Heat Exchangers

Larry Caretto
Mechanical Engineering 375

Heat Transfer

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Outline

- Basic ideas of heat exchangers
- Overall heat transfer coefficient
- Log-mean temperature difference method
- Effectiveness –NTU method
- Practical considerations

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Heat Exchangers

- Used to transfer energy from one fluid to another
- Typically one fluid is cooled while the other is heated
- May have phase change: temperature of one or both fluids is constant
- Simplest is double pipe heat exchanger
  - Parallel flow and counter flow

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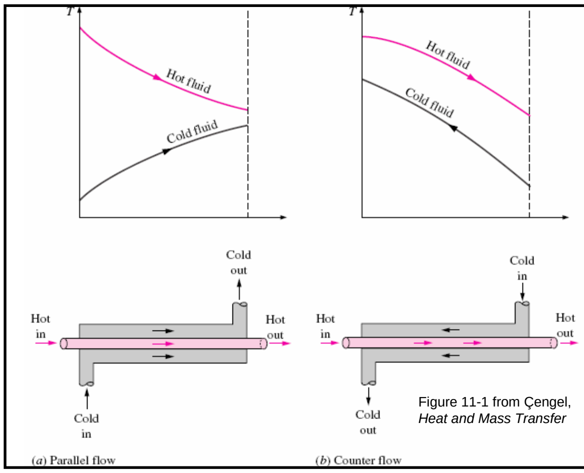

Figure 11-1 from Çengel, *Heat and Mass Transfer*

Compact Heat Exchangers

- Have large surface area for heat exchange per unit volume
  - Achieved by use of fins
  - Car and truck radiations best example
  - Operate in cross flow
  - Fluids said to be mixed or unmixed
    - Mixed: one flow passage for the fluid
    - Unmixed: several flow passages

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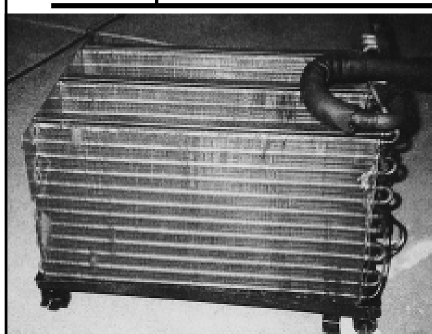
Compact Heat Exchangers II



- Compact heat exchanger in home air conditioner
  - Refrigerant in tubes, air through fins

Figure 11-2 from Çengel, *Heat and Mass Transfer*

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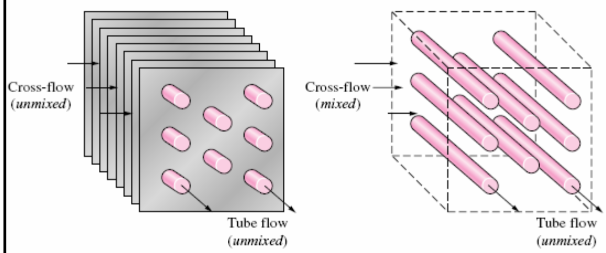
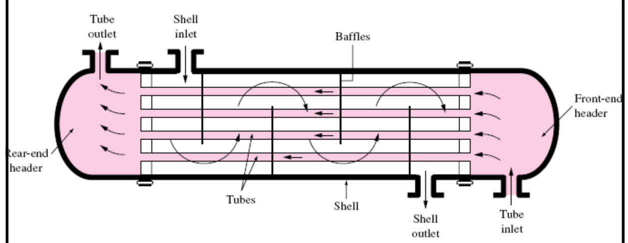
Compact Heat Exchangers III



(a) Both fluids unmixed (b) One fluid mixed, one fluid unmixed

Figure 11-3 from Çengel, *Heat and Mass Transfer* 7

Shell-and-Tube Exchanger



- Counter flow exchanger with larger surface area; baffles promote mixing

Figure 11-4 from Çengel, *Heat and Mass Transfer* 8

Tube and Shell Passes

- Previous chart showed one shell pass and one tube pass
  - No cases where flow changed direction completely
- Number of shell or tube passes is the number of times a fluid in the shell (or tubes) flows in a reverse direction
  - Examples next charts

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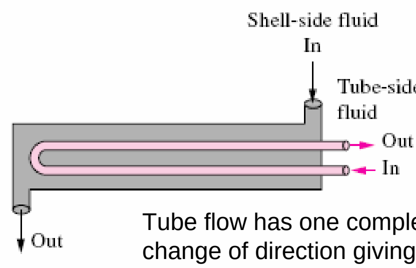
Shell and Tube Passes II

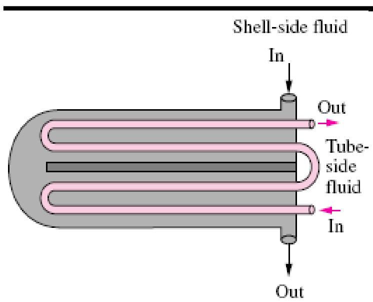


Tube flow has one complete change of direction giving two tube passes

(a) One-shell pass and two-tube passes

Figure 11-5(a) from Çengel, *Heat and Mass Transfer* 10

Shell and Tube Passes III



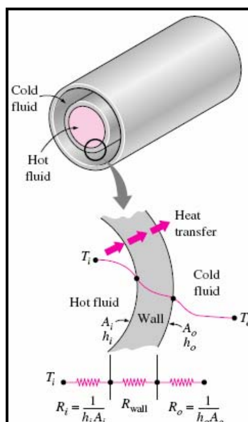
Tube flow has three complete changes of direction giving four tube passes

Shell flow changes direction to give two shell passes

(b) Two-shell passes and four-tube passes

Figure 11-5(b) from Çengel, *Heat and Mass Transfer* 11

Overall U



U is overall heat transfer coefficient

Analyzed here for double-pipe heat exchanger

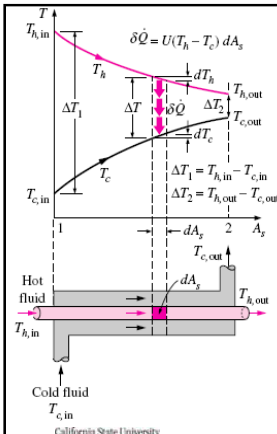
$$R = \frac{1}{h_i A_i} + R_{wall} + \frac{1}{h_o A_o}$$

$$= \frac{1}{U_o A_o} = \frac{1}{U_i A_i} = \frac{1}{UA}$$

Figure 11-7 from Çengel, *Heat and Mass Transfer* 12

Heat Exchange Analysis

- Heat transfer from hot to cold fluid $\dot{Q} = UA\Delta T$
- First law energy balances $\dot{Q} = \dot{m}_c c_{p_c}\left(T_{c,out} - T_{c,in}\right)$ $\dot{Q} = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right)$
- Assumes no heat loss to surroundings
  - Subscripts c and h denote cold and hot fluids, respectively
  - Alternative analysis for phase change



Analysis

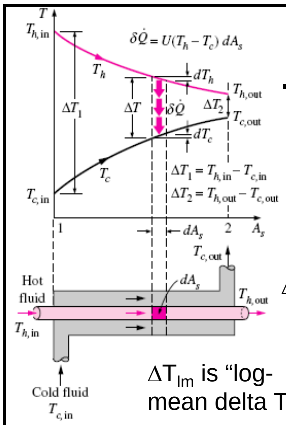
For no external heat transfer, combine three equations for differential area, dA, and integrate from 1 to 2

$$d\dot{Q} = U\left(T_h - T_c\right)dA$$

$$d\dot{Q} = \dot{m}_c c_{p_c} dT_c$$

$$d\dot{Q} = -\dot{m}_h c_{p_h} dT_h$$

Figure 11-14 from Çengel, *Heat and Mass Transfer*



Analysis Result

Parallel flow heat exchanger with no external heat transfer

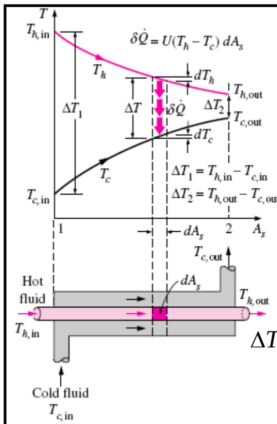
$$\dot{Q} = UA\Delta T_{lm}$$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

ΔT_{lm} is “log-mean delta T”

ΔT_1 and ΔT_2 are both $T_h - T_c$

Figure 11-14 from Çengel, *Heat and Mass Transfer*



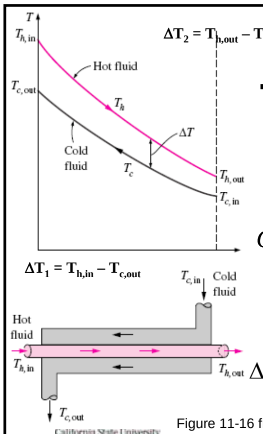
Analysis Result II

Parallel flow heat exchanger $\dot{Q} = UA\Delta T_{lm}$

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)}$$

$$\Delta T_{lm} = \frac{\left(T_{h,out} - T_{c,out}\right) - \left(T_{h,in} - T_{c,in}\right)}{\ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right)}$$

Figure 11-14 from Çengel, *Heat and Mass Transfer*



Counter Flow

Same basic equations

- Difference in ΔT_1 and ΔT_2 definitions

$$\dot{Q} = UA\Delta T_{lm} = UA\frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

$$\Delta T_{lm} = \frac{\left(T_{h,out} - T_{c,in}\right) - \left(T_{h,in} - T_{c,out}\right)}{\ln\left(\frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}}\right)}$$

Figure 11-16 from Çengel, *Heat and Mass Transfer*

What if $\Delta T_2 = \Delta T_1$?

- Apply l'Hopital's rule to show that $\Delta T_{lm} = \Delta T_1 = \Delta T_2$ in this case

$$\lim_{\Delta T_2 - \Delta T_1 \to 0} \Delta T_{lm} = \lim_{\Delta T_2 - \Delta T_1 \to 0} \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)} = \lim_{\Delta T_2 - \Delta T_1 \to 0} \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2 - \Delta T_1 + \Delta T_1}{\Delta T_1}\right)}$$

$$= \lim_{\Delta T_2 - \Delta T_1 \to 0} \frac{\Delta T_2 - \Delta T_1}{\ln\left(1 + \frac{\Delta T_2 - \Delta T_1}{\Delta T_1}\right)} = \lim_{\Delta T_2 - \Delta T_1 \to 0} \frac{1}{\frac{1}{1 + \frac{\Delta T_2 - \Delta T_1}{\Delta T_1}}\frac{1}{\Delta T_1}} = \Delta T_1$$

  - Last step takes derivative of numerator and denominator with respect to $\Delta T_2 - \Delta T_1$ for constant ΔT_1 in denominator

Heat Exchanger Problems

- With ΔT_{lm} method we want to find U or A when all temperatures are known
- If we know three temperatures, we can find the fourth by an energy balance with known mass flow rates (and c_p's)

$$\dot{Q} = \dot{m}_c c_{p_c}\left(T_{c,out} - T_{c,in}\right)$$
$$\dot{Q} = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right)$$

Can find $\dot{Q}$ from two temperatures for one stream and then find unknown temperature

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Problem

- An counterflow heat exchanger with U = 200 W/m²·°C is to be used to cool 1 kg/s of oil (c_p =2000 J/kg·°C) from 100°C to 30°C using 3 kg/s of water (c_p = 4184 J/kg·°C) at 20°C. What area is required?
- **Given:** $T_{h,in}$ = 100°C, $T_{h,out}$ = 30°C, $T_{c,in}$ = 20°C, U = 200 W/m²·°C, $c_{p,c}$ = 4184 J/kg·°C, $c_{p,h}$ = 2000 J/kg·°C, $\dot{m}_h$ = 1 kg/s, and $\dot{m}_c$ = 3 kg/s. **Find:** A

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Solution

- **Equations:**

$$\dot{Q} = UA\Delta T_{lm} = UA\frac{\left(T_{h,out} - T_{c,in}\right) - \left(T_{h,in} - T_{c,out}\right)}{\ln\left(\frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}}\right)}$$
$$= \dot{m}_c c_{p_c}\left(T_{c,out} - T_{c,in}\right) = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right)$$

$$\dot{Q} = \dot{m}_h c_{p_h}\left(T_{h,in} - T_{h,out}\right) = \frac{1\,kg}{s}\frac{2000\,J}{kg\cdot{}^oC}\left(100^oC - 30^oC\right) = 140000\,\frac{J}{s}$$

$$T_{c,out} = \frac{\dot{Q}}{\dot{m}_c c_{p_c}} + T_{c,in} = \frac{140000\,\frac{J}{s}}{\frac{3\,kg}{s}\frac{4184\,J}{kg\cdot{}^o\,C}} = 31.2\,{}^oC$$

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Solution II

$$\Delta T_{lm} = \frac{\left(T_{h,out} - T_{c,in}\right) - \left(T_{h,in} - T_{c,out}\right)}{\ln\left(\frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}}\right)} =$$
$$\frac{\left(30^oC - 20^oC\right) - \left(100^oC - 31.2^oC\right)}{\ln\left(\frac{30^oC - 20^oC}{100^oC - 31.2^oC}\right)} = 30.5^oC$$

$$A = \frac{\dot{Q}}{U\Delta T_{lm}} = \frac{140000\,\frac{J}{s}\frac{W\cdot s}{1\,J}}{\frac{200\,W}{m^2\cdot{}^oC}\left(30.5^oC\right)}$$

$$A = 22.9\,m^2$$

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Other Configurations

- Use correction factor $\Delta T_{lm} = F_{corr}\Delta T_{lm,CF}$
- $\Delta T_{lm,CF}$ = is ΔT_{lm} for counter flow

$$\Delta T_{lm,CF} = \frac{\left(T_{h,out} - T_{c,in}\right) - \left(T_{h,in} - T_{c,out}\right)}{\ln\left(\frac{T_{h,out} - T_{c,in}}{T_{h,in} - T_{c,out}}\right)}$$

- F_{corr} depends on temperatures on tube side (t_{in} and t_{out}) and shell side (T_{in} and T_{out}) through two parameters, R and P

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Correction Factor

- Correction factor parameters, R and P
  - Shell and tube definitions below

$$P = \frac{T_{tube,out} - T_{tube,in}}{T_{shell,in} - T_{tube,in}} = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_{shell,in} - T_{tube,in}}{T_{tube,out} - T_{tube,in}} = \frac{T_1 - T_2}{t_2 - t_1} = \frac{\left(\dot{m}c_p\right)_{tube}}{\left(\dot{m}c_p\right)_{shell}}$$

  - Correction factor charts show diagrams that illustrate the equations for P and R

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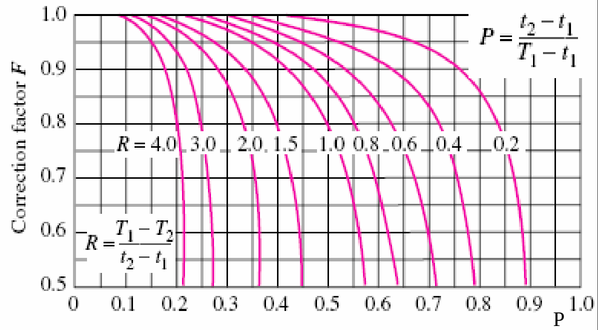


(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes

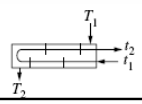
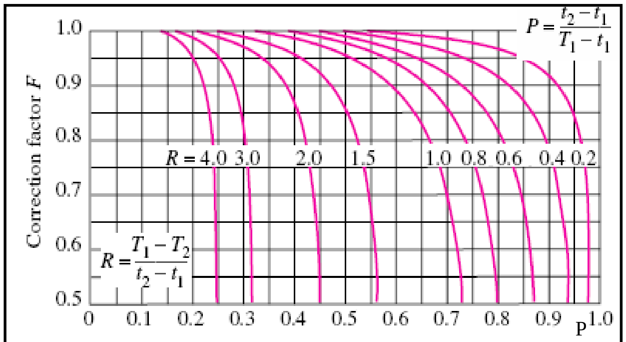
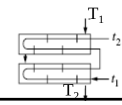


Figure 11-18 from Çengel, *Heat and Mass Transfer*



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



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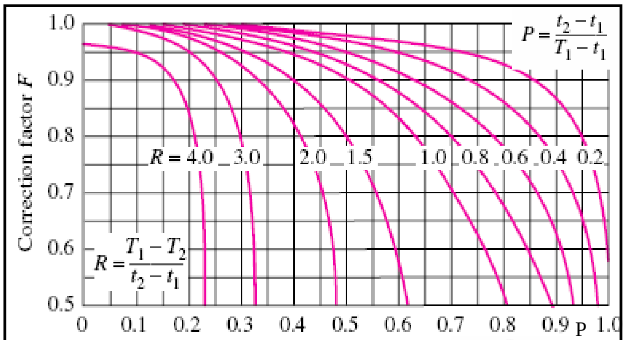


(*c*) Single-pass cross-flow with both fluids *unmixed*

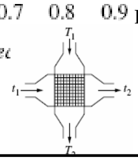
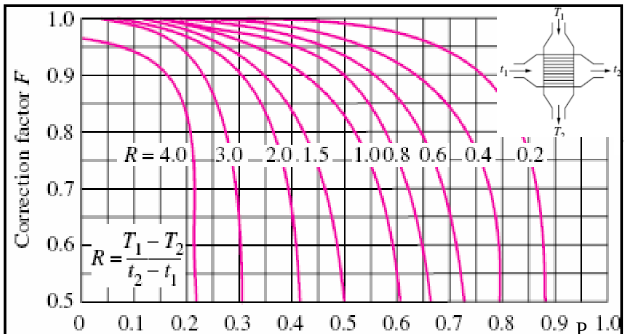


Figure 11-18 from Çengel, *Heat and Mass Transfer*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

$P = \frac{t_2 - t_1}{T_1 - t_1}$

Figure 11-18 from Çengel, *Heat and Mass Transfer*

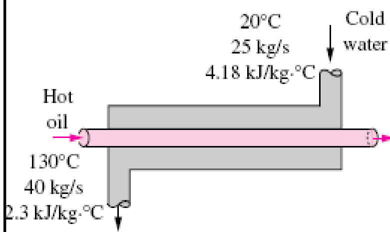
Effectiveness-NTU Method

- Used when not all temperatures are known
- Based on ratio of actual heat transfer to maximum possible heat transfer
- Maximum possible temperature difference, ΔT_{max} is $T_{h,in} - T_{c,in}$
  - Only one fluid, the one with the smaller value of $\dot{m}c_p$, can have ΔT_{max}
  - Define $C_c = (\dot{m}c_p)_c$ and $C_h = (\dot{m}c_p)_h$

Effectiveness, ε

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\dot{Q}}{C_{\min}\left(T_{h.in} - T_{c,in}\right)} \qquad C_{\min} = \min\left(C_h, C_c\right)$$

- In effectiveness-NTU method we find ε, then find $\dot{Q} = \varepsilon\dot{Q}_{max}$
  - Use $C_{min}\Delta T_{max}$ to find $\dot{Q}_{max}$ because $C_1\Delta T_1 = C_2\Delta T_2$ or $\Delta T_2 = C_1\Delta T_1/C_2$
  - If $\Delta T_2 = \Delta T_{max}$ and $C_1/C_2 > 1$, $\Delta T_2 > \Delta T_{max}$
  - $C_{min}\Delta T_{max}$ is maximum heat transfer that can occur without impossible T < Tc,in



$C_c = \dot{m}_c c_{pc} = 104.5$ kW/°C

$C_h = \dot{m}_c c_{ph} = 92$ kW/°C

$C_{min} = 92$ kW/°C

$\Delta T_{max} = T_{h,in} - T_{c,in} = 110$°C

$\dot{Q}_{max} = C_{min}\,\Delta T_{max} = 10{,}120$ kW

Max

In example, $\Delta T_{max} = \Delta T_{oil}$ and $\Delta T_{water} =$ (92) ΔT_{max} / 104.5

If we used $C_{max}\Delta T_{max}$ to get Q_{max} = (104.5)(110) = 11,495 kW then ΔT_{oil} = (104.5)(110)/(92) = 124.9°C > ΔT_{max}

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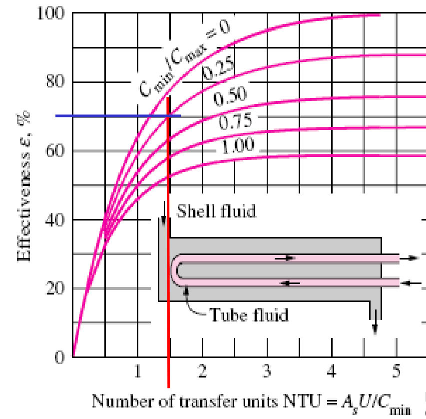

(c) One-shell pass and 2, 4, 6, … tube passes

Find ε

Example chart for finding effectiveness from NTU = UA/C_{min} and C_{min}/C_{max} ratio

For NTU = 1.5 and C_{min}/C_{max} = 0.25, ε = .7

Figure 11-26 from Çengel, *Heat and Mass Transfer*

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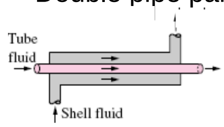
Effectiveness Equations

- Double pipe parallel flow

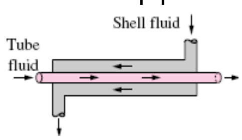
$$NTU = \frac{UA}{C_{min}}$$



$$\varepsilon = \frac{1 - e^{-NTU(1+c)}}{1+c}$$

- Double pipe counter flow

$$c = \frac{C_{min}}{C_{max}}$$

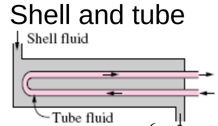


$$\varepsilon = \frac{1 - e^{-NTU(1-c)}}{1 - ce^{-NTU(1-c)}}$$

California State University Northridge Figures from Figure 11-26 from Çengel, *Heat and Mass Transfer* 33

More Effectiveness Equations

- Shell and tube



One shell pass and 2, 4, 6, … tube passes

$$c = \frac{C_{min}}{C_{max}}$$

$$\varepsilon = 2\left\{1 + c + \sqrt{1+c^2}\,\frac{1+e^{-NTU\sqrt{1+c^2}}}{1-e^{-NTU\sqrt{1+c^2}}}\right\}^{-1}$$

- Any geometry with c = 0

$$NTU = \frac{UA}{C_{min}}$$

$$\varepsilon = 1 - e^{-NTU}$$

California State University Northridge Figure from Figure 11-26 from Çengel, *Heat and Mass Transfer* 34

Problem

- An 25 m² counterflow heat exchanger with U = 200 W/m²·°C is to be used to cool 1 kg/s of oil (c_p = 2000 J/kg·°C) at 100°C using 3 kg/s of water (c_p = 4184 J/kg·°C) at 20°C. What is the oil outlet temperature.
- **Given:** $T_{h,in}$ = 100°C, $T_{c,in}$ = 20°C, U = 200 W/m²·°C, A = 25 m², $c_{p,c}$ = 4184 J/kg·°C, $c_{p,h}$ = 2000 J/kg·°C, $\dot{m}_h$ = 1 kg/s, and $\dot{m}_c$ = 3 kg/s. **Find:** $T_{h,out}$

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Solution

$$C_h = \dot{m}_h c_{p_h} = \frac{1\,kg}{s}\frac{2000\,J}{kg\cdot{}^oC} = \frac{2000\,J}{s\cdot{}^oC}$$

$$C_c = \dot{m}_c c_{p_c} = \frac{3\,kg}{s}\frac{4184\,J}{kg\cdot{}^oC} = \frac{12552\,J}{s\cdot{}^oC}$$

$$C_{min} = \frac{2000\,J}{s\cdot{}^oC}$$

$$NTU = \frac{UA}{C_{min}} = \frac{\frac{200\,W}{m^2\cdot{}^oC}\left(25\,m^2\right)}{\frac{2000\,J}{s\cdot{}^oC}\frac{W\cdot s}{1\,J}} = 2.5$$

$$c = \frac{C_{min}}{C_{max}} = \frac{\frac{2000\,J}{s\cdot{}^oC}}{\frac{12552\,J}{s\cdot{}^oC}} = 0.1593$$

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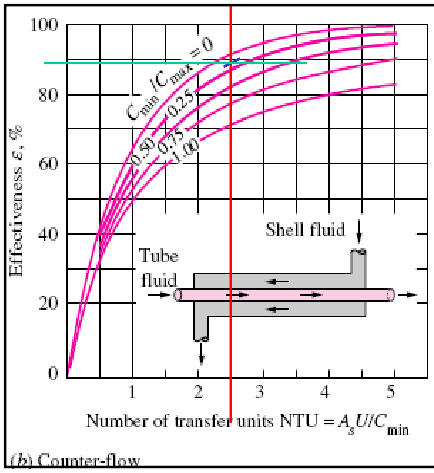
Chart ε



From chart for counter flow:

NTU = 2.5

C_{min}/C_{max} = 0.16

ε = 0.89

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Effectiveness Equation

- For counterflow heat exchangers (c = C_{min}/C_{max})

$$\varepsilon = \frac{1 - e^{-NTU(1-c)}}{1 - ce^{-NTU(1-c)}}$$

$$\varepsilon = \frac{1 - e^{-2.5(1-0.1593)}}{1 - 0.1593e^{-2.5(1-0.1593)}} = 0.895$$

$$\dot{Q} = \varepsilon \dot{Q}_{max} = \varepsilon C_{min}\left(T_{h.in} - T_{c,in}\right) = 0.895 \frac{2000\ J}{s \cdot {}^oC}\left(100^oC - 20^oC\right)$$

$$\boxed{\dot{Q} = 1.43x10^5 \frac{J}{s} = 143\ kW}$$

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Outlet Temperatures

- Use basic energy balance equations to find outlet temperatures from $\dot{Q}$

$$T_{c,out} = T_{c,in} + \frac{\dot{Q}}{\dot{m}_c c_{p_c}} = 20^oC + \frac{1.43x10^5 \frac{J}{s}}{\frac{3\ kg}{s}\frac{4184\ J}{kg \cdot {}^oC}} = 31.4^oC$$

$$T_{h,out} = T_{h,in} - \frac{\dot{Q}}{\dot{m}_c c_{p_c}} = 100^oC - \frac{1.43x10^5 \frac{J}{s}}{\frac{3\ kg}{s}\frac{4184\ J}{kg \cdot {}^oC}} = 28.4^oC$$

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Problem Comparison

- Comparing examples
  - The ΔT_{lm} and ε-NTU examples had the same mass flow rates, heat capacities and U values
  - The ε-NTU example had A = 25 m^2 compared to A = 22.9 m^2 for ΔT_{lm}
  - As expected, slightly larger area gives a slightly larger temperature change
    - Oil cools from 100°C to 30°C with A = 22.9 m^2 and to 28.4°C with A = 25 m^2

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