Pipe Flow

Larry Caretto
Mechanical Engineering 390

Fluid Mechanics

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California State University Northridge

Outline

- Laminar and turbulent flows
- Developing and fully-developed flows
- Laminar and turbulent velocity profiles: effects on momentum and energy
- Calculating head losses in pipes
  - Major losses from pipe only
  - Minor losses from fittings, valves, *etc*.
- Noncircular ducts

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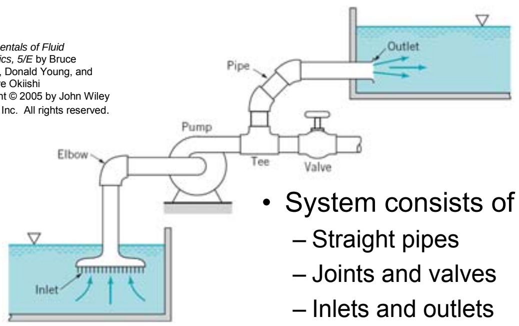
Piping System



- System consists of
  - Straight pipes
  - Joints and valves
  - Inlets and outlets
  - Work input/output

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What We Want to Do

- Determine losses from friction forces in straight pipes and joints/valves
  - Will be expressed as head loss or "pressure drop" $h_L = \Delta P/\gamma$
    - Will show that this is head loss in energy equation if variables other than pressure change
  - Losses in straight pipes are called "major" losses
  - Losses in fittings, joints, valves, etc. are called "minor" losses
  - Minor losses may be greater than major losses in some cases

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Pipe Cross Section

- Most pipes have circular cross section to provide stress resistance
- Main exception is air conditioning ducts
- Consider round pipes first then extend analysis to non-circular cross sections
  - Extension based on using same equations as for circular pipe by defining hydraulic diameter = 4 (area) / (perimeter), which is D for circular cross sections

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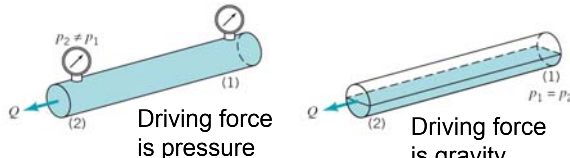
The Pipes are Full

- Consider only flows where the fluid completely fills the pipe
- Partially filled pipes are considered under open-channel flow



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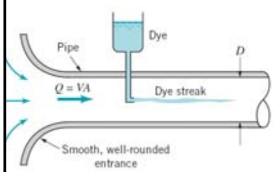
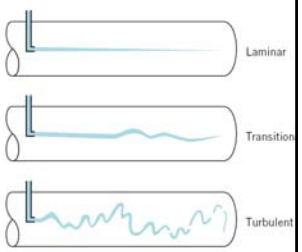
Laminar *vs*. Turbulent Flow





- Laminar flows have smooth layers of fluid
- Turbulent flows have fluctuations

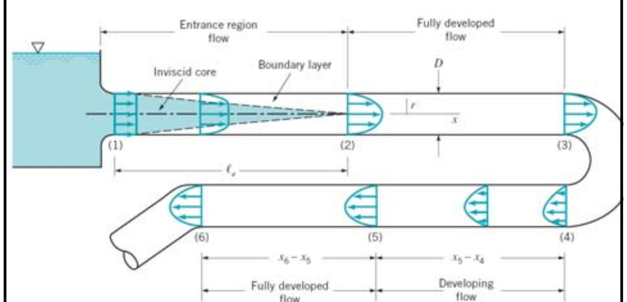
Laminar *vs*. Turbulent Flow II

- Most flows of engineering interest are turbulent
  - Analysis relies mainly on experimentation guided by dimensional analysis
  - Even advanced computer models, called computational fluid dynamics (CFD) rely on "turbulence models" that have large degree of empiricism
- Can get some (very limited) analytical results for laminar flows

Laminar *vs*. Turbulent Flow III

- Condition of flow as laminar or turbulent depends on Reynolds number
- For pipe flows
  - Re = $\rho VD/\mu < 2100$ is laminar
  - Re = $\rho VD/\mu > 4000$ is turbulent
  - 2100 < Re < 4000 is transition flow
- Other flow geometries have different characteristics in Re = $\rho VL_c/\mu$ and different values of Re for laminar and turbulent flow limits

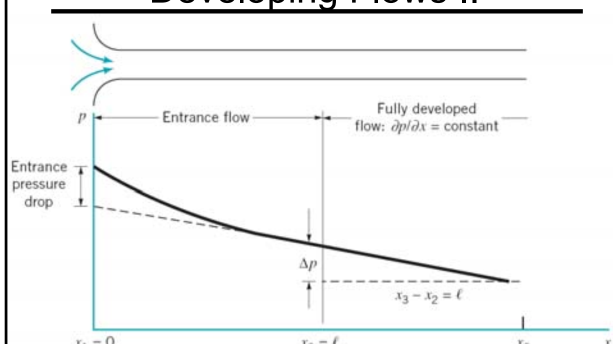
Flow Development



Developing Flows

- Entrance regions and bends create changing flow patters with different head losses
- Once flow is "fully developed" the head loss is proportional to the distance
- Entrance pressure drop is complex
  - Complete entrance region treated under minor losses
  - Will not treat partial entrance region here

Developing Flows II



Developing Flows III

- After development region, pressure drop (head loss) is proportional to pipe length
- Equations for entrance region length, ℓ_e
  - Laminar flow: $\frac{\ell_e}{D} = 0.06\,\text{Re}$
  - Turbulent flow: $\frac{\ell_e}{D} = 4.4\,\text{Re}^{1/6}$
  - Turbulent flow rule of thumb $\ell_e \approx 10D$

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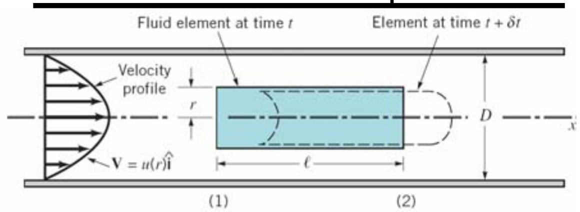
Fluid Element in Pipe Flow



- Look at arbitrary element, with length ℓ, and radius r, in fully developed flow
- What are forces on this element?

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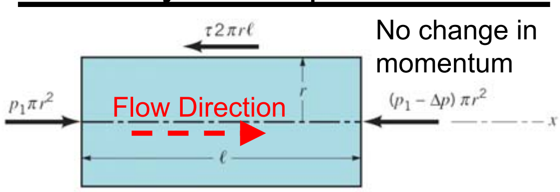
Fully Developed Flow



No change in momentum

$$\sum F_x = \pi r^2 p_1 - \pi r^2 (p_1 - \Delta p) - \tau 2\pi r \ell = 0$$

$$\Delta p = \frac{2\tau\ell}{r}$$

- Pressure drop is due to viscous stresses

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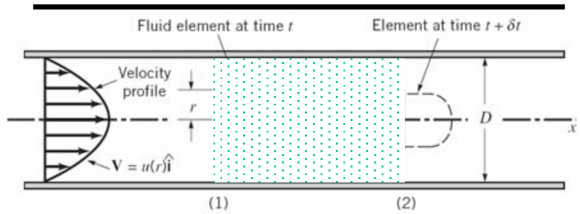
Extend Relation to Wall



- Have $\Delta p = 2\tau\ell/r$ for any r: $0 < r < R = D/2$
- For wall $r = R = D/2$ and $\tau = \tau_w$ = wall shear stress: $\Delta p = 2\tau_w\ell/R = 4\tau_w\ell/D$

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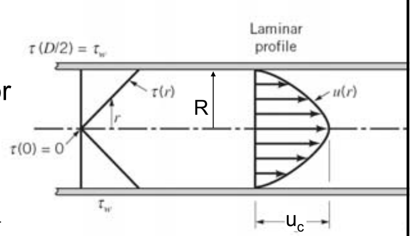
Fully Developed Laminar Flow

- Can get exact equation for pressure drop



$$\Delta p = \frac{128\mu\ell Q}{\pi D^4}$$

- Laminar velocity profile $u = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]$

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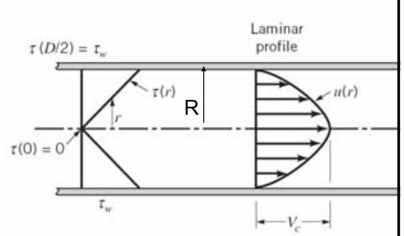
Fully Developed Laminar Flow II

$$u = u_c\left[1 - \left(\frac{r}{R}\right)^2\right]$$



- Laminar shear stress profile found from $\tau = \mu\frac{du}{dr}$

$$\tau = \mu\frac{du}{dr} = \mu u_c \frac{2r}{R^2} = \frac{8\mu u_c}{D^2} r$$

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Fully Developed Laminar Flow III

- What is centerline velocity, u_c?

$$Q = VA = V\pi R^2 = \int_A u\,dA = \int_0^R u 2\pi r\,dr = \int_0^R u_c\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r\,dr$$

$$Q = 2\pi u_c\left[\int_0^R r\,dr - \int_0^R \frac{r^3}{R^2}dr\right] = 2\pi u_c\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R = 2\pi u_c\frac{R^2}{4}$$

$$Q = \pi u_c\frac{R^2}{2} \quad\Rightarrow\quad u_c = \frac{2Q}{\pi R^2} = \frac{2VA}{\pi R^2} = \frac{2V\pi R^2}{\pi R^2} = 2V$$

Centerline u_c is twice the mean velocity, V

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Effect of Velocity Profile

- Momentum and kinetic energy flow for mean velocity, V
  - $Flow_{Momentum} = \dot{m}V = \rho VAV = \rho V^2(\pi R^2)$
  - $Flow_{KE} = \dot{m}V^2/2 = \rho VAV^2/2 = \rho V^3(\pi R^2)/2$
- Accurate representation uses profile

$$Flow_{Momentum} = \int_A \rho u\,du\,A = \int_0^R \rho\left[u_c\left(1-\frac{r^2}{R^2}\right)\right]^2 2\pi r\,dr = \frac{4}{3}\rho V^2 A$$

$$Flow_{KE} = \int_A \rho u\,dA\frac{u^2}{2} = \frac{1}{2}\int_0^R \rho\left[u_c\left(1-\frac{r^2}{R^2}\right)\right]^3 2\pi r\,dr = 2\rho A\frac{V^3}{2}$$

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Turbulent Flow

- For laminar and turbulent flows, the velocity at the wall is zero
  - This is called the no-slip condition
  - Momentum is maximum in the center of the flow and zero at the wall
    - Laminar flows: momentum transport from wall to center is by viscosity, $\tau = \mu du/dr$
    - Turbulent flows: random fluctuations exchange eddies of high momentum from the center with low momentum flow from near-wall regions

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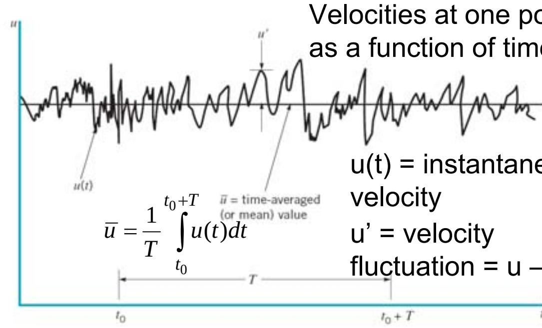
Turbulent Flow Quantities

Velocities at one point as a function of time



$$\bar{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(t)\,dt$$

u(t) = instantaneous velocity

u' = velocity fluctuation = u – $\bar{u}$

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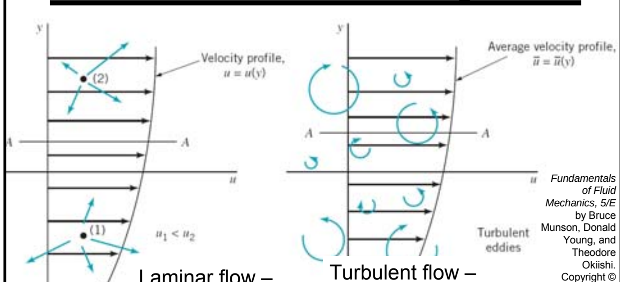
Momentum Exchange



Laminar flow – random molecular motion

Turbulent flow – eddies have structure

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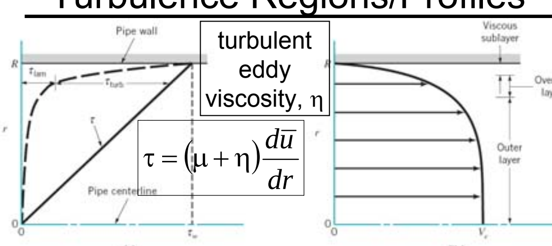
Turbulence Regions/Profiles



turbulent eddy viscosity, η

$$\tau = (\mu + \eta)\frac{d\bar{u}}{dr}$$

- Very thin viscous sublayer next to wall
  - 0.13% of R = 3 in for H_20 at $\bar{u}$ = 5 ft/s
- Flat velocity profile in center of flow

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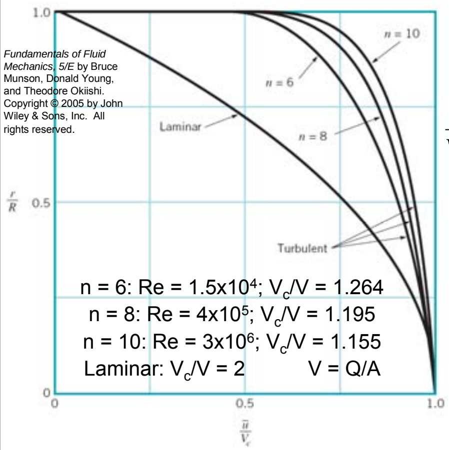

n = 6: Re = 1.5×10^4; V_c/V = 1.264
n = 8: Re = 4×10^5; V_c/V = 1.195
n = 10: Re = 3×10^6; V_c/V = 1.155
Laminar: V_c/V = 2   V = Q/A

Profile

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

Turbulent velocity profiles with n a function of Reynolds number

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Effect of Velocity Profile

- Analysis similar to one used for laminar flow profile
  - Determine momentum and kinetic energy flow for mean velocity
  - Correction factor multiplies average V results to give integrated u^2 and u^3 values

n	Re	Momentum	KE
6	1.5×10^4	1.027	1.077
8	4×10^5	1.016	1.046
10	3×10^6	1.011	1.031

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Pipe Roughness

- Effect of rough walls on pressure drop may depend on surface roughness of pipe
- Typical roughness values for different materials expressed as roughness length, ε, with units of feet or meters
- Only turbulent flows depend on roughness length, laminar flows do not

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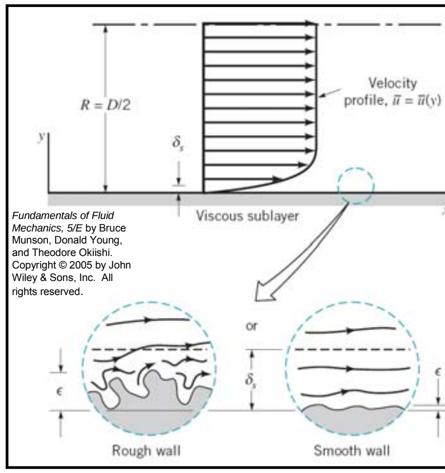

Pipe roughness effects in viscous sublayer affects pressure drop in turbulent flow

No effect on laminar flow

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TABLE 8.1
Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

Pipe	Equivalent Roughness, ε Feet	Equivalent Roughness, ε Millimeters
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

Use this table (p 433 of text) to find ε

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Energy Equation

- Energy equation between inlet (1) and outlet (2)

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

- Previous applications allowed us to compute the head loss from all other data in this equation
  - Call this the ***measured*** head loss
    - We can compute it, but we have no way of knowing its cause

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Pressure Drop/Head Loss

- We now seek a design calculation for h_L
- Use level pipe ($z_1 = z_2$) with constant area ($V_1 = V_2$) and no shaft head ($h_s = 0$)

$$z_2 + \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = z_1 + \frac{p_1}{\gamma} + \frac{V_1^2}{2g} + h_s - h_L$$

$$h_L = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{\Delta p}{\gamma}$$

- Calculated Δp for $z_1 = z_2$, $V_1 = V_2$, and $h_s = 0$ gives h_L for more general flows

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Pressure Drop/Head Loss

- We now seek a design calculation for h_L
- Use level pipe ($z_1 = z_2$) with constant area ($V_1 = V_2$) and no shaft head ($h_s = 0$)
- Will use friction factor f for Δp in such flows, but we are really getting h_L
  - Extend to more general flows later

$$h_L = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{\Delta p}{\gamma}$$

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Head Loss in Pipes

- Dimensional analysis shows that dimensionless pressure drop, $\Delta p/\rho V^2$, is a function of Reynolds number, $\rho VD/\mu$, the ℓ/D ratio and relative roughness, ε/D
- Expressed in terms of friction factor, f

$$f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = f\left(\frac{\rho VD}{\mu}, \frac{\varepsilon}{D}\right)$$

$$f = \frac{\gamma h_L}{\frac{1}{2}\frac{\ell}{D}\rho V^2} \Rightarrow h_L = f\frac{\ell}{D}\frac{\rho V^2}{2g}$$

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How do we get f?

- Have said that $f = f(\mathrm{Re}, \varepsilon/D)$
- What is form of this relationship?
- For laminar flow we will later show that $f = 64/\mathrm{Re}$
- Relationship determined experimentally with empirical fit to equations for turbulent flows
- Results expressed as Moody diagram

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Moody Diagram

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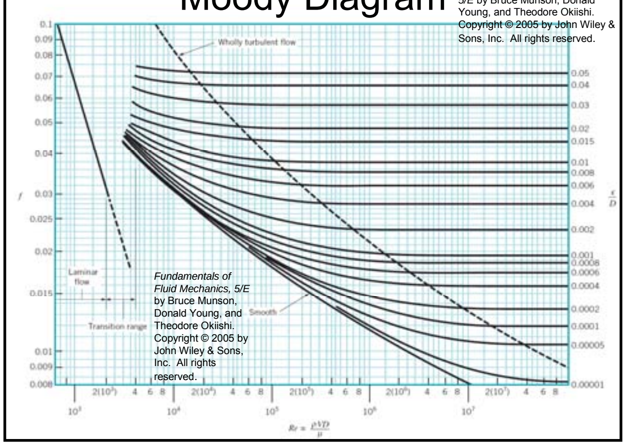

Moody Diagram Equations

- Colebrook equation (turbulent) $$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$
- Haaland equation (turbulent) $$\frac{1}{\sqrt{f}} \approx -1.8\log_{10}\left(\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right)$$
- Laminar

$$f = \frac{\Delta p}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{128\mu\ell Q}{\pi D^4}}{\frac{1}{2}\frac{\ell}{D}\rho V^2} = \frac{\frac{256\mu}{\pi D^3}V\frac{\pi}{4}D^2}{\rho V^2} = \frac{64}{\frac{\rho VD}{\mu}} = \frac{64}{\mathrm{Re}}$$

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Wholly Turbulent Flows

- Large Reynolds numbers: f independent of Re depends only on ε/D

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2} \qquad V = \frac{Q}{A} = \frac{Q}{\frac{\pi}{4}D^2} \quad \Rightarrow \quad V^2 = \frac{16}{\pi^2}\frac{Q^2}{D^4}$$

$$\Delta p = f\frac{\ell}{D}\frac{\rho}{2}V^2 = f\frac{\ell}{D}\frac{\rho}{2}\frac{16}{\pi^2}\frac{Q^2}{D^4} = \frac{8f\ell}{\pi^2}\frac{\rho Q^2}{D^5} = \frac{8f\ell}{\pi^2}\frac{\dot{m}^2}{\rho D^5}$$

- Pressure drop varies as D^{-5}
  - Similar to D^{-4} dependence in laminar flow

Pressure Drop Problems

- Find the pressure drop given fluid data, pipe dimensions, ε, and flow (volume flow, mass flow, or velocity)
  - Get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Find ρ and μ for fluid at given T and P
  - Compute $Re = \rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar f = 64/Re; Colebrook for turbulent
  - Compute $\Delta p = f(\ell/D)\rho V^2/2$

Sample Problem

- You have been asked to size a pump for an airport fuel delivery system. JP-4 fuel (ρ = 1.50 slug/ft^3, $\mu = 1.2\text{x}10^{-5}$ slug/ft·s) has to travel 0.5 mi through commercial steel, schedule 40 pipe with a nominal 6 in diameter. The flow rate is 5 slug/s. What is the head loss?
- Schedule 40 pipe: OD = 6.625 in; thickness = 0.280 in; ID = 6.065 in

Sample Problem Solution

$$D = (6.065\ in)\frac{ft}{12\ in} = 0.5054\ ft \qquad A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.5054\ ft)^2 = 0.2006\ ft^2$$

$$V = \frac{\dot{m}}{\rho A} = \frac{\frac{5\ slug}{s}}{\frac{1.50\ slug}{ft^3}\left(0.2006\ ft^3\right)} = \frac{16.61\ ft}{s}$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\frac{1.50\ slug}{ft^3}\frac{16.61\ ft}{s}(0.5054\ ft)}{\frac{1.2x10^{-5}\ slug}{ft\cdot s}} = 1.05x10^6 > 4100$$

Since Re > 4,100, flow is turbulent

Sample Problem Solution II

$$\frac{\varepsilon}{D} = \frac{0.00015\ ft}{0.5054\ ft} = 0.000297$$

ε = 0.00015 for commercial steel (Table 8.1, page 433)

Find f from Moody diagram (page 434)

$$f(\mathrm{Re} = 1.05x10^6, \varepsilon/D = 0.000297) = 0.0155$$

Check f value with Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) = -2.0\log_{10}\left(\frac{0.000297}{3.7} + \frac{2.51}{1.05x10^6\sqrt{0.0155}}\right)$$

$$\frac{1}{\sqrt{f}} = 8.005 \quad \Rightarrow \quad f = \frac{1}{8.005^2} = 0.0156$$

Use f = 0.0156

Sample Problem Solution III

$$\Delta P = f\frac{\ell}{D}\frac{\rho V^2}{2} = \frac{0.0156}{2}\frac{0.5\ mi}{0.5054\ ft}\frac{5280\ ft}{mi}\frac{1.50\ slug}{ft^3}\frac{1\ lb_f\cdot s^2}{slug\cdot ft}\left(\frac{16.61\ ft}{s}\right)^2$$

$$\Delta P = \frac{16876\ lb_f}{ft^2} = \frac{117.2\ lb_f}{in^2} = 117.2\ psi$$

- For shaft head to overcome this lead loss

$$\frac{\dot{W}_{\substack{shaft\\net\ in}}}{\dot{m}g} = h_s \ge h_L = \frac{\Delta P}{\gamma} = \frac{\Delta P}{\rho g} \quad \Rightarrow \quad \dot{W}_{\substack{shaft\\net\ in}} \ge \frac{\dot{m}\Delta P}{\rho}$$

$$\dot{W}_{\substack{shaft\\net\ in}} \ge \frac{\dot{m}\Delta P}{\rho} = \frac{\frac{5\ slug}{s}\frac{16876\ lb_f}{ft^2}}{\frac{1.50\ slug}{ft^3}}\frac{hp\cdot s}{550\ ft\cdot lb_f} = 102\ hp$$

Pressure Drop Problems II

- Find the diameter for a given pressure drop given fluid data, ε, and flow (volume flow, mass flow, or velocity)
  - Find ρ and μ for fluid at given T and P
  - Guess D; get $A = \pi D^2/4$
  - Get $V = Q/A$ or $V = \dot{m}/\rho A$ if not given V
  - Compute $Re = \rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar f = 64/Re; Colebrook for turbulent
  - Compute $\Delta p_{calculated} = f (\ell/D) \rho V^2/2$
  - Iterate on D until $\Delta p_{calculated} = \Delta p_{required}$

A Harder Problem

- You have a 200 hp pump to deliver 5 slug/s of JP-4 fuel (ρ = 1.50 slug/ft³, μ = 1.2x10⁻⁵ slug/ft·s) over 0.5 mi. What diameter of commercial steel, schedule 40 pipe should be used?
- Compute required Δp

$$\Delta p_{required} = \frac{\rho \dot{W}_{\substack{shaft \\ net\ in}}}{\dot{m}} = \frac{\frac{1.50\ slug}{ft^3}(200\ hp)\frac{550\ ft\cdot lb_f}{hp\cdot s}}{\frac{5\ slug}{s}} = \frac{33000\ lb_f}{ft^2}$$

Iterative Solution

- The calculation we just did for D = 6.065 in gave Δp = 16876 psf an error of 16876 psf – 33000 psf = –16124 psf

Count	D_{guess} (in)	$\Delta p_{computed}$ (psf)	Error (psf)
1	6.065	16876	–16124

- Take second guess of D = 5 in and repeat calculations done previously to find $\Delta p_{computed}$

Iterative Problem Solution

$$D = (5\ in)\frac{ft}{12\ in} = 0.4167\ ft \qquad A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.4167\ ft)^2 = 0.1364\ ft^2$$

$$V = \frac{\dot{m}}{\rho A} = \frac{\frac{5\ slug}{s}}{\frac{1.50\ slug}{ft^3}(0.1364\ ft^3)} = \frac{24.45\ ft}{s}$$

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{\frac{1.50\ slug}{ft^3}\frac{24.45\ ft}{s}(0.1364\ ft)}{\frac{1.2x10^{-5}\ slug}{ft\cdot s}} = 1.27x10^6 > 4100$$

Since Re > 4,100, flow is turbulent

Iterative Problem Solution II

$$\frac{\varepsilon}{D} = \frac{0.00015\ ft}{0.4167\ ft} = 0.00036$$

ε = 0.00015 for commercial steel (Table 8.1, page 433)

Find f from Moody diagram (page 434)

$$f(\mathrm{Re} = 1.27x10^6, \varepsilon/D = 0.000297) = 0.0159$$

Check f value with Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) = -2.0\log_{10}\left(\frac{0.00036}{3.7} + \frac{2.51}{1.27x10^6\sqrt{0.0159}}\right)$$

$$\frac{1}{\sqrt{f}} = 7.894 \quad\Rightarrow\quad f = \frac{1}{7.894^2} = 0.0160$$

Use f = 0.0160

Iterative Problem Solution III

$$\Delta P = f\frac{\ell}{D}\frac{\rho V^2}{2} = \frac{0.0160}{2}\frac{0.5\ mi}{0.4167\ ft}\frac{5280\ ft}{mi}\frac{1.50\ slug}{ft^3}\frac{1\ lb_f\cdot s^2}{slug\cdot ft}\left(\frac{24.45\ ft}{s}\right)^2$$

$$\Delta P = \frac{45564\ lb_f}{ft^2} = \frac{316.4\ lb_f}{in^2} = 316.4\ psi$$

- We now have two iterations

Count	D_{guess} (in)	$\Delta p_{computed}$ (psf)	Error (psf)
1	6.065	16876	–16124
2	5	45564	12564

Iterative Problem Solution IV

- Use linear interpolation to get new guess, D_{i+1} that sets error e_{i+1} to zero

$$D_{i+1} = D_i + \frac{D_i - D_{i-1}}{e_i - e_{i-1}}\left(e_{i+1} - e_i\right) = D_i - e_i\frac{D_i - D_{i-1}}{e_i - e_{i-1}}$$

(e_{i+1} → 0)

$$D_{i+1} = D_i - e_i\frac{D_i - D_{i-1}}{e_i - e_{i-1}} = 5 - 12564\frac{5 - 6.065}{12564 - (-16124)} = 5.466$$

Count	D_{guess} (in)	$\Delta p_{computed}$ (psf)	Error (psf)
1	6.065	16876	–16124
2	5	45564	12564

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Iterative Problem Solution V

- Continue iterations until error is "small"

Count	D_{guess} (in)	$\Delta p_{computed}$ (psf)	Error (psf)
1	6.065	16876	–16124
2	5	45564	12564
3	5.466	28780	–4219
4	5.349	32176	–823
5	5.321	33072	72
6	5.323	32999	–1

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Iterations and Reality

- Commercial pipe and tubing only comes in fixed sizes
  - Extra iterations not needed once the minimally acceptable commercial size is found
  - In this case available nominal diameters are 5 in and 6 in with actual inside diameters of 5.047 in and 6.065 in (for Schedule 40)
  - Only choice is 6 in (nominal)

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Pressure Drop Problems III

- Find the flow rate for a given pressure drop given fluid data, ε, and diameter
  - Get $A = \pi D^2/4$
  - Find ρ and μ for fluid at given T and P
  - Guess V
  - Compute Re = $\rho VD/\mu$ and ε/D
  - Find f from diagram or equation
    - Laminar f = 64/Re; Colebrook for turbulent
  - Compute $\Delta p_{calculated} = f\,\ell/D\,\rho V^2/2$
  - Iterate on V until $\Delta p_{calculated} = \Delta p_{required}$
  - Compute Q or $\dot{m}$ as desired

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Different Friction Factors

- The friction factor definition we are using here is the common one
  - Called the Darcy friction factor if clarification is needed
- Another definition, called the Fanning friction factor is sometimes seen
  - Fanning factor = $\tau_w / (\rho V^2/2)$
    - From the relationship that $\tau_w = D\Delta p/4\ell$ we get the result that the Fanning factor is one fourth of the Darcy factor

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Minor Losses

- Determine pressure drop (head loss) in a variety of flow passages
  - Entrance into a piping system
  - Exit from a piping system
  - Expansion in a piping system
  - Contraction in a piping system
  - Valves of various types (with different opening fractions)
  - Fittings (elbows, tees, bends, unions)

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Minor Losses

- Fittings in pipe systems modeled as loss coefficients, K_L

$$h_L = K_L \frac{V^2}{2g} \quad \Rightarrow \quad \frac{\Delta p_L}{\rho g} = K_L \frac{V^2}{2g} \quad \Rightarrow \quad \Delta p_L = K_L \frac{\rho V^2}{2}$$

- K_L depends on geometry and Re
  - For flows dominated by inertia effects K_L is a function of geometry only
- Alternative process, not given here, uses equivalent length for minor losses

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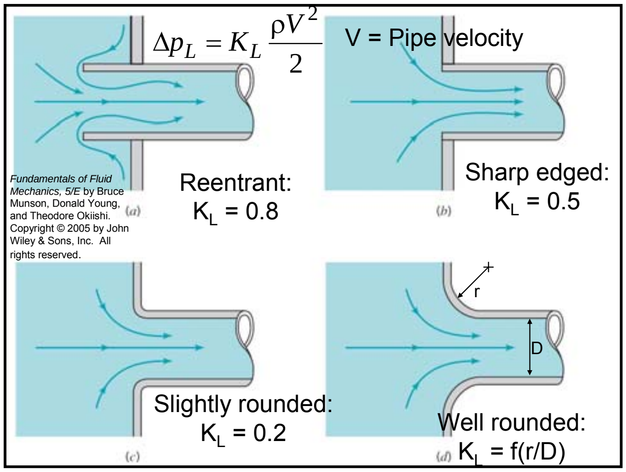
$$\Delta p_L = K_L \frac{\rho V^2}{2}$$

V = Pipe velocity



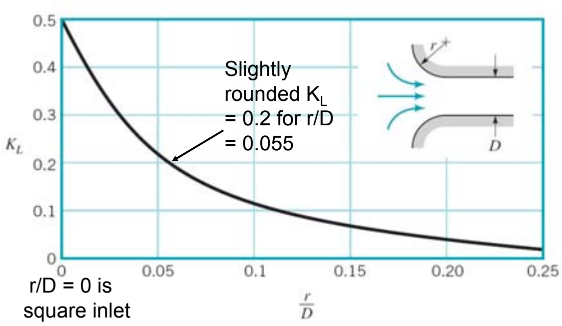
Reentrant: K_L = 0.8

Sharp edged: K_L = 0.5

Slightly rounded: K_L = 0.2

Well rounded: K_L = f(r/D)

Rounded Inlet K_L



Slightly rounded K_L = 0.2 for r/D = 0.055

r/D = 0 is square inlet

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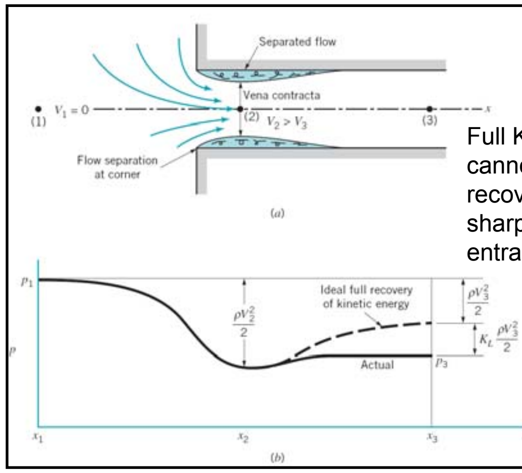

Full KE loss cannot be recovered in sharp-edged entrance

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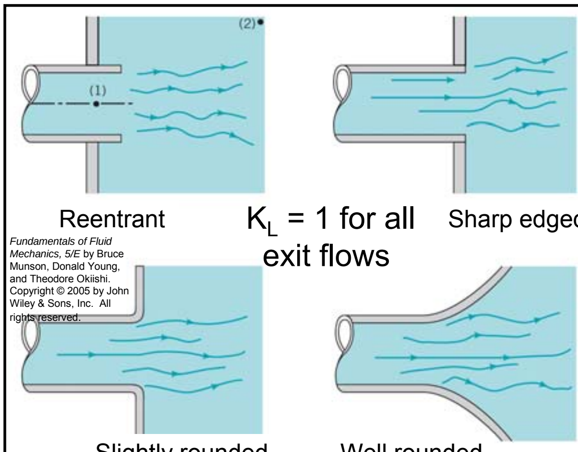


Reentrant

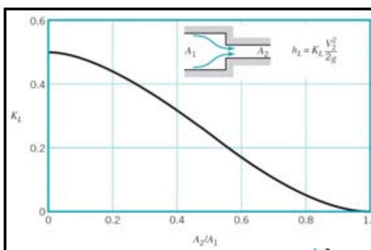
Sharp edged

K_L = 1 for all exit flows

Slightly rounded

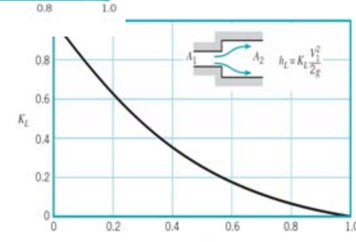
Well rounded

New Area



Sudden contraction (left)

For sudden expansion (right) $K_L = (1 – A_1/A_2)^2$



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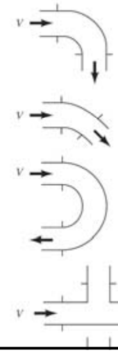
■ TABLE 8.2

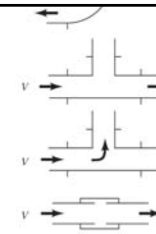
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$ (Data from Refs. 5, 10, 27)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0



Component	K_L
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
[a]e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210

[a]See Fig. 8.32 for typical valve geometry.



Problem with Minor Losses

- 4 kg/s of oil with SG = 0.82 and μ = 0.05 kg·m/s² is pumped from one tank to another. The line of 2-in Schedule-40 pipe has a total length of 40 m, with two gate valves and six elbows (regular flanged 90°). The entrance is rounded with an r/D ratio of 0.1.
- Find pressure loss with both valves open
- 2-in schedule 40 pipe has OD = 2.375 in and thickness = 0.154 in, so ID = 2.067 in = 0.05250 m

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Problem Solution

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.05250\ m)^2 = 0.002165\ m^2$$

$$\rho = (SG)\rho_{ref} = (0.82)\frac{999\ kg}{s} = \frac{819.2\ kg}{s}$$

$$V = \frac{\dot{m}}{\rho A} = \frac{\frac{4\ kg}{s}}{\frac{819.2\ kg}{m^3}\left(0.002165\ m^3\right)} = \frac{2.26\ m}{s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\frac{819.2\ kg}{m^3}\frac{2.26\ m}{s}(0.05250\ m)}{\frac{0.050\ kg}{m \cdot s}} = 1940 < 2100$$

Since Re < 2,100, flow is laminar

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Problem Solution II

Find Δp_{major} directly from laminar flow equation

$$Q = \frac{\dot{m}}{\rho} = \frac{4\ kg}{s}\frac{m^3}{819.2\ kg} = \frac{0.04883\ m^3}{s}$$

Could also use f = 64/Re

$$\Delta p_{major} = \frac{128\mu \ell Q}{\pi D^4} = \frac{(128)\frac{0.05\ N \cdot s}{m^2}(40\ m)\frac{0.004883\ m^3}{s}}{\pi(0.05250\ m)^4} = \frac{52369\ N}{m^2} = 52.369\ kPa$$

Minor losses coefficients: rounded entrance (r/D = 0.1), K_L = 0.12; exit, K_L = 1; fully open gate valve, K_L = 0.15; 6 elbows, K_L = 6(0.3) = 1.8. Total K_L = 0.12 + 1 + 0.15 + 1.8 = 3.07

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Problem Solution III

Δp_{minor} = (Loss coefficient sum) times $\rho V^2/2$

$$\Delta p_{\mathbf{minor}} = \left(\sum K_L\right)\frac{\rho V^2}{2} = \frac{3.07}{2}\frac{819\ kg}{m^3}\left(\frac{2.26\ m}{s}\right)^2\frac{1\ N \cdot s^2}{kg \cdot m} = \frac{6{,}397\ N}{m^2}$$

$$\Delta p_{total} = \Delta p_{major} + \Delta p_{\mathbf{minor}} = \frac{52{,}369\ N}{m^2} + \frac{6{,}397\ N}{m^2}$$

$$\Delta p_{total} = \frac{58{,}766\ N}{m^2} = 58.8\ kPa$$

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Noncircular Ducts

- Define hydraulic diameter, $D_h = 4A/P$
  - A is cross-sectional area for flow
  - P is wetted perimeter
  - For a circular pipe where $A = \pi D^2/4$ and $P = \pi D$, $D_h = 4(\pi D^2/4) / (\pi D) = D$
- For turbulent flows use Moody diagram with D replaced by D_h in Re, f, and ε/D
- For laminar flows, f = C/Re (both based on D_h) – see next slide for C values

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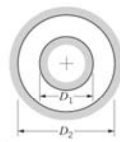
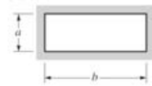
TABLE 8.3
Friction Factors for Laminar Flow in Noncircular Ducts (Data from Ref. 18)

Shape	Parameter	$C = f\,\mathrm{Re}_h$
I. Concentric Annulus $D_h = D_2 - D_1$	D_1/D_2	
	0.0001	71.8
	0.01	80.1
	0.1	89.4
	0.6	95.6
	1.00	96.0
II. Rectangle $D_h = \frac{2ab}{a+b}$	a/b	
	0	96.0
	0.05	89.9
	0.10	84.7
	0.25	72.9
	0.50	62.2
	0.75	57.9
	1.00	56.9





$$D_h = \frac{4A}{P}$$

$$\mathrm{Re}_h = \frac{\rho V D}{\mu}$$

$$\Delta P = f\frac{\ell}{D_h}\frac{\rho V^2}{2}$$

$$\Delta P = \frac{2C}{\pi}\frac{\mu \ell Q}{D_h^4}$$

Problem

- An 10-in-square, commercial steel air conditioning duct contains air at 80°F and atmospheric pressure and has a flow rate of 125 ft^3/min. Find the pressure drop per unit duct length
- Property data at 80°F (Table B.3) ρ = 0.002286 slug/ft^3; $\nu = 1.69 \times 10^{-4}$ ft^2/s
- Solution: find Re_h to see if flow is laminar or turbulent then find f and Δp

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Solution

$$D_h = \frac{4A}{P} = \frac{4L^2}{4L} = L = 10\ in = 0.8333\ ft$$

$$V = \frac{Q}{A} = \frac{\dfrac{125\ ft^3}{\min}\dfrac{\min}{60\ s}}{(10\ in)^2\dfrac{ft^2}{144\ in^2}} = \frac{3\ ft}{s}$$

$$\mathrm{Re}_h = \frac{VD_h}{\nu} = \frac{\dfrac{3\ ft}{s}(0.8333\ ft)}{\dfrac{1.69x10^{-4}\ ft^2}{s}} = 1.78x10^5$$

Turbulent flow for Re_h > 4100

$$\frac{\varepsilon}{D} = \frac{0.00015\ ft}{0.8333\ ft} = 0.00018$$

ε = 0.00015 for commercial steel (Table 8.1, page 433)

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Solution II

Find f from Moody diagram (page 434)

$$f\,(\mathrm{Re} = 1.27x10^6, \varepsilon/D = 0.00018) = 0.0172$$

Check f value with Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) = -2.0\log_{10}\left(\frac{0.00018}{3.7} + \frac{2.51}{1.78x10^5\sqrt{0.0172}}\right)$$

$$\frac{1}{\sqrt{f}} = 7.611 \Rightarrow f = \frac{1}{7.611^2} = 0.0173$$

Use f = 0.0173

$$\frac{\Delta P}{\ell} = f\frac{1}{D}\frac{\rho V^2}{2} = \frac{0.0173}{2}\frac{1}{0.8333\ ft}\frac{0.00229\ slug}{ft^3}\frac{1\ lb_f \cdot s^2}{slug \cdot ft}\left(\frac{3\ ft}{s}\right)^2 = \frac{1.78x10^{-5}\ lb_f}{ft^3}$$

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Recommended Air Velocity

Air Ducts	Air Velocity m/s	Air Velocity ft/s
Combustion air ducts	12 - 20	40 - 66
Air inlet to boiler room	1 - 3	3.3 - 9.8
Warm air for house heating	0.8 - 1.0	2.6 - 3.3
Vacuum cleaning pipe	8 - 15	26 - 49
Compressed air pipe	20 - 30	66 - 98
Ventilation ducts (hospitals)	1.8 - 4	5.9 - 13
Ventilation ducts (offices)	2.0 - 4.5	6.5 - 15

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http://www.engineeringtoolbox.com/flow-velocity-air-ducts-d_388.html