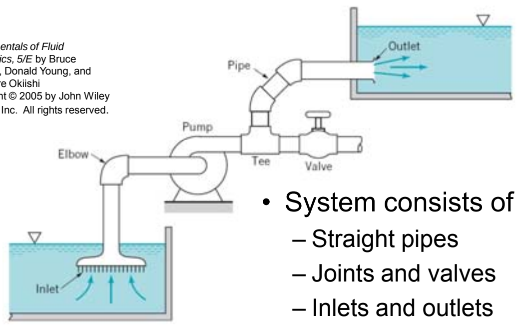

- System consists of
  - Straight pipes
  - Joints and valves
  - Inlets and outlets
  - Work input/output

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What We Want to Do

- Determine losses from friction forces in straight pipes and joints/valves
  - Will be expressed as head loss or "pressure drop" – $h_L = \Delta P/\gamma$
  - Losses in straight pipes are called "major" losses
  - Losses in fittings, joints, valves, etc. are called "minor" losses
  - Minor losses may be greater than major losses in some cases

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Pipe Cross Section

- Most pipes have circular cross section to provide stress resistance
- Main exception is air conditioning ducts
- Consider round pipes first then extend analysis to non-circular cross sections
  - Extension based on using same equations as for circular pipe by defining hydraulic diameter = 4 (area) / (perimeter), which is D for circular cross sections

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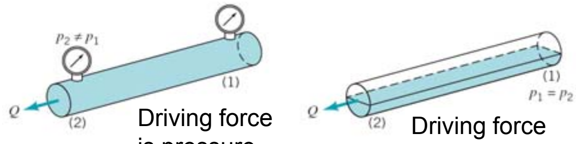
The Pipes are Full

- Consider only flows where the fluid completely fills the pipe
- Partially filled pipes are considered under open-channel flow



Driving force is pressure

Driving force is gravity

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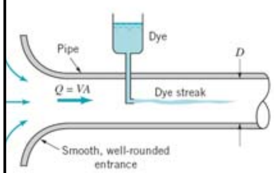
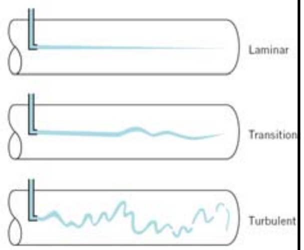
Laminar *vs*. Turbulent Flow





- Laminar flows have smooth layers of fluid
- Turbulent flows have fluctuations

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Laminar *vs*. Turbulent Flow II

- Most flows of engineering interest are turbulent
  - Analysis relies mainly on experimentation guided by dimensional analysis
  - Even advanced computer models, called computational fluid dynamics (CFD) rely on “turbulence models” that have large degree of empiricism
- Can get some (very limited) analytical results for laminar flows

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Laminar *vs*. Turbulent Flow III

- Condition of flow as laminar or turbulent depends on Reynolds number
- For pipe flows
  - Re = $\rho VD/\mu < 2100$ is laminar
  - Re = $\rho VD/\mu > 4000$ is turbulent
  - $2100 < Re < 4000$ is transition flow
- Other flow geometries have different characteristics in Re = $\rho VL_c/\mu$ and different values of Re for laminar and turbulent flow limits

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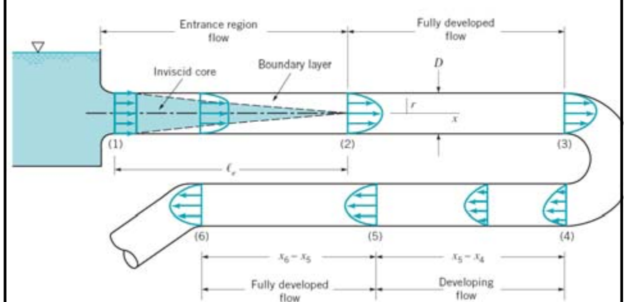
Flow Development



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Developing Flows

- Entrance regions and bends create changing flow patters with different head losses
- Once flow is “fully developed” the head loss is proportional to the distance
- Entrance pressure drop is complex
  - Complete entrance region treated under minor losses
  - Will not treat partial entrance region here

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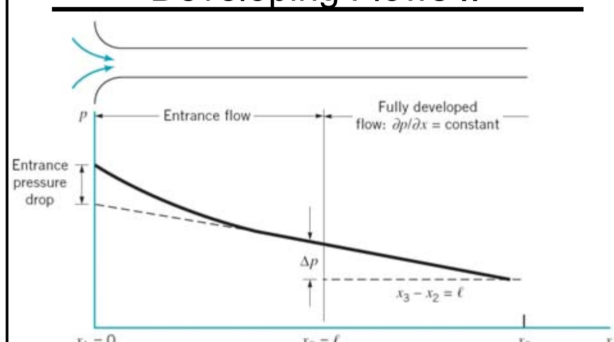
Developing Flows II



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Developing Flows III

- After development region, pressure drop (head loss) is proportional to pipe length
- Equations for entrance region length, ℓ_e
  - Laminar flow: $\dfrac{\ell_e}{D} = 0.06\,\mathrm{Re}$
  - Turbulent flow: $\dfrac{\ell_e}{D} = 4.4\,\mathrm{Re}^{1/6}$
  - Turbulent flow rule of thumb $\ell_e \approx 10D$

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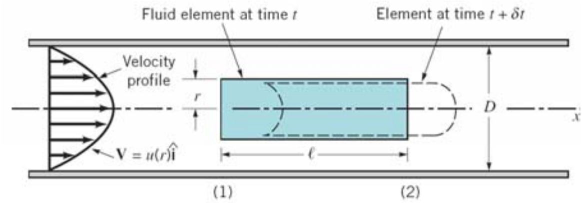
Fluid Element in Pipe Flow



- Look at arbitrary element, with length ℓ, and radius r, in fully developed flow
- What are forces on this element?

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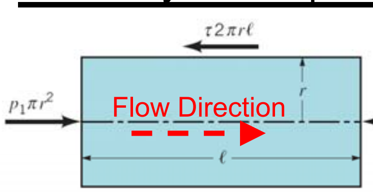
Fully Developed Flow



No change in momentum

$$\sum F_x = \pi r^2 p_1 - \pi r^2 (p_1 - \Delta p) - \tau 2\pi r \ell = 0$$

$$\Delta p = \frac{2\tau\ell}{r}$$

- Pressure drop is due to viscous stresses

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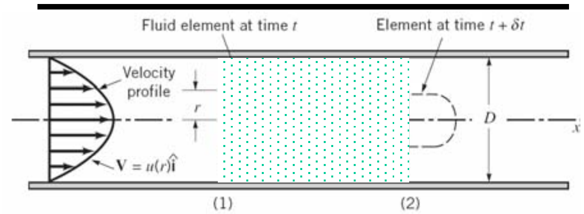
Extend Relation to Wall



- Have $\Delta p = 2\tau\ell/r$ for any r: $0 < r < R = D/2$
- For wall $r = R = D/2$ and $\tau = \tau_w$ = wall shear stress: $\Delta p = 2\tau_w\ell/R = 4\tau_w\ell/D$

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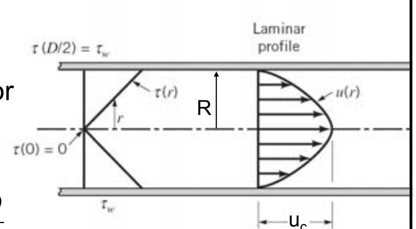
Fully Developed Laminar Flow

- Can get exact equation for pressure drop



$$\Delta p = \frac{128\mu\ell Q}{\pi D^4}$$

- Laminar velocity profile $u = u_c\left[1 - \left(\dfrac{r}{R}\right)^2\right]$

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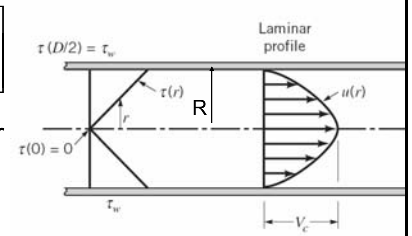
Fully Developed Laminar Flow II

$$u = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]$$



- Laminar shear stress profile found from $\tau = \mu \dfrac{du}{dr}$

$$\tau = \mu\frac{du}{dr} = \mu u_c \frac{2r}{R^2} = \frac{8\mu u_c}{D^2} r$$

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Fully Developed Laminar Flow III

- What is centerline velocity, u_c?

$$Q = VA = V\pi R^2 = \int_A u\,dA = \int_0^R u 2\pi r dr = \int_0^R u_c\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r dr$$

$$Q = 2\pi u_c\left[\int_0^R r dr - \int_0^R \frac{r^3}{R^2}dr\right] = 2\pi u_c\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = 2\pi u_c\frac{R^2}{4}$$

$$Q = \pi u_c\frac{R^2}{2} \quad\Rightarrow\quad u_c = \frac{2Q}{\pi R^2} = \frac{2VA}{\pi R^2} = \frac{2V\pi R^2}{\pi R^2} = 2V$$

Centerline u_c is twice the mean velocity 19

Effect of Velocity Profile

- Momentum and kinetic energy flow for mean velocity, V
  - $Flow_{Momentum} = \dot{m}V = \rho VAV = \rho V^2(\pi R^2)$
  - $Flow_{KE} = \dot{m}V^2/2 = \rho VAV^2/2 = \rho V^3(\pi R^2)/2$
- Accurate representation uses profile

$$Flow_{Momentum} = \int_A \rho u dA u = \int_0^R \rho\left[u_c\left(1-\frac{r^2}{R^2}\right)\right]^2 2\pi r dr = \frac{4}{3}\rho V^2 A$$

$$Flow_{KE} = \int_A \rho u dA\frac{u^2}{2} = \int_0^R \rho\left[u_c\left(1-\frac{r^2}{R^2}\right)\right]^3 2\pi r dr = 2\rho A\frac{V^3}{2}$$

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Turbulent Flow

- For laminar and turbulent flows, the velocity at the wall is zero
  - This is called the no-slip condition
  - Momentum is maximum in the center of the flow and zero at the wall
    - Laminar flows: momentum transport from wall to center is by viscosity, $\tau = \mu du/dr$
    - Turbulent flows: random fluctuations exchange eddies of high momentum from the center with low momentum flow from near-wall regions

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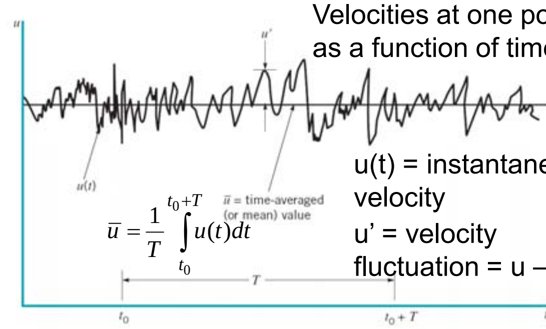
Turbulent Flow Quantities

Velocities at one point as a function of time



$$\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)dt$$

u(t) = instantaneous velocity

u' = velocity fluctuation = u – $\bar{u}$

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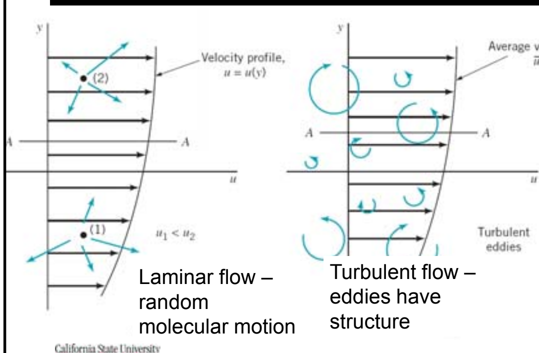
Momentum Exchange



Laminar flow – random molecular motion

Turbulent flow – eddies have structure

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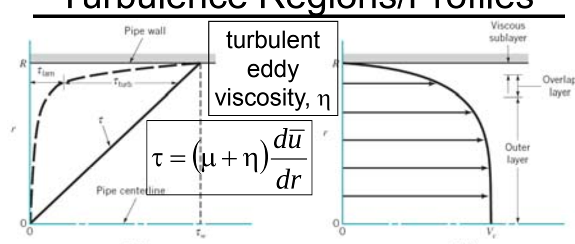
Turbulence Regions/Profiles



$$\tau = (\mu + \eta)\frac{d\bar{u}}{dr}$$

- Very thin viscous sublayer next to wall
  - 0.13% of R = 3 in for H_20 at $\bar{u}$ = 5 ft/s
- Flat velocity profile in center of flow

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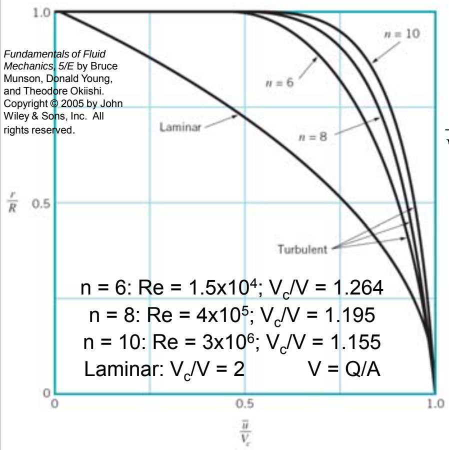

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n = 6: Re = 1.5×10^4; V_c/V = 1.264
n = 8: Re = 4×10^5; V_c/V = 1.195
n = 10: Re = 3×10^6; V_c/V = 1.155
Laminar: V_c/V = 2 V = Q/A

Profile

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

Turbulent velocity profiles with n a function of Reynolds number

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Effect of Velocity Profile

- Analysis similar to one used for laminar flow profile
  - Determine momentum and kinetic energy flow for mean velocity
  - Correction factor multiplies average V results to give integrated u^2 and u^3 values

n	Re	Momentum	KE
6	1.5×10^4	1.027	1.077
8	4×10^5	1.016	1.046
10	3×10^6	1.011	1.031

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Pipe Roughness

- Effect of rough walls on pressure drop may depend on surface roughness of pipe
- Typical roughness values for different materials expressed as roughness length, ε, with units of feet or meters
- Only turbulent flows depend on roughness length, laminar flows do not

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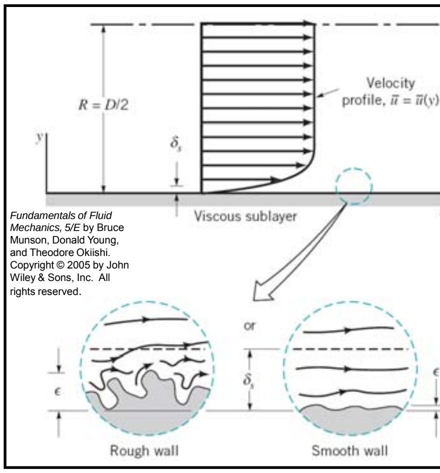

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Pipe roughness effects in viscous sublayer affects pressure drop in turbulent flow

No effect on laminar flow

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TABLE 8.1
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	Feet	Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Use this table (p 433 of text) to find ε

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Energy Equation

- Energy equation between inlet (i) and outlet (o)

$$\frac{p_o}{\gamma} + \frac{V_o^2}{2g} + z_o = \frac{p_i}{\gamma} + \frac{V_i^2}{2g} + z_i + h_s - h_L$$

- Previous applications allowed us to compute the head loss from all other data in this equation
  - Call this the measured head loss
    - We can compute it but do not know its cause

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Pressure Drop/Head Loss

- We now seek a design calculation for h_L
- Use level pipe ($z_1 = z_2$) with constant area ($V_1 = v_2$) and no shaft head ($h_s = 0$)

$$\frac{p_o}{\gamma} + \cancel{\frac{V_o^2}{2g}} + \cancel{z_o} = \frac{p_i}{\gamma} + \cancel{\frac{V_i^2}{2g}} + \cancel{z_i} + \cancel{h_s} - h_L$$

$$h_L = \frac{p_i}{\gamma} - \frac{p_o}{\gamma} = \frac{\Delta p}{\gamma}$$

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Pressure Drop/Head Loss II

- Calculated Δp for $z_1 = z_2$, $V_1 = V_2$, and $h_s = 0$ gives h_L for more general flows
- Will later define friction factor, f, such that $f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} \Rightarrow \Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$
- Will use this to define head loss

$$h_L = \frac{\Delta p}{\gamma} = \frac{\Delta p}{\rho g} = \frac{f\frac{1}{2}\frac{\ell}{D}\rho V^2}{\rho g} = f\frac{\ell}{D}\frac{V^2}{2g}$$

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Head Loss in Pipes

- Dimensional analysis shows that dimensionless pressure drop, $\Delta p/\rho V^2$, is a function of Reynolds number, $\rho VD/\mu$, the ℓ/D ratio and relative roughness, ε/D
- Expressed in terms of friction factor, f

$$f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = f\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

- What is the form of f(Re, ε/D)?

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Moody Diagram

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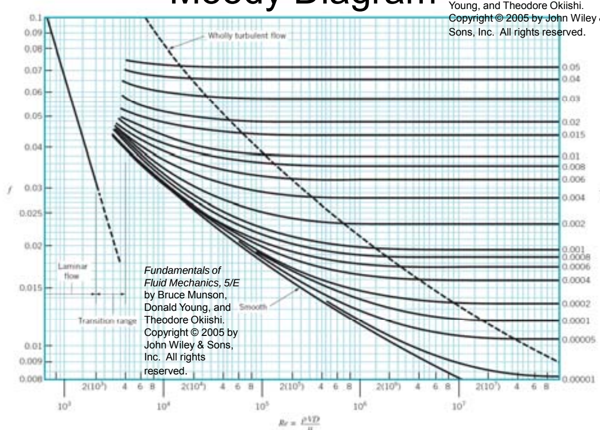

Moody Diagram Equations

- Colebrook equation (turbulent) $\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$
- Haaland equation (turbulent) $\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$
- Laminar

$$f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{128\mu\ell Q}{\pi D^4}}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{256\mu}{\pi D^3}V\frac{\pi}{4}D^2}{\rho V^2} = \frac{64}{\frac{\rho VD}{\mu}} = \frac{64}{\mathrm{Re}}$$

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Wholly Turbulent Flows

- Large Reynolds numbers: f independent of Re depends only on ε/D

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2} \qquad V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} \Rightarrow V^2 = \frac{16}{\pi^2}\frac{Q^2}{D^4}$$

$$\Delta p = f\frac{\ell}{D}\frac{\rho}{2}V^2 = f\frac{\ell}{D}\frac{\rho}{2}\frac{16}{\pi^2}\frac{Q^2}{D^4} = \frac{8f\ell}{\pi^2}\frac{\rho Q^2}{D^5} = \frac{8f\ell}{\pi^2}\frac{\dot{m}^2}{\rho D^5}$$

- Pressure drop varies as D^{-5}
  - Similar to D^{-4} dependence in laminar flow

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Pressure Drop Problems

- Find the pressure drop given fluid data, pipe dimensions, ε, and flow (volume flow, mass flow, or velocity)
  - Get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Find ρ and μ for fluid at given T and P
  - Compute $Re = \rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar f = 64/Re; Colebrook for turbulent
  - Compute $\Delta p = f (\ell/D) \rho V^2/2$

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