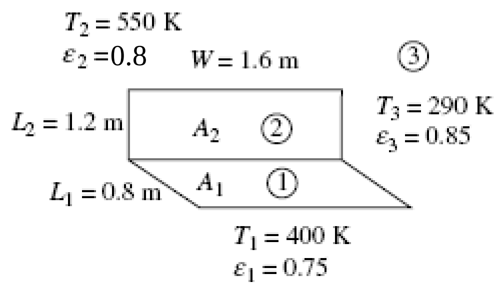
California State University Northridge

College of Engineering and Computer Science
Mechanical Engineering Department

Mechanical Engineering 375
Heat Transfer

Spring 2007 Number 17629 Instructor: Larry Caretto

Solutions to Quiz Twelve –Radiation Heat Transfer



Consider a triangular enclosure that has the two rectangular surfaces shown at the right. These two surfaces are perpendicular to each other with a common edge which is 1.6 m long. The horizontal surface is 0.8 m wide and the vertical surface is 1.2 m high. The horizontal surface has an emissivity of 0.75 and is maintained at 400 K. The vertical surface is maintained at 550 K and has an emissivity of 0.8. The back sides of the surfaces are insulated. The remaining three sides of the triangular enclosure (one flat surface that joins the outer edges of the horizontal and vertical surfaces shown and two triangular surfaces at the ends) are at 290 K and can be considered to have an emissivity of 0.85. Determine the net rate of radiation heat transfers between the two surfaces, and between the horizontal surface and the surroundings. (Problem and figure P13-45 from Çengel, H*eat and Mass Transfer*.)

The equations for a three surface enclosure are taken from slides 29 and 30 of the May 2 lecture presentation.

$$\left(1+\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}+\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}\right)J_1-\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}J_2-\frac{1-\varepsilon_1}{\varepsilon_1}F_{13}J_3=\sigma T_1^4$$

$$-\frac{1-\varepsilon_2}{\varepsilon_2}F_{21}J_1+\left(1+\frac{1-\varepsilon_2}{\varepsilon_2}F_{21}+\frac{1-\varepsilon_2}{\varepsilon_2}F_{23}\right)J_2-\frac{1-\varepsilon_2}{\varepsilon_2}F_{23}J_3=\sigma T_2^4$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}J_1-\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}J_2+\left(1+\frac{1-\varepsilon_3}{\varepsilon_3}F_{31}+\frac{1-\varepsilon_3}{\varepsilon_3}F_{32}\right)J_3=\sigma T_3^4$$

We can start by computing the black body emissive power for the three surfaces that appear on the right-hand sides of our equations.

$$E_{b1}=\sigma T_1^4=\frac{5.670x10^{-8}W}{m^2\cdot K^4}(400\ K)^4=\frac{1452\,W}{m^2}\qquad E_{b2}=\sigma T_2^4=\frac{5.670x10^{-8}W}{m^2\cdot K^4}(550\ K)^4=\frac{5188\,W}{m^2}$$

$$E_{b3}=\sigma T_3^4=\frac{5.670x10^{-8}W}{m^2\cdot K^4}(290\ K)^4=\frac{401.0\,W}{m^2}$$

The remainder of the coefficients will involve view factors and areas. The areas of the two flat surfaces in the enclosure are $A_1 = L_1W$ = (0.8 m) (1.6 m) = 1.28 m^2 and $A_2 = L_2W$ = (1.2 m) (1.6 m) = 1.92 m^2; the area of the third radiation surface is the area of a single diagonal surface, $W(L_1^2 + L_2^2)^{1/2}$ plus the area two triangles enclosing the ends: $2[(1/2)L_1L_2]$. This third surface area is thus (1.6 m)[(0.8 m)^2 +(1.2 m)^2]$^{1/2}$ + (0.8 m)(1.2 m) = 3.268 m^2.

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We have to find six view factors F_{12}, F_{13}, F_{21}, F_{23}, F_{31}, and F_{32}. We can find F_{12} from the view factor graph in Figure 13-6. For this problem we have L_1/W = (0.8 m) / (1.6 m) = 0.5 and L_2/W = (1.2 m) / (1.6 m) = 0.75; for these values I read a view factor of F_{12} = 0.28 from Figure 13-6.

We can find F_{13} from the summation rule: $F_{11} + F_{12} + F_{13} = 1$, where $F_{11} = 0$ because surface 1 is a flat surface. This gives F_{13} = 1 – 0 – 0.2749 = 0.72.

We can use the reciprocity rule to find F_{21} and F_{31}: $A_2F_{21} = A_1F_{12}$; so, $F_{21} = A_1F_{12}/A_2$ = (1.28 m^2)(0.28)/ (1.92 m^2) = 0.1867; $A_3F_{31} = A_1F_{13}$; so, $F_{31} = A_1F_{13}/A_3$ = (1.28 m^2)(0.72)/ (3.268 m^2) = 0.2820.

Since surface 2 is a flat surface, $F_{22} = 0$, and we can use the summation rule, $F_{21} + F_{22} + F_{23} = 1$ to find $F_{23} = 1 - F_{21} - F_{22}$ = 1 – 0.1867 – 0 = 0.8133.

Finally, we can use the reciprocity rule to find $F_{32} = A_2F_{23}/A_3$ = (1.92 m^2)(0.8133)/ (3.268 m^2) = 0.4799.

The emissivity factors in the equations that we have to solve are $(1 - \varepsilon_1)/\varepsilon_1$ = (1 – 0.75)/0.75 = 1/3, $(1 - \varepsilon_2)/\varepsilon_2$ = (1 – 0.8)/0.8 = 1/4, and $(1 - \varepsilon_3)/\varepsilon_3$ = (1 – 0.85)/0.85 = 0.1765. We can now compute the coefficients and right-hand-side terms in our system of two equations.

We now have all the data required to compute the coefficients on the left-hand sides of the three equations. Note that the calculations of diagonal terms use the results of the previous calculations for the other terms in the same equation.

$$-\frac{1-\varepsilon_1}{\varepsilon_1}F_{12}\frac{1}{3}(0.28) = -0.09333 \qquad -\frac{1-\varepsilon_1}{\varepsilon_1}F_{13} = \frac{1}{3}(0.72) = -0.24$$

$$1-\frac{1-\varepsilon_1}{\varepsilon_1}F_{12} - \frac{1-\varepsilon_1}{\varepsilon_1} = 1 + 0.09333 + 0.24 = 1.3333$$

$$-\frac{1-\varepsilon_2}{\varepsilon_2}F_{21} = \frac{1}{4}(0.1967) = -0.04667 \qquad -\frac{1-\varepsilon_2}{\varepsilon_2}F_{23} = \frac{1}{4}(0.8133) = -0.2033$$

$$1+\frac{1-\varepsilon_2}{\varepsilon_2}F_{21} + \frac{1-\varepsilon_2}{\varepsilon_2}F_{23} = 1 + 0.04667 + 0.20333 = 1.25$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3}F_{31} = (0.1765)(0.2820) = -0.049773 \qquad -\frac{1-\varepsilon_3}{\varepsilon_3}F_{32} = (0.1765)(0.4779) = -0.08434$$

$$1+\frac{1-\varepsilon_3}{\varepsilon_3}F_{31} + \frac{1-\varepsilon_3}{\varepsilon_3}F_{32} = 1 + 0.049773 + 0.08434 = 1.1341$$

We have the following system of equations to solve.

$$1.3333J_1 - 0.093333J_2 - 0.24J_3 = \frac{1452\,W}{m^2}$$

$$-0.046667J_1 + 1.253J_2 - 0.20333J_3 = \frac{5188\,W}{m^2}$$

$$-0.049773J_1 - 0.084337J_1 - 1.1341J_3 = \frac{401.0\,W}{m^2}$$

We see that the left-hand side coefficients are dimensionless and the right-hand side terms have units of W/m^2 so our results for radiosity will have units of W/m^2. Solving these simultaneous equations gives J_1 = 1525 W/m^2, J_2 = 4238 W/m^2 and J_3 = 742.4 W/m^2. The heat transfer from the vertical surface (2) to the horizontal surface (1) is

$$\dot{Q}_{21} = A_2F_{21}(J_2 - J_1) = (1.92\ m^2)(0.1867)\left(\frac{4328\,W}{m^2} - \frac{1525\,W}{m^2}\right) = \boxed{\textbf{1005 W/m}^2}$$