Heat Transfer with Fins

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Mechanical Engineering 375

Heat Transfer

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California State University Northridge

Outline

- Review previous material
- What is a fin and why do we use them?
- Examples of fins
- Analysis of fins
- Fin effectiveness

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Review Parallel Resistances

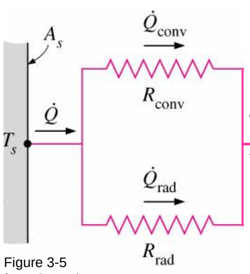

Figure 3-5 from Çengel, *Heat and Mass Transfer*

$$\frac{1}{R_{total}} = \frac{1}{R_{conv}} + \frac{1}{R_{rad}}$$

$$T_\infty = T_{surr} \qquad \frac{1}{R_{total}} = A_s h_{conv} + A_s h_{rad}$$

Define total heat transfer coefficient, h_{total}

$$\dot{Q} = \dot{Q}_{conv} + \dot{Q}_{rad} \qquad h_{total} = \frac{1}{A_s R_{total}} = h_{conv} + h_{rad}$$

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Review Slab with Convection

A is area normal to heat flow

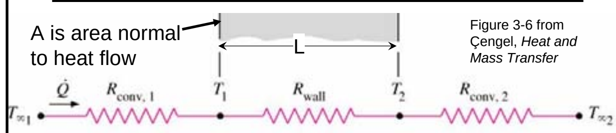

Figure 3-6 from Çengel, *Heat and Mass Transfer*

First find heat flux from h_1, h_2, L, k, $T_{\infty1}$, and $T_{\infty2}$

$$\dot{q} = \frac{\dot{Q}}{A} = \frac{T_{\infty1} - T_{\infty2}}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$$

Once heat flux is known, find T_1 and T_2

$$T_1 = T_{\infty1} - \frac{\dot{q}}{h_1} \qquad T_2 = T_{\infty2} + \frac{\dot{q}}{h_2}$$

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Review Composite Cylindrical Shell

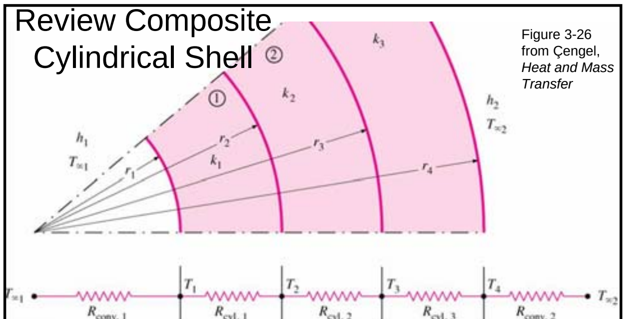

Figure 3-26 from Çengel, *Heat and Mass Transfer*

$$\dot{Q} = \frac{T_{\infty1} - T_{\infty2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{1}{2\pi k_1 L}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2\pi k_2 L}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{2\pi k_3 L}\ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_2 2\pi r_4 L}}$$

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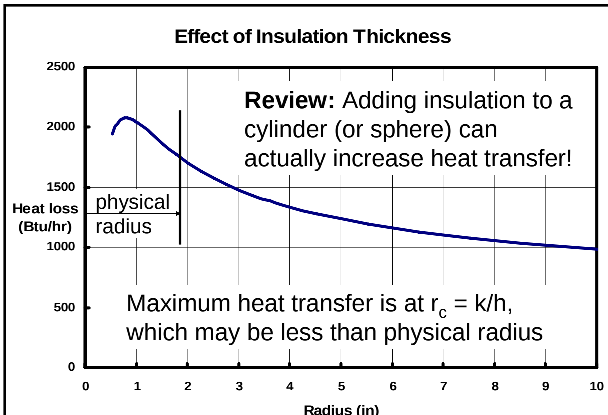

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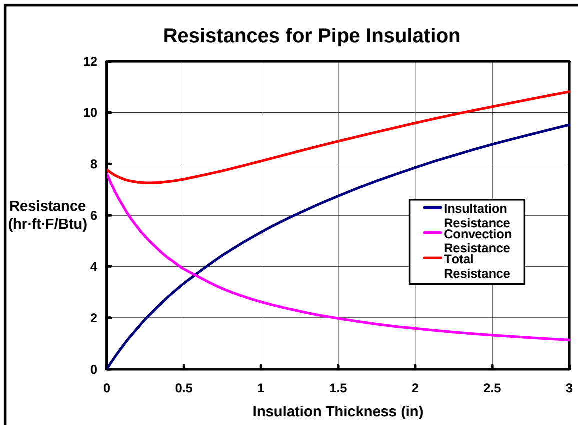

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Review Conduction Shape Factor

- Simplified analysis
  - for multidimensional geometries with each surface at a uniform temperature
  - Use shape factor, S, whose equation is found from tables like Çengel Table 3-7
  - Basic equation: $\dot{Q} = kS(T_1 - T_2)$
  - S must have dimensions of length
    - Equations for S depend on parameters in the different geometries

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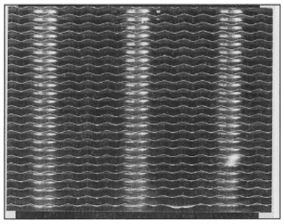
What is a Fin?

- A fin is a extended surface to increase area for convection heat transfer

$$\dot{Q} = hA_S\left(T_S - T_\infty\right)$$

- Goal is to increase A_s to increase $\dot{Q}$



- Automobile radiator is example of fin

Figure 3-33 from Çengel, *Heat Transfer*

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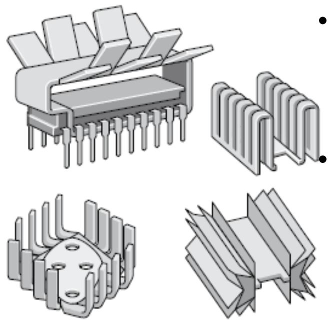
Innovative Fin Designs



- Important application is cooling of power electronics
- How do we analyze fin effectiveness?
  - Start with simple 1D fin

Figure 3-34 from Çengel, *Heat Transfer*

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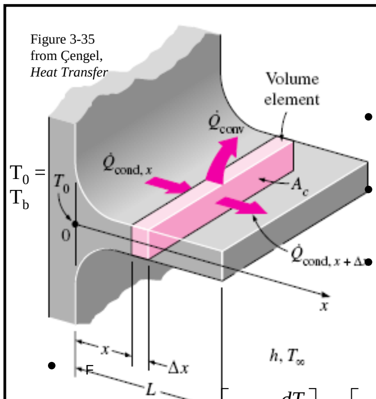
Figure 3-35 from Çengel, *Heat Transfer*



Analysis

- Conduction in x-direction
- Convection from fin surface
- Heat balance over differential volume element
  - p = perimeter

$$\left[-kA_c\frac{dT}{dx}\right]_x = \left[-kA_c\frac{dT}{dx}\right]_{x+\Delta x} + hp\Delta x\left(T - T_\infty\right)$$

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What is Fin Shape?

- Analysis does not consider shape of cross section, only area and perimeter
  - Derivation of fin equation assumes constant cross section
    - Fin may be in the shape of a uniform cylinder, but not a cone
- Is fin two dimensional?
  - Yes, but one-dimensional analysis is accurate if $h\delta/k < 0.2$, where δ = D for circular fins, thickness for rectangular fins

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Fin Equation and Solution

- $T_0 = T_b$ = base temperature at x = 0
- Define $\theta = T - T_\infty$ so $\theta_b = T_b - T_\infty$
- The fin equation for constant k and A_c is

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad m^2 = \frac{hp}{kA_c}$$

- The solution to this differential equation is:

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

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Boundary Conditions for C_1, C_2

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

- First condition: base temperature is T_b so $\theta = \theta_b = T_b - T_\infty$ at x = 0
- Alternatives for second condition
  - Infinitely long fin (requires $C_1 = 0$)
  - Negligible heat transfer from end at x = L
  - Convection and radiation heat transfer coefficient for end at x = L

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Infinitely Long Fin

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

- $C_1 = 0$ to keep solution finite
- At x = 0, $\theta = \theta_b$ = requires $C_2 = \theta_b$

$$\theta = \theta_b e^{-mx} \quad \Rightarrow \quad T - T_\infty = (T_b - T_\infty) e^{-x\sqrt{hp/kA_c}}$$

$$\dot{q} = -k\frac{dT}{dx} = -k(T_b - T_\infty)e^{-x\sqrt{hp/kA_c}}\left(-\sqrt{hp/kA_c}\right)$$

$$\dot{Q}_{x=0} = A_c\dot{q}_{x=0} = kA_c(T_b - T_\infty)\sqrt{hp/kA_c} = \sqrt{kA_c hp}\,(T_b - T_\infty)$$

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Zero Heat Transfer at x = L

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \quad \frac{d\theta}{dx} = \frac{d(T - T_b)}{dx} = \frac{dT}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

- At x = 0, $\theta = \theta_b$; at x = L, $d\theta/dx = 0$
- Algebraic details at end $\theta = \theta_b \frac{\cosh m(L-x)}{\cosh mL}$

$$\dot{q} = -k\frac{d\theta}{dx} = -k\theta_b\frac{-m\sinh m(L-x)}{\cosh mL} = \frac{k\theta_b m\sinh m(L-x)}{\cosh mL}$$

$$\dot{Q}_{x=0} = A_c\dot{q}_{x=0} = \frac{kA_c\theta_b m\sinh mL(L-x)}{\cosh mL} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL$$

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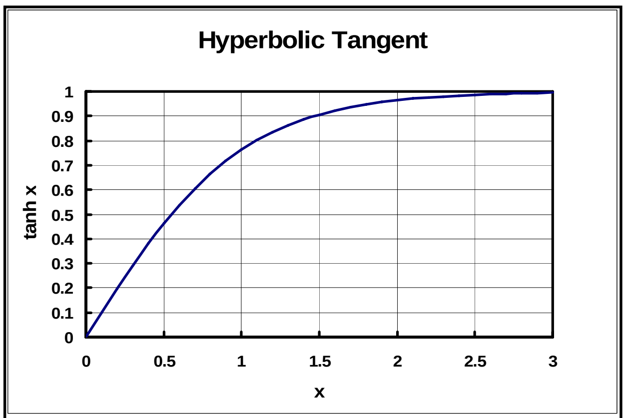

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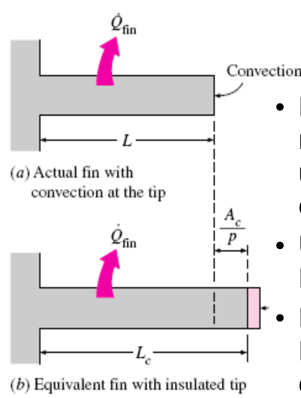
Convection



- For convection (+ radiation) at x = L use approximation of extra length
- Use adjusted length, L_c, to give area A_c
- Modified length, $L_c = L + A_c/p$, will give convection from end

California State University Northridge Figure 3-39 from Çengel, *Heat Transfer* 18

Problem

- A pin (cylindrical) fin has a diameter of 4 mm, a length of 5 cm, and a thermal conductivity of 200 W/m·K. If the heat transfer coefficient is 70 W/m²·K, with a surface temperature of 50ºC and an air temperature of 20ºC, what is the heat transfer with and without the fin?
- **Given:** k = 200 W/m·K, h = 70 W/m²·K, D = 0.004 m, L = 0.05 m, T_s = 50ºC, T_∞ = 20ºC, **Find:** $\dot{Q}$ with and without fin

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Solution

- Use equation for finite length fin with end correction for convection

$$\dot{Q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL_c \quad m = \sqrt{hp/kA_c} \quad L_c = L + A_c/p$$

$$p = \pi D = \pi(0.004\ m) = 0.01257\ m$$

$$A_c = \pi D^2/4 = \pi(0.004\ m)^2/4 = 0.00001257\ m^2$$

$$L_c = L + A_c/p = 0.05\,m + (0.00001257 m^2)/(0.01257\,m) = 0.051\,m$$

$$m = \sqrt{\frac{hp}{kA_c}} = \sqrt{\frac{70\,W}{m^2 \cdot K}\frac{m \cdot K}{200\,W}\frac{0.01257\,m}{0.00001257 m^2}} = 18.71\,m^{-1}$$

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Solution II

$$\dot{Q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL_c = (50-20)K \bullet$$

$$\sqrt{\frac{70\,W}{m^2 \cdot K}(0.01257\,m)(0.00001257\,m^2)\frac{200\,W}{m \cdot K}}\tanh\left[\frac{18.71}{m}(0.051\,m)\right]$$

$$\boxed{\dot{Q}_{fin} = 1.05\,W}$$

- Without fin available area is A_c

$$\dot{Q}_{no\ fin} = A_c h(T_b - T_\infty) = \frac{70\,W}{m^2 \cdot K}(0.00001257\,m^2)(30\,K) = 0.0264\,W$$

- Increase by factor of 39.64 (effectiveness, ε)

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Increase of 40 times?

- Factor of 40 increase in heat transfer is just for area of fin
  - A practical installation of small fins like this one would have several fins on a surface
  - Consider two areas on original surface
    - Area where pins occur will have increase by factor shown in fin analysis
    - Area without fins has usual flat surface result
  - Will consider this difference in determining fin effectiveness

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Fin Effectiveness

- Effectiveness, ε_{fin}, is ratio of heat transfer with fin to heat transfer with no fin

$$\varepsilon_{fin} = \frac{\dot{Q}_{fin}}{\dot{Q}_{no\ fin}} = \frac{\dot{Q}_{fin}}{hA_b(T_b - T_\infty)}$$

- A_b is fin area at base, same as A_c for fin with constant cross sectional area
- Want fin effectiveness greater than one to get additional heat transfer

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Constant A_c Fin Effectiveness

- Infinitely long fin $\dot{Q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)$
- Fin with end convection $\dot{Q}_{x=0} = \sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL$

$$\varepsilon_{\infty fin} = \frac{\dot{Q}_{x=0}}{hA_c(T_b - T_\infty)} = \frac{\sqrt{kA_c hp}\,(T_b - T_\infty)}{hA_c(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}$$

$$\varepsilon_{convfin} = \frac{\sqrt{kA_c hp}\,(T_b - T_\infty)\tanh mL}{hA_c(T_b - T_\infty)} = \sqrt{\frac{kp}{hA_c}}\tanh mL$$

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What makes a fin effective?

$$\varepsilon_{\infty fin} = \sqrt{\frac{kp}{hA_c}} \qquad \varepsilon_{convfin} = \sqrt{\frac{kp}{hA_c}}\tanh mL = \sqrt{\frac{kp}{hA_c}}\tanh\left(\sqrt{\frac{hp}{kA_c}}L\right)$$

- For infinite fin high k, low h and high ratio of p/A_c make a fin effective
- For pin fin, $p = \pi D$ and $A_c = \pi D^2/4$, so $p/A_c = 4/D$; small D gives better ε
- Effect of variables on fin with convection at end is not clear

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What makes a fin effective? II

$$\varepsilon_{convfin} = \sqrt{\frac{kp}{hA_c}}\tanh mL = \sqrt{\frac{kp}{hA_c}}\tanh\left(\sqrt{\frac{hp}{kA_c}}L\right)$$

- Choose a base case and vary each parameter from 0.1 to 10 times base value
  - Rectangular fin with width, w, and thickness, t, so that $A_c = tw$ and $p = 2(t + w)$
- Base case has k = 200 W/m·K, w = 0.03 m, t = 0.005 m, h = 70 W/m²·K, L = 0.08 m
  - Base mL_c = 1.05 and ε = 28.5

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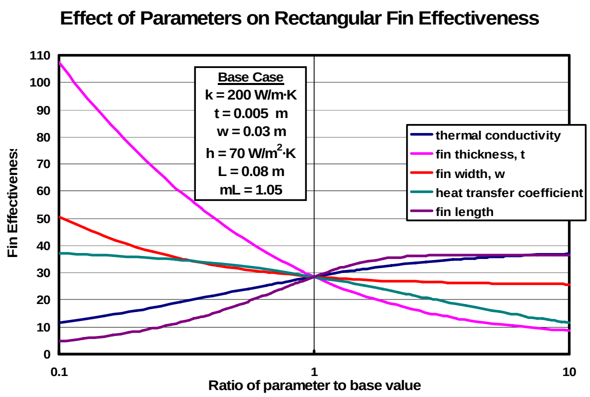

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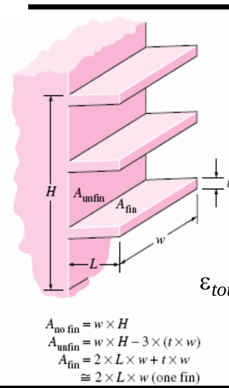
Overall Fin Effectiveness



- Original area, A = (area with fins, A_{fin}) + (area without fins, A_{unfin})

$$\frac{\dot Q_{fin}}{\dot Q_{no\ fin}} = \frac{h(\varepsilon_{fin}A_{fin} + A_{unfin})(T_b - T_\infty)}{hA(T_b - T_\infty)}$$

$$\varepsilon_{total} = \frac{\dot Q_{fin}}{\dot Q_{no\ fin}} = \left(\varepsilon_{fin}\frac{A_{fin}}{A} + \frac{A_{unfin}}{A}\right)$$

Figure 3-45 from Çengel, *Heat Transfer*

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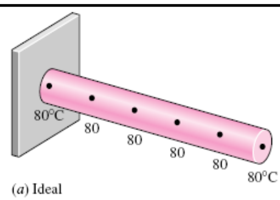
Fin Efficiency



for uniform cross section

$$A_{fin} = L_c p$$

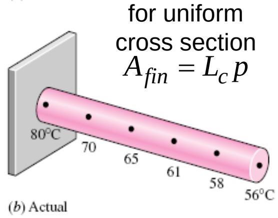


Figure 3-39 from Çengel, *Heat Transfer*

- Compare actual heat transfer to ideal case where entire fin is at base temperature

$$\eta_{fin} = \frac{\dot Q_{fin}}{\dot Q_{fin,\max}} = \frac{\dot Q_{fin}}{hA_{fin}(T_b - T_\infty)}$$

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Fin Efficiency II

- Relation to effectiveness

$$\eta_{fin} = \frac{\dot Q_{fin}}{\dot Q_{fin,\max}} = \frac{\varepsilon_{fin} hA_c(T_b - T_\infty)}{hA_{fin}(T_b - T_\infty)} = \frac{\varepsilon_{fin}A_c}{A_{fin}}$$

- Recall pin fin problem with D = 0.004 m and L_c = 0.051 m so $A_{fin} = pL_c = \pi DL_c$ = π(0.004 m)(0.051 m) = 0.0006409 m²
  - Previously showed ε = 39.64 so η = (39.64)($pD^2/4$)/(0.0006409 m²) = 0.777

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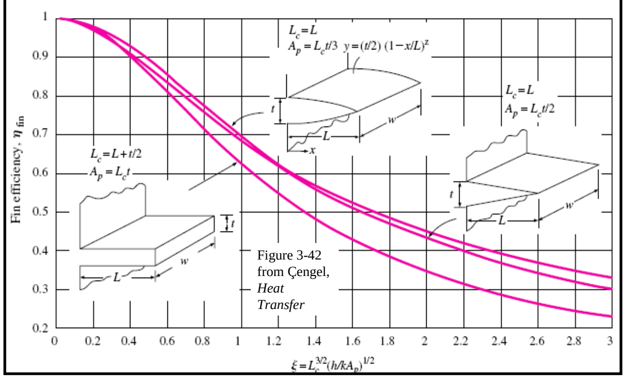
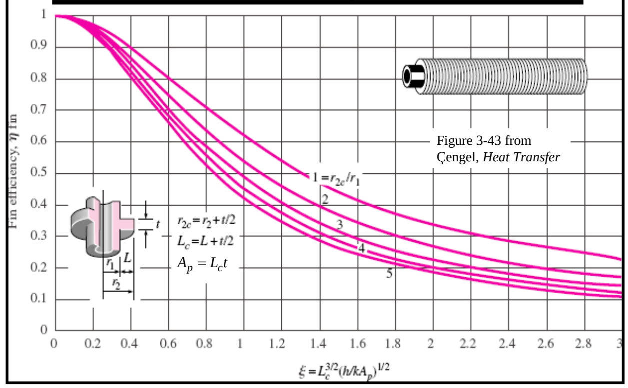
Fin Efficiency Data



Fin Efficiency Data II



Electronic Heat Sinks

- Designed to protect equipment like power transistors from overheating
- Characterized by thermal resistance, R, such that $\dot{Q}$ = power dissipation = $(T_{device} - T_\infty)/R$
- See Table 3-6 in Çengel for examples with R values
  - First part of table shown on next chart

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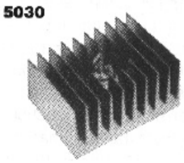
TABLE 3-6 From Çengel, *Heat Transfer*

Combined natural convection and radiation thermal resistance of various heat sinks used in the cooling of electronic devices between the heat sink and the surroundings. All fins are made of aluminum 6063T-5, are black anodized, and are 76 mm (3 in) long.

HS 5030		R = 0.9°C/W (vertical) R = 1.2°C/W (horizontal) Dimensions: 76 mm × 105 mm × 44 mm Surface area: 677 cm²
HS 6065		R = 5°C/W Dimensions: 76 mm × 38 mm × 24 mm Surface area: 387 cm²

Problem

- A power transistor that dissipates 120 W has a maximum operating temperature of 70°C. Cooling air is available at 25°C. Are any of the heat sinks on the previous page suitable for this transistor?
- **Given:** $\dot{Q}$ = 120 W, T_{surf} = 70°C, T_∞ = 25°C
- **Find:** Do devices have sufficient R

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Solution

$$\dot{Q} = \frac{T_{surf} - T_\infty}{R} \quad \Rightarrow \quad R = \frac{T_{surf} - T_\infty}{\dot{Q}} = \frac{70^oC - 25^oC}{40\,W} = \frac{1.125^oC}{W}$$

- Only the HS 5030 mounted vertically (with R = 0.9°C/W) will satisfy this cooling requirement
- The R values for the other heat sinks are too large (1.2°C/W and 5°C/W)

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Extra Charts

- The following charts show the details of the fin equation solution for no convection (dT/dx = 0) at x = L

Zero Heat Transfer at x = L

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \qquad \frac{d\theta}{dx} = \frac{d(T - T_b)}{dx} = \frac{dT}{dx} = mC_1 e^{mx} - mC_2 e^{-mx}$$

- At x = 0, $\theta = \theta_b$; at x = L, $d\theta/dx = dT/dx = 0$

$$\theta_b = C_1 + C_2 \qquad \left.\frac{d\theta}{dx}\right|_{x=L} = \left.\frac{dT}{dx}\right|_{x=L} = mC_1 e^{mL} - mC_2 e^{-mL} = 0$$

$$\theta_b = C_1 + C_1 e^{2mL} \qquad C_2 = C_1 e^{2mL}$$

$$C_1 = \frac{\theta_b}{1 + e^{2mL}} \qquad C_2 = C_1 e^{2mL} = \frac{\theta_b e^{2mL}}{1 + e^{2mL}}$$

Zero Heat Transfer at x = L II

- Combine results from last chart

$$\theta = C_1 e^{mx} + C_2 e^{-mx} \qquad C_1 = \frac{\theta_b}{1 + e^{2mL}} \qquad C_2 = \frac{\theta_b e^{2mL}}{1 + e^{2mL}}$$

$$\theta = \frac{\theta_b}{1 + e^{2mL}} e^{mx} + \frac{\theta_b e^{2mL}}{1 + e^{2mL}} e^{-mx} = \frac{\theta_b e^{mx} + \theta_b e^{2mL} e^{-mx}}{1 + e^{2mL}}$$

- Multiply fraction top and bottom by e^{-mL}

$$\theta = \frac{\theta_b}{1 + e^{2mL}} e^{mx} + \frac{\theta_b e^{2mL}}{1 + e^{2mL}} e^{-mx} = \frac{\theta_b e^{-mL} e^{mx} + \theta_b e^{mL} e^{-mx}}{e^{-mL} + e^{mL}}$$

Zero Heat Transfer at x = L III

- Rearrange exponential terms on last chart and introduce hyperbolic cosine, $\cosh(x) = (e^z + e^{-z})/2$

$$\theta = \frac{\theta_b e^{-mL} e^{mx} + \theta_b e^{mL} e^{-mx}}{e^{-mL} + e^{mL}} = \theta_b \frac{e^{m(x-L)} + e^{-m(x-L)}}{e^{-mL} + e^{mL}}$$

$$= \frac{2\theta_b \cosh m(L - x)}{2\cosh mL} = \frac{\theta_b \cosh m(L - x)}{\cosh mL}$$

- Final result: $\theta = \dfrac{\theta_b \cosh m(L - x)}{\cosh mL}$