Solution to Fourth Quiz – Fins

You are asked to evaluate two possible designs for fin cooling. One design uses a single rectangular fin that is 4.1 cm wide, 0.5 cm thick and has a length of 3 cm. An alternative design, which requires the same space, but less material, consists of four cylindrical fins with a diameter of 0.5 cm and a separation distance between cylinders of 0.7 cm. Both designs use aluminum (k = 237 W/m·°C), have a heat transfer coefficient of 35 W/m²·°C, a base temperature of 100°C, and an ambient temperature of 30°C. Which design has the higher heat transfer?

For the single rectangular fin with \( w = 0.041 \) m, and \( t = 0.005 \) m, the perimeter is \( 2(w+t) = 2(0.041 \text{ m} + 0.005 \text{ m}) = 0.092 \text{ m} \). The cross-sectional area, \( A_c = tw = (0.041 \text{ m})(0.005 \text{ m}) = 0.000205 \text{ m}^2 \). The corrected length \( L_c = L + A_c/p = 0.03 \text{ m} + (0.000205 \text{ m}^2)/(0.092 \text{ m}) = 0.03223 \text{ m} \). With these data and the values of \( k \) and \( h \) given in the problem statement, we can compute the values of \( m \) and \( mL_c \) for the rectangular fin.

\[
m = \sqrt{\frac{h}{kA_c}} = \sqrt{\frac{35 \text{ W}}{237 \text{ W}^2 \cdot \text{m}^{-2} \cdot \text{C}}} \left(\frac{0.092 \text{ m}}{0.000205 \text{ m}^2}\right) = 8.141 \text{ m} \quad mL_c = \frac{8.141 \text{ m}}{m}(0.03223 \text{ m}) = 0.2624
\]

The heat transfer from this rectangular fin can now be found.

\[
\dot{Q}_{\text{rect fin}} = \sqrt{kA_c/hp} (T_b - T_x) \tanh mL = \sqrt{\frac{35 \text{ W}}{237 \text{ W}^2 \cdot \text{m}^{-2} \cdot \text{C}}} \left(\frac{0.000205 \text{ m}^2}{0.03 \text{ m}}\right)(100\text{°C} - 30\text{°C}) \tanh(0.2624) = 7.1023 \text{ W}
\]

Each of the four cylindrical fins has a cross sectional area \( A_c = \pi(0.005 \text{ m})^2/4 = 19.635 \times 10^{-6} \text{ m}^2 \) and the perimeter \( p = \pi(0.005 \text{ m}) = 0.01571 \text{ m} \). The corrected length, \( L_c = L + A_c/p = 0.03 \text{ m} + (19.635 \times 10^{-6} \text{ m}^2) / (0.01571 \text{ m}) = 0.03125 \text{ m} \). With these data and the values for \( k \) and \( h \) given above we can compute the parameters \( m \) and \( mL_c \) for each cylindrical fin.

\[
m = \sqrt{\frac{h}{kA_c}} = \sqrt{\frac{35 \text{ W}}{237 \text{ W}^2 \cdot \text{m}^{-2} \cdot \text{C}}} \left(\frac{0.01571 \text{ m}}{10.635 \times 10^{-6} \text{ m}^2}\right) = 10.87 \text{ m} \quad mL_c = \frac{10.87 \text{ m}}{m}(0.03125 \text{ m}) = 0.3397
\]

The heat transfer from one cylindrical fin is given by the following equation.

\[
\dot{Q}_{\text{one cyl}} = \sqrt{kA_c/hp} (T_b - T_x) \tanh mL = \sqrt{\frac{35 \text{ W}}{19.635 \times 10^{-6} \text{ m}^2 \cdot \text{C}}} \left(\frac{237 \text{ W}}{0.01571 \text{ m}}\right)(100\text{°C} - 30\text{°C}) \tanh(0.3397) = 1.158 \text{ W}
\]

Compared to base area of the rectangular fins, the cylindrical fins will have an unfinned area of \( tw - 4[\pi(D^2)/4] = 0.000205 \text{ m}^2 - 4(19.635 \times 10^{-6} \text{ m}^2) = 0.000126 \text{ m}^2 \) from which there will be convection. Thus the total convection from the cylindrical fins will be the sum of the heat transfer from the four fins plus the heat transfer from the unfinned base.
\[ \dot{Q} = \dot{Q}_{\text{finned}} + \dot{Q}_{\text{unfinned}} = N_{\text{fin}} \dot{Q}_{\text{one fin}} + hA_{\text{unfinned}}(T_b - T_w) \]
\[ = (4)(1.158 \, W) + \frac{35 \, W}{m^2 \cdot ^\circ C} \left(0.000126 \, m^2 \right) \left(100^\circ C - 30^\circ C \right) = 4.942 \, W \]

The single rectangular fin has the greater heat transfer.