Radiation Exchange

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Mechanical Engineering 375

Heat Transfer

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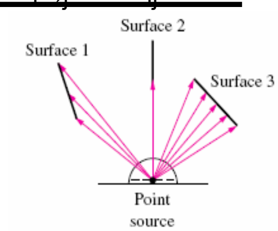
Outline

- Basics of radiation interchange
- Limit considerations to diffuse surfaces
- Definition of view factor
- Finding view factors
- Use of view factor in radiation exchange
- Exchange among several diffuse surfaces

View Factor, $F_{i\rightarrow j}$ or F_{ij}

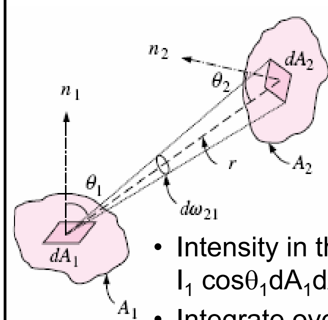
- $F_{i\rightarrow j}$ or F_{ij} is the fraction of radiation, leaving surface i, that strikes surface j
  – Use easier to write notation F_{ij} or more descriptive, $F_{i\rightarrow j}$



- Fraction of radiation from point striking surfaces 1 to 3 varies with orientation

Figure 13-1 from Çengel, *Heat and Mass Transfer*

Deriving F_{ij} Equation



- Intensity leaving surface dA_1 at angle θ_1 is $I_1 \cos\theta_1 dA_1$
- Solid angle $d\omega_{21} = dA_2\cos\theta_2/r^2$
- Intensity in this direction in $d\omega_{21}$ is $I_1 \cos\theta_1 dA_1 dA_2 \cos\theta_2/r^2$
- Integrate over both areas

Figure 13-2 from Çengel, *Heat and Mass Transfer*

View Factor F_{ij}

$$F_{12} = \frac{1}{A_1}\int_{A_2}\int_{A_1}\frac{\cos\theta_1\cos\theta_2}{\pi r^2}dA_1 dA_2$$

- Looking at radiation from surface 2 to surface 1 produces similar result

$$F_{21} = \frac{1}{A_2}\int_{A_1}\int_{A_2}\frac{\cos\theta_2\cos\theta_1}{\pi r^2}dA_2 dA_1$$

- **Important result:** $A_1F_{12} = A_2F_{21}$
  – Note that $F_{12} \neq F_{21}$

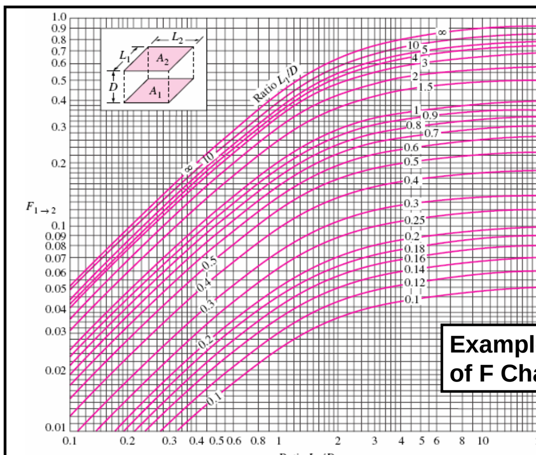
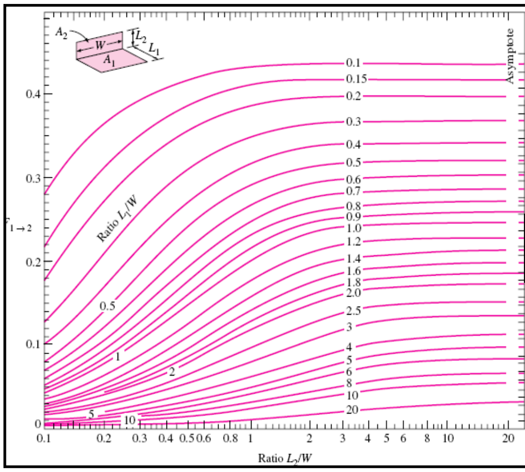


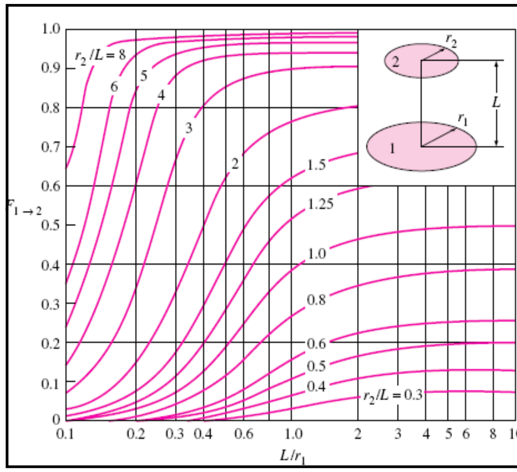
Figure 13-5 from Çengel, *Heat and Mass Transfer*

Example of F Chart



Another F Chart

Figure 13-6 from Çengel, *Heat and Mass Transfer*

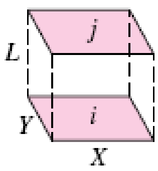


Yet Another F Chart

Figure 13-7 from Çengel, *Heat and Mass Transfer*

View Factor Equations

- Example for aligned parallel rectangles ($\bar{X} = X/L$, $\bar{Y} = Y/L$)
  - Because $A_i = A_j$, $F_{ij} = F_{ji}$ for this case



$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\frac{1}{2}\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{(1+\bar{X}^2+\bar{Y}^2)}\right] + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

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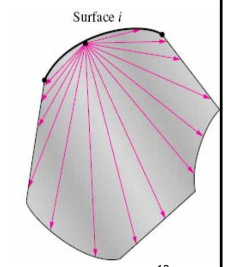
Figure from Table 13-1 from Çengel, *Heat and Mass Transfer*

Working with View Factors

- Equations in Tables 13-1 and 13-2
  - Can program into your calculators
- Charts in Figures 13-5 to 13-8 of text
- Reciprocity relation $F_{ji} = A_iF_{ij}/A_j$
- Summation rule for each surface in an enclosure

$$\sum_{j=1}^{N} F_{ij} = 1$$



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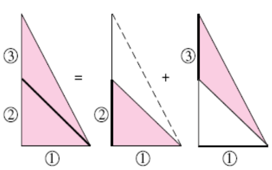
Working with View Factors II

- Superposition Rule
- $F_{1\to 2+3} = F_{1\to 2} + F_{1\to 3}$
- Use to get unknown shape factors



- In this example we can find $F_{1\to 2+3}$ and $F_{1\to 2}$ from Figure 13-6 then compute $F_{1\to 3}$
- Can extend superposition to multiple areas: $F_{1\to 2+3+4} = F_{1\to 2} + F_{1\to 3} + F_{1\to 4}$

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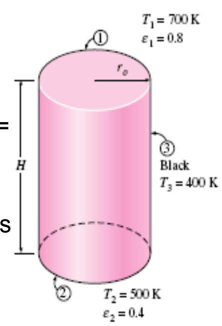
View-Factor Problem

- Find all view factors for the cylindrical enclosure with $H = r_0 = 1$ m
  - From chart for $r_1/L = r_2/L = 1$, $F_{12} = F_{21} = 0.38$
  - $F_{11} = F_{22} = 0$
  - Summation of view factors in enclosure: $F_{11} + F_{12} + F_{13} = 1$, so $F_{13} = 1 - 0.38 - 0 = 0.62$



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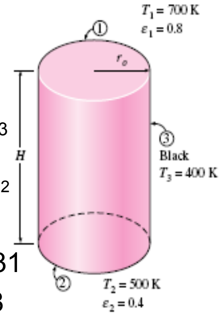
View-Factor Problem II

- Now have $F_{12} = F_{21} = 0.38$, $F_{11} = F_{22} = 0$, and $F_{13} = 0.62$
- Reciprocal: $A_3F_{31} = A_1F_{13}$
  - $A_1 = \pi(1\text{ m})2 = \pi\text{ m}^2$
  - $A_3 = 2\pi(1\text{ m})(1\text{ m}) = 2\pi\text{ m}^2$
  - $F_{31} = A_1F_{13}/A_3 = 0.31$
- Symmetry $F_{32} = F_{31} = 0.31$
- $F_{33} = 1 - F_{31} - F_{32} = 0.38$



Radiation Exchange

- First consider black surfaces then extend to gray diffuse surfaces
- Heat transfer from surface 1 reaching surface 2 is $A_1F_{12}\sigma T_1^4$
- Heat transfer from surface 2 reaching surface 1 is $A_2F_{21}\sigma T_2^4 = A_1F_{12}\sigma T_2^4$
- Net heat transfer from surface 1 is $A_1F_{12}\sigma(T_1^4 - T_2^4)$
  - Negative value indicates heat into surface 1

Black Enclosures

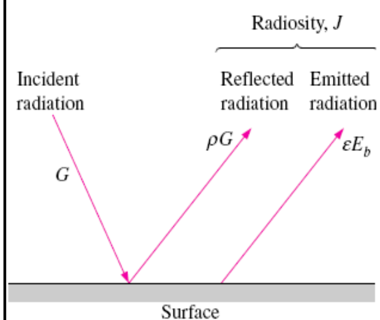
- In an enclosure all heat leaving a surface strikes one of the other surfaces
- Net heat from surface i to surface j from previous slide is $A_iF_{ij}\sigma(T_i^4 - T_j^4)$
- Net heat from surface i to all other surfaces is sum of terms like this

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i\to j} = \sum_{j=1}^{N} A_i F_{ij}\sigma\left(T_i^4 - T_j^4\right)$$

Gray Diffuse Opaque Enclosures

- For a gray, diffusive surface Kirchoff's law applies to the total hemispherical quantities: $\alpha = \varepsilon$
- For opaque surfaces $\tau = 0$ so $\alpha + \rho = 1$
- For gray, diffusive, opaque surfaces then $\rho = 1 - \alpha = 1 - \varepsilon$
- For nonblack surfaces have to consider emitted and reflected radiation

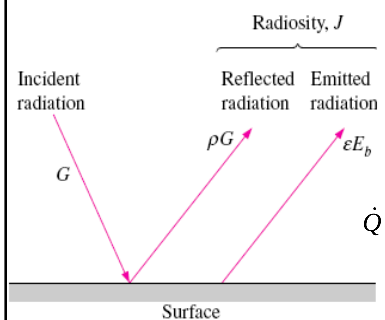
Radiosity, $J = \rho G + \varepsilon E_b$



- Radiosity, the total radiation leaving the surface, is the sum of emitted and reflected radiation

 Figure 13-20 from Çengel, *Heat and Mass Transfer*

Net Radiation Leaving Surface



- $\dot{Q} = A(J - G)$
- $J = \rho G + \varepsilon E_b = (1 - \varepsilon)G + \varepsilon E_b$

$$G = \frac{J - \varepsilon E_b}{1 - \varepsilon}$$

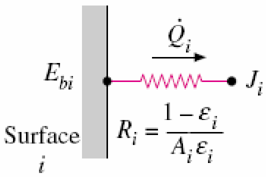
$$\dot{Q} = A\left(J - \frac{J - \varepsilon E_b}{1 - \varepsilon}\right)$$

$$\boxed{\dot{Q} = \frac{A\varepsilon}{1 - \varepsilon}\left(E_b - J\right)} \quad \text{A}$$

 Figure 13-20 from Çengel, *Heat and Mass Transfer*

Circuit Analogs

- Equation A on previous slide can be viewed as circuit
  - E_{bi} and J_i are potentials
  - Heat transfer from surface i is flow
  - Resistance is $(1 - \varepsilon_i)/A_i\varepsilon_i$



$$\dot{Q}_i = \frac{A_i\varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right)$$

$$\dot{Q}_i = \frac{E_{bi} - J_i}{\frac{1-\varepsilon_i}{A_i\varepsilon_i}} = \frac{E_{bi} - J_i}{R_i}$$

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Radiation Exchange

- Net heat transfer from surface i to surface j is radiation leaving i that strikes j ($A_iF_{ij}J_i$) minus radiation leaving j that strikes i ($A_jF_{ji}J_j = A_iF_{ij}J_j$)

$$\dot{Q}_{i\leftrightarrow j} = \dot{Q}_{ij} = A_iF_{ij}J_i - A_jF_{ji}J_j = A_iF_{ij}J_i - A_iF_{ij}J_j = A_iF_{ij}\left(J_i - J_j\right)$$

- Net heat transfer from surface i to all other surfaces is sum of all these terms

$$\dot{Q}_i = \sum_{j=1}^{N}\dot{Q}_{ij} = \sum_{j=1}^{N}A_iF_{ij}\left(J_i - J_j\right) \quad \boxed{B}$$

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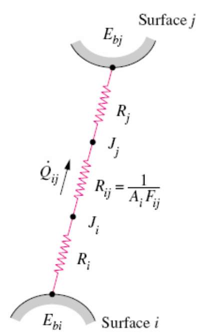
Another Circuit Analogy

$$\dot{Q}_{i\leftrightarrow j} = \dot{Q}_{ij} = A_iF_{ij}\left(J_i - J_j\right) = \frac{J_i - J_j}{R_{ij}}$$

- Next step in circuit analog is flow between radiosity J_i and J_j
- Here $R_{ij} = 1/A_iF_{ij}$
- With previous resistance between E and J have a complete circuit



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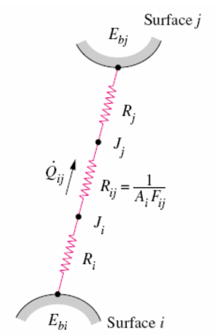
Using Circuit Analogy

- Look at simple enclosure with only two surfaces
- Apply circuit analogy with total resistance

$$\dot{Q}_{12} = \frac{E_{b1} - E_{b2}}{R_{Total}} = \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1\varepsilon_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_2}{A_2\varepsilon_2}}$$



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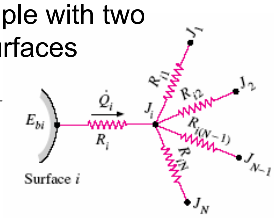
One More Circuit Analogy

- Extend previous example with two surfaces to multiple surfaces

$$\dot{Q}_{i\leftrightarrow j} = \dot{Q}_{ij} = A_iF_{ij}\left(J_i - J_j\right) = \frac{J_i - J_j}{R_{ij}}$$

$$R_{ij} = \frac{1}{A_iF_{ij}}$$



- Each surface will have similar resistance between surface (at potential E_{bi}) and corresponding radiosity, J_i

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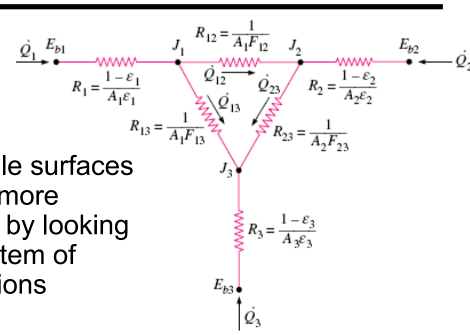
Three-Surface Circuit

- Multiple surfaces done more easily by looking at system of equations



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Radiation Exchange Equations

- Equation A $\dot{Q}_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right)$
- Equation B $\dot{Q}_i = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right)$

$$\frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{bi} - J_i\right) = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right)$$

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Radiation Exchange Equations II

- Eliminate $\dot{Q}_i$ from equations A and B

$$\dot{Q}_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{b_i} - J_i\right) = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) \quad i = 1, \ldots, N$$

- This gives N simultaneous equations to solve for N values of J_i
- We first find the areas, A_i, and view factors, F_{ij}, and emissivities, ε_i
- Know $E_{bi} = \sigma T_i^4$

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Radiation Exchange Solution

- Two possible surface conditions: (1) known temperature, (2) known $\dot{Q}_i$

$$\dot{Q}_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{b_i} - J_i\right) = \sum_{j=1}^{N} A_i F_{ij}\left(J_i - J_j\right) \quad i = 1, \ldots, N$$

(1) $J_i\left(1 + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N_T} F_{ij}\right) - \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N_T} F_{ij} J_j = E_{b_i} = \sigma T_i^4$

(2) $\left(\sum_{j=N_T+1, j\neq i}^{N} A_i F_{ij}\right) J_i - \sum_{j=N_T+1, j\neq i}^{N} A_i F_{ij} J_j = \dot{Q}_i$

Solve this set of N simultaneous equations for N values of J_i

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Radiation Exchange Solution II

- Once all J_i values are known we can compute unknown values of T_i and $\dot{Q}_i$
  - For known T_i

$$\dot{Q}_i = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(E_{b_i} - J_i\right) = \frac{A_i \varepsilon_i}{1-\varepsilon_i}\left(\sigma T_i^4 - J_i\right)$$

  - For known $\dot{Q}_i$

$$E_{b_i} = J_i + \frac{1-\varepsilon_i}{A_i \varepsilon_i}\dot{Q}_i \quad \Rightarrow \quad T_i = \frac{1}{\sigma}\sqrt[4]{J_i + \frac{1-\varepsilon_i}{A_i \varepsilon_i}\dot{Q}_i}$$

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Three Surface Enclosure

- Have three equations (known T case)

$$J_i\left(1 + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij}\right) - \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij} J_j = E_{b_i} = \sigma T_i^4$$

$$\left[1 + \frac{1-\varepsilon_1}{\varepsilon_1}\left(F_{12} + F_{13}\right)\right] J_1 - \frac{1-\varepsilon_1}{\varepsilon_1} F_{12} J_2 - \frac{1-\varepsilon_1}{\varepsilon_1} F_{13} J_3 = \sigma T_i^4$$

$$-\frac{1-\varepsilon_2}{\varepsilon_2} F_{21} J_1 + \left[1 + \frac{1-\varepsilon_2}{\varepsilon_2}\left(F_{21} + F_{23}\right)\right] J_2 - \frac{1-\varepsilon_2}{\varepsilon_2} F_{23} J_3 = \sigma T_2^4$$

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Three Surface Enclosure II

- General equation and third equation

$$J_i\left(1 + \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij}\right) - \frac{1-\varepsilon_i}{\varepsilon_i}\sum_{j=1, j\neq i}^{N} F_{ij} J_j = E_{b_i} = \sigma T_i^4$$

$$-\frac{1-\varepsilon_3}{\varepsilon_3} F_{31} J_1 - \frac{1-\varepsilon_3}{\varepsilon_3} F_{32} J_2 + \left[1 + \frac{1-\varepsilon_3}{\varepsilon_3}\left(F_{31} + F_{32}\right)\right] J_3 = \sigma T_3^4$$

- Find coefficients and use linear equation solver to find J_1, J_2, and J_3

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