California State University **Northridge**	**College of Engineering and Computer Science** **Mechanical Engineering Department** ***Mechanical Engineering 375*** ***Heat Transfer***
	Spring 2007 Number 17629 Instructor: Larry Caretto

February 14 Homework Solutions

2.48 Consider an aluminum pan used to cook stew on top of an electric range. The bottom section of the pan is *L* = 0.25 cm thick and has a diameter of *D* = 18 cm. The electric heating unit on the range top consumes 900 W of power during cooking, and 90 percent of the heat generated in the heating element is transferred to the pan. During steady operation, the temperature of the inner surface of the pan is measured to be 108°C. Assuming temperature-dependent thermal conductivity and one-dimensional heat transfer, express the mathematical formulation (the differential equation and the boundary conditions) of this heat conduction problem during steady operation. Do not solve (Figure from Çengel, *Heat and Mass Transfer*)

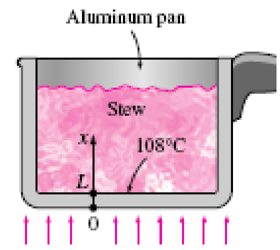


FIGURE P2–48

We can start with the general heat conduction equation.

$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \dot{e}_{gen}$$

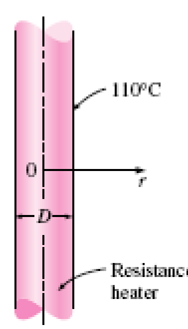
CWe are told that the process is steady and one-dimensional so we can ignore the time, y and z derivative terms. We are also told that there is a heat input of 900 W from an electric heater of which 90% enters the bottom of the pan. This is a boundary condition heat flux of (90%)(900 W)/[(p/4)(0.18 m)^2] = 31,831 W/m^2. There is no heat generation with the bottom of the pan so we can set the heat generation term to zero. With these assumptions, the differential equation becomes

$$\frac{d}{dx} k \frac{dT}{dx} = 0 \qquad \Rightarrow \qquad k \frac{dT}{dx} = Const$$

The boundary conditions are the heat flux found in the previous paragraph at x = 0 and a temperature of 108^{o}C at x = L. We can write these as follows.

$$-k \frac{dT}{dx} = 0 \ at \ x = 0 \quad and \quad T = 108^o C \ at \ x = L$$

2.79 A 2-kW resistance heater wire with thermal conductivity of *k* = 20 W/m·°C, a diameter of *D* = 4 mm, and a length of *L* = 0.9 m is used to boil water. If the outer surface temperature of the resistance wire is *Ts* = 110°C, determine the temperature at the center of the wire.



We can use the equation for the temperature profile in a solid cylinder and set r = 0 to get the temperature at the center of the wire. To apply this equation we must first compute the heat source term, which is the heat generated per

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unit volume. We are given the total heat generation of 2 kW and we divide by the volume of the wire to get the heat generation term.

$$\dot{e}_{gen} = \frac{\dot{Q}_{gen}}{V} = \frac{2\ kW}{\pi R^2 L} = \frac{2000\ W}{\pi(0.002\ m)^2(0.09\ m)} = 1.768x10^8\ \frac{W}{m^3}$$

We now apply the equation for the temperature in a solid cylinder with heat generation using r = 0 to get the center temperature.

$$T = T_{outer\ surface} + \frac{\dot{e}_{gen}\left(R^2 - r^2\right)}{4k} = 110^oC + \frac{\frac{1.768x10^8\ W}{m^3}\left[(0.002\ m)^2 - 0\right]}{4\frac{20\ W}{m\cdot{}^oC}} = 118.8^oC$$

2.80 Consider a long solid cylinder of radius r_o = 4 cm and thermal conductivity k = 25 W/m·°C. Heat is generated in the cylinder uniformly at a rate of $\dot{e}_{gen}$ =35W/cm^3. The side surface of the cylinder is maintained at a constant temperature of T_s = 80°C. The variation of temperature in the cylinder is given by

$$T = T_s + \frac{r_o^2\dot{e}_{gen}}{4k}\left[1 - \left(\frac{r}{r_o}\right)^2\right]$$

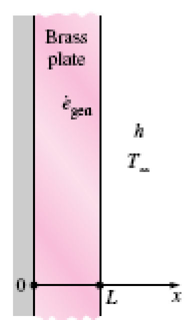
Based on this relation, determine (*a*) if the heat conduction is steady or transient, (*b*) if it is one-, two-, or three-dimensional, and (*c*) the value of heat flux on the side surface of the cylinder at $r = r_o$.

(a and b) The equation for temperature has a single independent variable, r. There is no time in the equation. Hence we conclude that the process is (a) steady and (b) one-dimensional.

Taking the first derivative of this equation gives the heat flux.

$$\dot{q} = -k\frac{dT}{dr} = \frac{r\dot{e}_{gen}}{2k} = \frac{1}{2}\frac{35\ W}{cm^3}\left(\frac{100\ cm}{m}\right)^3(0.02\ m) = \frac{3.5x10^5\ W}{m^2}$$

2.87 Consider a large 5-cm-thick brass plate (k = 111 W/m · °C) in which heat is generated uniformly at a rate of 2 x 10^5 W/m^3. One side of the plate is insulated while the other side is exposed to an environment at 25°C with a heat transfer coefficient of 44 W/m^2· °C. Explain where in the plate the highest and the lowest temperatures will occur, and determine their value. (Figure at right taken from Çengel, *Heat and Mass Transfer*.)



Since no heat leaves from the plate at x = 0 we expect the maximum temperature to occur there. The lowest temperature should occur at the other side of the plate. The total heat generation is the product of the heat generation times the volume, AL. Since all this heat is convected away from the plate at x = L, we can use the heat balance at that location to determine the surface temperature, T_L at x = L.

$$\dot{Q}_{conv} = hA(T_L - T_\infty) = \dot{e}_{gen} AL \quad \Rightarrow \quad T_L = T_\infty + \frac{\dot{e}_{gen} L}{h} = 25^o C + \frac{\frac{2x10^5 W}{m^3}(0.05\ m)}{\frac{44\ W}{m^2}} 252.3^o C$$

We can use the equation for the heat flux and apply the result that the heat flux is 0 at x = 0 to get an equation for the temperature at this point. Substituting the given data provides the desired result.

$$\dot{q} = \dot{e}_{gen} x - \frac{k(T_{x=L} - T_{x=0})}{L} - \frac{\dot{e}_{gen} L}{2} \quad \Rightarrow \quad T_{x=0} = T_{x=L} + \frac{L}{k}\left(\dot{q}_{x=0} + \frac{\dot{e}_{gen} L}{2}\right)$$

$$T_{x=0} = 252.3^o C + \frac{0.05\ m}{\frac{111 W}{m \cdot {}^o C}} \left(0 + \frac{\frac{2x10^5\ W}{m^3}(0.05\ m)}{2} \right) = 254.6^o C$$

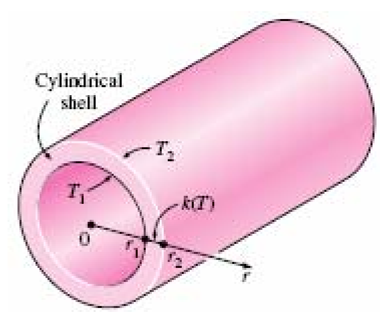
2.101C When the thermal conductivity of a medium varies linearly with temperature, is the average thermal conductivity always equivalent to the conductivity value at the average temperature?

A linear variation has the form k = a + bT. Using the average temperature gives a midpoint thermal conductivity value of a + b(T_1 + T_2)/2. Using the definition of average thermal conductivity with k = a + bT gives the following result.

$$\bar{k} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} k dT = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} (a + bT) dT = \frac{a(T_2 - T_1) + b\left(T_2^2 - T_1^2\right)/2}{T_2 - T_1} = a + b\frac{T_2 + T_1}{2}$$

Thus the average thermal conductivity is always the same as the thermal conductivity at the average temperature if the thermal conductivity varies linearly with temperature.

2.103 Consider a cylindrical shell of length *L*, inner radius r_1, and outer radius r_2 whose thermal conductivity varies linearly in a specified temperature range as $k(T) = k_0(1 + \beta T)$ where k_0 and β are two specified constants. The inner surface of the shell is maintained at a constant temperature of T_1, while the outer surface is maintained at T_2. Assuming steady one-dimensional heat transfer, obtain a relation for (*a*) the heat transfer rate through the wall and (*b*) the temperature distribution *T*(*r*) in the shell.



In this problem we can follow the basic process in the notes on heat generation to solve the basic differential equation for one-dimensional heat transfer in a cylindrical shell with heat generation. That differential equation is.

$$\frac{1}{r}\frac{d}{dr}rk\frac{dT}{dr}+\dot{e}_{gen}=0 \qquad [1]$$

Multiplying by r and doing an indefinite integration one time gives

$$\int d\left(rk\frac{dT}{dr}\right)+\int \dot{e}_{gen}r^n dr + C_1 = rk\frac{dT}{dr}+\int \dot{e}_{gen}rdr + C_1 = 0 \qquad [2]$$

If the heat generation is not a function of distance the heat generation term simply becomes $\dot{e}_{gen}r^2/2$, and a second integration (after multiplying by dr/r) gives

$$\int k_0(1+\beta T)dT+\frac{1}{2}\int \dot{e}_{gen}rdr + C_1\int\frac{dr}{r} = k_0\left(T+\beta\frac{T^2}{2}\right)+\frac{\dot{e}_{gen}r^2}{4}+C_1\ln r + C_2 = 0 \qquad [3]$$

We can evaluate the two constants of integration by applying the boundary conditions that the temperatures at r = r_1 and r = r_2 are T_1 and T_2, respectively.

$$k_0\left(T_1+\beta\frac{T_1^2}{2}\right)+\frac{\dot{e}_{gen}r_1^2}{4}+C_1\ln r_1 + C_2 = 0 \qquad [4]$$

$$k_0\left(T_2+\beta\frac{T_2^2}{2}\right)+\frac{\dot{e}_{gen}r_2^2}{4}+C_1\ln r_2 + C_2 = 0 \qquad [5]$$

Subtracting equation [4] from equation [5] gives

$$k_0\left(T_2-T_1+\beta\frac{T_2^2-T_1^2}{2}\right)+\frac{\dot{e}_{gen}\left(r_2^2-r_1^2\right)}{4}+C_1\left(\ln r_2-\ln r_1\right)=0 \qquad [6]$$

We can solve this equation for C_1.

$$C_1=-\frac{k_0\left(T_2-T_1+\beta\frac{T_2^2-T_1^2}{2}\right)+\frac{\dot{e}_{gen}\left(r_2^2-r_1^2\right)}{4}}{\ln\frac{r_2}{r_1}} \qquad [7]$$

Substituting this result into equation [5] or [6] gives the value of C_2. Once the constants are known, the temperature profile is given by the following modification of equation [3].

$$\frac{k_0\beta}{2}T^2+k_0T(+)+\left(\frac{\dot{e}_{gen}r^2}{4}+C_1\ln r + C_2\right)=0 \qquad [8]$$

This quadratic equation in temperature can be solved for T for any given value of r.

$$T=\frac{-k_0\pm\sqrt{k_0{}^2-4\frac{k_0\beta}{2}\left(\frac{\dot{e}_{gen}r^2}{4}+C_1\ln r+C_2\right)}}{k_0\beta} \qquad [9]$$

The heat transfer at any value of r can be found from equation [2] and the definition of heat transfer.

$$rk\frac{dT}{dr} = -\frac{\dot{e}_{gen}r^2}{2} - C_1 \quad \Rightarrow \quad \dot{Q} = A\dot{q} = 2\pi rL\left(-k\frac{dT}{dr}\right) = 2\pi L\left(\frac{\dot{e}_{gen}r^2}{2} + C_1\right) \qquad [10]$$

We can apply this equation for any value of r between r_1 and r_2,.using the value of C_1 given by equation [7]

If the heat generation term is zero, our equation for C_1 becomes

$$C_1 = -\frac{k_0\left(T_2 - T_1 + \beta\frac{T_2^2 - T_1^2}{2}\right)}{\ln\frac{r_2}{r_1}} = -\frac{k_0(T_2 - T)\left(1 + \beta\frac{T_2^2 - T_1^2}{2(T_2 - T)}\right)}{\ln\frac{r_2}{r_1}} = -\frac{\bar{k}(T_2 - T)}{\ln\frac{r_2}{r_1}} \qquad [11]$$

Substituting this value for C_1 into equation [10] with the heat generation term set to zero gives the heat flow as

$$\dot{Q} = -\frac{2\pi L\bar{k}(T_2 - T)}{\ln\frac{r_2}{r_1}} \qquad [12]$$

Substituting the value for C_1 from equation [11] into equation [4] with the heat generation term set to zero gives

$$k_0\left(T_1 + \beta\frac{T_1^2}{2}\right) - \frac{\bar{k}(T_2 - T)}{\ln\frac{r_2}{r_1}}\ln r_1 + C_2 = 0 \quad \Rightarrow \quad C_2 = \frac{\bar{k}(T_2 - T)}{\ln\frac{r_2}{r_1}}\ln r_1 - k_0\left(T_1 + \beta\frac{T_1^2}{2}\right) \qquad [13]$$

The temperature for no heat generation is found by substituting equations [] and [13] into equation [9]

$$T = \frac{-k_0 \pm \sqrt{k_0{}^2 - 4\frac{k_0\beta}{2}\left(-\frac{\bar{k}(T_2 - T_1)}{\ln\frac{r_2}{r_1}}\ln r + \frac{\bar{k}(T_2 - T_1)}{\ln\frac{r_2}{r_1}}\ln r_1 - k_0\left(T_1 + \beta\frac{T_1^2}{2}\right)\right)}}{k_0\beta} \qquad [14]$$

$$T = -\frac{1}{\beta} \pm \sqrt{\frac{1}{\beta^2} + \frac{2}{k_0\beta}\left(-\frac{\bar{k}(T_2 - T_1)}{\ln\frac{r_2}{r_1}}\ln\frac{r}{r_1} + k_0\left(T_1 + \beta\frac{T_1^2}{2}\right)\right)} \qquad [15]$$