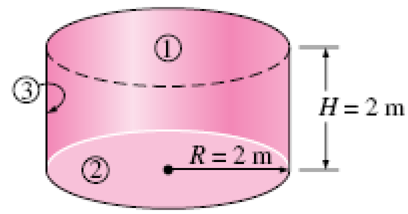
Solutions to Exercise Twelve –Radiation Heat Transfer

1. **A furnace is of cylindrical shape with R = H = 2 m. The base, top, and side surfaces are all black and are maintained at uniform temperatures of 500 K, 700 K, and 1400 K, respectively. Determine the net rate of radiation heat transfer to or from the top surface during steady operation. (Problem and figure P13-30 from Çengel, H*eat and Mass Transfer*.)**



Since all the surfaces are black, the heat transfer from the top surface (surface 1) is given by the following equation.

$$\dot{Q}_1 = \dot{Q}_{12} + \dot{Q}_{13} = A_1F_{12}\sigma\left(T_1^4 - T_2^4\right) + A_1F_{13}\sigma\left(T_1^4 - T_3^4\right)$$

We have all the data required except for the view factors. We can find the view factor from surface 1 to surface 2 from Figure 13.7 with L/r_1 = H/R = (2 m)/(2 m) = 1and r_2/L = (2 m)/(2 m) = 1. I read a value of F_{12} = 0.38 from Figure 13-7. Alternatively we can find the view factor from the following equation for two coaxial, circular surfaces with radii r_i and r_j separated by a distance L from Table 13-1.

$$F_{12} = \frac{1}{2}\left(S - \sqrt{S^2 - 4\left(\frac{r_2}{r_1}\right)^2}\right) \quad where \quad S = 1 + \frac{1 + (r_2/L)^2}{(r_1/L)^2}$$

For this problem $r_1 = r_2$ = 2 m and L = 2 m, so $r_1/L = r_2/L$ = (2 m)/(2 m) =1. This gives

$$S = 1 + \frac{1 + (r_2/L)^2}{(r_1/L)^2} = 1 + \frac{1 + (1)^2}{(1)^2} = 3$$

$$F_{12} = \frac{1}{2}\left(S - \sqrt{S^2 - 4\left(\frac{r_2}{r_1}\right)^2}\right) = \frac{1}{2}\left(3 - \sqrt{3^2 - 4\left(\frac{2\ m}{2\ m}\right)^2}\right) = 0.3820$$

We can find F_{13} from the summation rule: $F_{11} + F_{12} + F_{13}$ = 1. Since surface 1 is flat, F_{11} = 0 and we have F_{13} = 1 – 0 – 0.3820 = 0.6280. We can now solve for the heat transfer from surface 1 whose area is $\pi R^2 = \pi(2\text{ m})^2 = 12.566\text{ m}^2$.

$$\dot{Q}_1 = A_1\sigma\left[F_{12}\left(T_1^4 - T_2^4\right) + F_{13}\left(T_1^4 - T_3^4\right)\right] =$$

$$\left(12566m^2\right)\frac{5.670x10^{-8}W}{m^2 \cdot K^4}\left\{(0.3820)\left[(700K)^4 - (500K)^4\right] - (0.6280)\left[(700K)^4 - (1400K)^4\right]\right\}$$

$$\boxed{\dot{Q}_1 = \mathbf{-1.538x10^6\ W}}$$

2. Consider a 20-cm diameter hemispherical enclosure. The dome is maintained at 600 K and heat is supplied from the base at a rate of 50 W while the base surface with an emissivity of 0.55 is maintained at 400 K. Determine the emissivity of the dome.

This is a two surface problem for which we can find the heat transfer from the following equation.

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}}$$

Define surface 1 as the flat surface (the base) and surface 2 as the hemispherical dome. Since the bottom surface is flat, all its radiation reaches the hemispherical dome so F_{12} = 1. The area of the base, $A_1 = \pi D^2/4 = \pi(0.2\text{ m})^2/4 = 0.03142\text{ m}^2$. The area of the hemisphere is half the area of a sphere: $A_2 = \pi D^2/2 = \pi(0.2\text{ m})^2/2 = 0.06283\text{ m}^2$ From the formulas we see that the area ratio A_1/A_2 = 1/2. Substituting F_{12} = 1 and A_1/A_2 = 1/2 into our heat transfer equation and solving for e_2 gives.

$$\dot{Q}_{12} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{A_1}{A_2}\frac{1-\varepsilon_2}{\varepsilon_2}} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} - 1 + \frac{1}{1} + \frac{1}{2}\left(\frac{1}{\varepsilon_2} - 1\right)} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{2\varepsilon_2} - \frac{1}{2}}$$

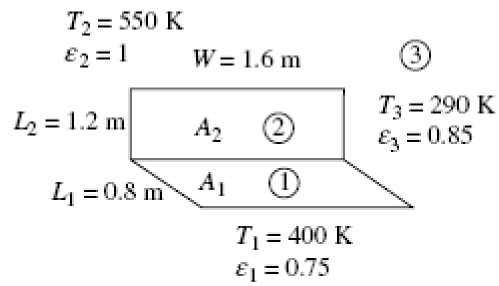
$$\frac{1}{\varepsilon_1} + \frac{1}{2\varepsilon_2} - \frac{1}{2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dot{Q}_{12}} \quad\Rightarrow\quad \frac{1}{2\varepsilon_2} = \frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dot{Q}_{12}} - \frac{1}{\varepsilon_1} + \frac{1}{2} \quad\Rightarrow\quad \varepsilon_2 = \frac{\frac{1}{2}}{\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dot{Q}_{12}} - \frac{1}{\varepsilon_1} + \frac{1}{2}}$$

Substituting the given data for all the terms on the right side of this equation gives the desired emissivity.

$$\varepsilon_2 = \frac{\frac{1}{2}}{\frac{A_1\sigma\left(T_1^4 - T_2^4\right)}{\dot{Q}_{12}} - \frac{1}{\varepsilon_1} + \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{\left(0.03142\ m^2\right)\frac{5.670x10^{-8} W}{m^2 \cdot K^4}\left[\left(400\ K\right)^4 - \left(600\ K\right)^4\right]}{-50\ W} - \frac{1}{0.55} + \frac{1}{2}}$$

$$\boxed{\varepsilon_2 = 0.209}$$

3. Consider a triangular enclosure that has the two rectangular surfaces shown at the right. These two surfaces are perpendicular to each other with a common edge which is 1.6 m long. The horizontal surface is 0.8 m wide and the vertical surface is 1.2 m high. The horizontal surface has an emissivity of 0.75 and is maintained at 400 K. The vertical surface is black and is maintained at 550 K. The back sides of the surfaces are insulated. The remaining three sides of the triangular enclosure (one flat surface that joins the outer edges of the horizontal and vertical surfaces shown and two triangular surfaces at the ends) are at 290 K and can be considered to have an emissivity of 0.85. Determine the net rate of radiation heat transfers between the two surfaces, and between the horizontal surface and the surroundings. (Problem and figure P13-45 from Çengel, H*eat and Mass Transfer*.)



The equations for a three surface enclosure are taken from slides 29 and 30 of the May 2 lecture presentation.

$$\left(1+\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}+\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}\right)J_1-\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}J_2-\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}J_3=\sigma T_1^4$$

$$-\frac{1-\varepsilon_2}{\varepsilon_2}F_{21}J_1+\left(1+\frac{1-\varepsilon_2}{\varepsilon_2}F_{21}+\frac{1-\varepsilon_2}{\varepsilon_2}F_{23}\right)J_2-\frac{1-\varepsilon_2}{\varepsilon_2}F_{23}J_3=\sigma T_2^4$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}J_1-\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}J_2+\left(1+\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}+\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}\right)J_3=\sigma T_3^4$$

Because surface 2 is a black body, $e_2 = 1$, and the second equation reduces to $J_2 = \sigma T_2^4$, and, because we know T_2, our system of three equations in three unknowns becomes a system of two equations in two unknowns.

$$\left(1+\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}+\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}\right)J_1-\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}J_3=\sigma T_1^4+\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}\sigma T_2^4$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}J_1+\left(1+\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}+\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}\right)J_3=\sigma T_3^4+\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}\sigma T_2^4$$

The areas of the two flat surfaces in the enclosure are $A_1 = L_1W = (0.8\text{ m})(1.6\text{ m}) = 1.28\text{ m}^2$ and $A_2 = L_2W = (1.2\text{ m})(1.6\text{ m}) = 1.92\text{ m}^2$; the area of the third radiation surface is the area of a single diagonal surface, $W(L_1^2 + L_2^2)^{1/2}$ plus the area two triangles enclosing the ends: $2[(1/2)L_1L_2]$. This third surface area is thus $(1.6\text{ m})[(0.8\text{ m})^2 + (1.2\text{ m})^2]^{1/2} + (0.8\text{ m})(1.2\text{ m}) = 3.268\text{ m}^2$.

We have to find four view factors F_{12}, F_{13}, F_{31}, and F_{32}. We can find F_{12} from the view factor graph in Figure 13-6. For this problem we have $L_1/W = (0.8\text{ m}) / (1.6\text{ m}) = 0.5$ and $L_2/W = (1.2\text{ m}) / (1.6\text{ m}) = 0.75$; for these values I read a view factor of 0.27 from Figure 13-6. Using the equation in Table 13-1, the view factor is found to be $F_{12} = 0.2749$, which will be used in subsequent calculations.

We can find F_{13} from the summation rule: $F_{11} + F_{12} + F_{13} = 1$, where $F_{11} = 0$ because surface 1 is a flat surface. This gives $F_{13} = 1 – 0 – 0.2749 = 0.7251$.

We can use the reciprocity rule to find F_{31}: $A_3F_{31} = A_1F_{13}$; so, $F_{31} = A_1F_{13}/A_3 = (1.28\ m^2)(0.7251)/(3.268\ m^2) = 0.2840$.

We still need F_{32}, but we cannot use the summation rule to find it. Surface 3 is not a flat surface so we cannot say that $F_{33} = 0$. If we knew F_{23} we could use the reciprocity rule to find F_{32}. We can find F_{23} from the summation rule for surface 2, where $F_{22} = 0$: $F_{23} = 1 – F_{21} – F_{22} = 1 – F_{21}$. And we can use the reciprocity rule to find $F_{21}= A_1F_{12}/A_2 = (1.28\ m^2)(0.2749)/ (1.92\ m^2) = 0.1833$. So we have $F_{23} = 1 – F_{21} = 1 – 0.1833 = 0.8167$, and $F_{32} = A_2F_{23}/A_3 = (1.92\ m^2)(0.8167)/ (3.268\ m^2) = 0.4799$.

The emissivity factors in the equations that we have to solve are $(1 – \varepsilon_1)/ \varepsilon_1 = (1 – 0.75)/0.75 = 1/3$ and $(1 – \varepsilon_3)/ \varepsilon_3 = (1 – 0.85)/0.85 = 0.1765$. We can now compute the coefficients and right-hand-side terms in our system of two equations. We start by computing the black body emissive power for the three surfaces.

$$E_{b1} = \sigma T_1^4 = \frac{5.670x10^{-8} W}{m^2 \cdot K^4}(400\ K)^4 = \frac{1452\ W}{m^2} \qquad E_{b2} = \sigma T_2^4 = \frac{5.670x10^{-8} W}{m^2 \cdot K^4}(550\ K)^4 = \frac{5188\ W}{m^2}$$

$$E_{b3} = \sigma T_3^4 = \frac{5.670x10^{-8} W}{m^2 \cdot K^4}(290\ K)^4 = \frac{401.0\ W}{m^2}$$

Using these emissive powers we can compute the right-hand side terms of our equations.

$$\sigma T_1^4 + \frac{1-\varepsilon_1}{\varepsilon_1}\sigma T_2^4 = \frac{1451\ W}{m^2} + \frac{1}{3}\frac{5188\ W}{m^2} = \frac{1927\ W}{m^2} \qquad \sigma T_3^4 + \frac{1-\varepsilon_3}{\varepsilon_3}\sigma T_2^4 = \frac{401.0\ W}{m^2} + (0.1765)\frac{5188\ W}{m^2} = \frac{840.4\ W}{m^2}$$

We next compute the left-hand side terms of the two equations.

$$\frac{1-\varepsilon_1}{\varepsilon_1}F_{13} = \frac{1}{3}(0.7251) = -0.2417 \qquad 1 + \frac{1-\varepsilon_1}{\varepsilon_1}(F_{12} + F_{13}) = 1 + \frac{1}{3}(0.2749) + 0.2417 = 1.333$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3}F_{31} = -(0.1765)(0.2840) = -0.05013 \qquad 1 + \frac{1-\varepsilon_3}{\varepsilon_3}(F_{31} + F_{32}) = 1 + 0.05013 + (0.1765)(0.4799) = 1.135$$

We have the following system of equations to solve.

$$1.333 J_1 - 0.2417 J_3 = \frac{1927\ W}{m^2}$$
$$-0.005013 J_1 - 1.135 J_3 = \frac{840.4\ W}{m^2}$$

We see that the left-hand side coefficients are dimensionless and the right-hand side terms have units of W/m^2 so our results for radiosity will have units of W/m^2. Solving these equations gives $J_1 = 1592\ W/m^2$ and $J_3 = 810.9\ W/m^2$. (Recall that we previously had $J_2 = E_{b2} = 5188\ W/m^2$.) We can now find the desired heat transfers. The heat transfer from the vertical surface (2) to the horizontal surface (1) is

$$\dot{Q}_{21} = A_2F_{21}(J_2 - J_1) = (1.92\ m^2)(0.1833)\left(\frac{5188\ W}{m^2} - \frac{1592\ W}{m^2}\right) = \boxed{\mathbf{1265\ W/m^2}}$$

The heat transfer from the horizontal surface (1) to the surroundings (3) is

$$\dot{Q}_{13} = A_1F_{13}(J_1 - J_3) = (1.28\ m^2)(0.1833)\left(\frac{1592\ W}{m^2} - \frac{810.9\ W}{m^2}\right) = \boxed{\mathbf{725.1\ W/m^2}}$$