Unit Twelve – Refrigeration and Air Standard Cycles

Mechanical Engineering 370

Thermodynamics

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Outline

- Two somewhat related topics this week
- Use assumptions for cycle analysis developed in the unit on Rankine cycles
- Refrigeration cycle fluids that exist in both a liquid and a gas during the cycle
- Air-standard cycles model combustion engines gases as air (an ideal gas) as the working fluid
  - Internal *vs*. external combustion

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Review Cycle Analysis Basics

- Basic assumptions
  - No line losses (output state of one device is input to the next device)
  - Work devices are isentropic ($w = \Delta h$)
  - Heat transfer has no work and $\Delta P = 0$
  - Exit from two-phase device is saturated
- Use actual data if available
- Account for different mass flow rates

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Unit Twelve Goals

- As a result of studying this unit you should be able to
  - calculate heat, work and coefficient of performance for refrigeration cycles
  - understand the assumptions for air-standard cycles
  - compute heat, work, and efficiency for otto, brayton, diesel and similar cycles

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Refrigeration Cycle

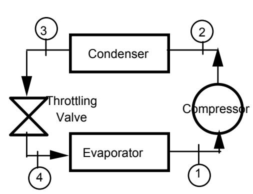
- Only required input is pressure (or temperature) of evaporator and condenser

Evaporator outlet is saturated vapor

Isentropic compressor

No work and $\Delta P = 0$ in condenser and evaporator

$\Delta h = 0$ for throttling valve



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Refrigeration Calculations

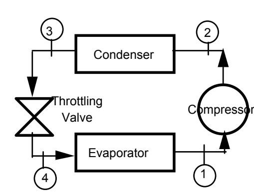
- $P_{evaporator} = P_1 = P_4 = P_{sat}(T_4 = T_1)$
- $P_{condenser} = P_2 = P_3 = P_{sat}(T_3 < T_2)$

State 1 is saturated vapor at $P_{evaporator}$

State 2, gas, $s_2 = s_1$ and $P_2 = P_{condenser}$

State 3 is saturated liquid at $P_{condenser}$

State 4, mixed, $h_4 = h_3$ and $P_4 = P_{evaporator}$



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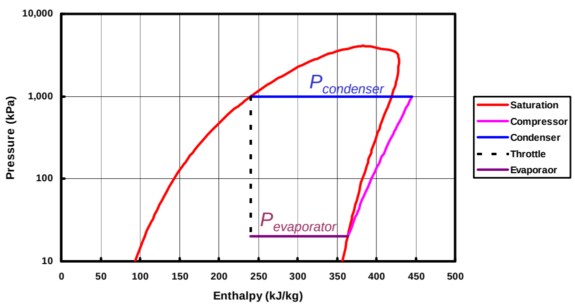
Refrigeration Cycle for R-134a



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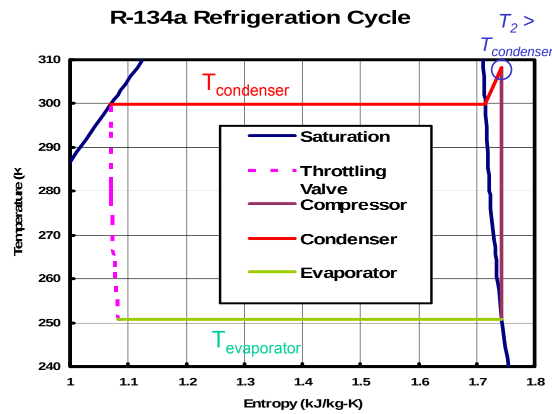

Refrigeration Calculations

- Given $P_{evaporator}$ and $P_{condenser}$
- $h_1 = h_g(P_{evaporator})$ and $s_1 = s_g(P_{evaporator})$
- $h_2 = h(P_{condenser}, s_2 = s_1)$
- $h_4 = h_3 = h_f(P_{condenser})$
- $|q_L| = q_{evaporator} = h_1 - h_4$
- $|w| = |w_{compressor}| = h_2 - h_1$
- COP = $|q_L| / |w|$

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Refrigeration Calculations II

- Given $T_{evaporator}$ and $T_{condenser}$
- $h_1 = h_g(T_{evaporator})$ $s_1 = s_g(T_{evaporator})$
- $P_2 = P_3 = P_{condenser} = P_{sat}(T_{condenser})$
- $h_2 = h(P_{condenser}, s_2 = s_1)$
- $h_4 = h_3 = h_f(T_{condenser})$
- $|q_L| = q_{evaporator} = h_1 - h_4$
- $|w| = |w_{compressor}| = h_2 - h_1$
- COP = $|q_L| / |w|$

Do not use $T_{condenser}$ to find h_2. $T_2 > T_{condenser}$

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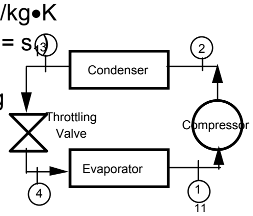
R-134a Example Calculation

- $P_{evaporator}$ = 100 kPa; $P_{condenser}$ =1 MPa
- Find: Coefficient of Performance, COP

$h_1 = h_g$(100 kPa) = 234.44 kJ/kg
$s_1 = s_g$(100 kPa) = .95183 kJ/kg•K
$h_2 = h(P_2 = P_{cond}$ = 1 MPa, $s_2 = s_1$) = 282.53 kJ/kg
$h_3 = h_f$(1 MPa) = 107.32 kJ/kg
$h_4 = h_3$= 107.32 kJ/kg



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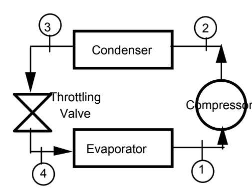
R-134a Example Answers

- $|w| = |w_{comp}| = |h_1 - h_2|$ = |231.35 kJ/kg – 279.14 kJ/kg = 48.09 kJ/kg
- $|q_L| = |q_{evap}| = |h_1 - h_4|$ = |231.35 kJ/kg – 105.29 kJ/kg = 129.15 kJ/kg

$$COP = \frac{|q_{evap}|}{|w_{comp}|} = \frac{h_1 - h_4}{h_2 - h_1}$$

$$COP = \frac{129.15\ kJ/kg}{48.09\ kJ/kg} = 2.69$$



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P_{evap} = 140 kPa; P_{cond} = 0.9 MPa

- $h_1 = h_g(P_{evaporator})$ $s_1 = s_g(P_{evaporator})$
- If $T_{evaporator}$ is given, $P_{evaporator} = P_{sat}$ at $T = T_{evaporator}$ so that $h_1 = h_g(T_{evaporator})$ and $s_1 = s_g(T_{evaporator})$
- $h_2 = h(P_{condenser}, s_2 = s_1)$
- Note if $T_{condenser}$ is given, you must find $P_{condenser} = P_{sat}(T_3)$.
- $h_4 = h_3 = h_f(P_{condenser})$

COP = ?

$$COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{239.16\frac{kJ}{kg} - 101.61\frac{kJ}{kg}}{280.21\frac{kJ}{kg} - 239.16\frac{kJ}{kg}} = 3.35$$

$$COP = \frac{|q_{evap}|}{|w_{comp}|} = \frac{h_1 - h_4}{h_2 - h_1}$$

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Air-Standard Cycles

- Otto cycle is model for spark-ignition (gasoline powered) engine
- Diesel (compression-ignition) cycle
- Brayton cycle for gas turbines
- Stirling cycle efficient but problems in implementation
  - Suggested use with solar collectors
- Others: Ericksson, Atkinson, Dual
- All have similar analysis

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Air-Standard Cycle Analysis

- Use air properties as ideal gas with variable or constant heat capacity
- Model chemical energy release as heat addition (~1,200 Btu/lb_m or 2,800 kJ/kg for Otto cycle engine)
- Heat addition at constant pressure, volume or temperature
- Isentropic work
- Closed system except Brayton Cycle

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Otto Cycle Definition

- Start at initial point with piston at bottom of cylinder filled with air at $v = v_1$
- Isentropic compression to v_2
  - Compression ratio, $CR = v_1/v_2$
- Constant volume heat addition at $v_2 = v_3$
- Isentropic expansion to $v_4 = v_1$
- Model exhaust as constant volume heat rejection

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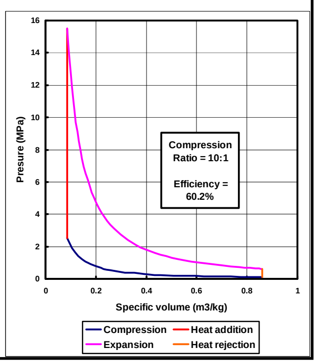
Otto Cycle

- **Given:** CR, P_1, T_1, q_H
- $v_1 = RT_1/P_1$
- $v_2 = v_1/CR$
- Isentropic compression to P_2
- $P_2 = P_1(CR)^k$
- $T_2 = P_2v_2/R$



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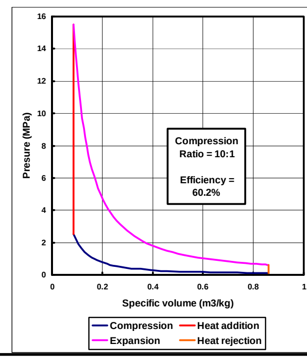
Otto Cycle Continued

- $T_3 = T_2 + q_H/c_v$
- $v_3 = v_2$
- $P_3 = RT_3/v_3$
- $v_4 = v_1 = (CR)v_3$
- $P_4 = P_3/(CR)^k$
- $T_4 = P_4 v_4/R$
- $|q_L| = c_v|T_1 – T_4|$
- $\eta = 1 - |q_L| / |q_H|$
- $\eta = 1 – (CR)^{1-k}$



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Otto Cycle Calculations

- **Given:** P_1 = 100 kPa; T_1 = 300 K; CR = 10; q_H = 2,800 kJ/kg
- For air, R = 0.287 kPa•m^3/kg•K
- $v_1 = RT_1/P_1$ = (0.287 kPa•m^3/kg•K) (300 K)/(100 kPa) = 0.861 m^3/kg
- $v_2 = v_1$ / CR = 0.0861 m^3/kg
- Continue with parallel computations for constant and variable heat capacity

Otto Cycle Calculations II

- Constant c_v

$P_2 = P_1(CR)^k$ = (100) (10)$^{1.4}$ = 2,510 kPa

$T_2 = P_2v_2/R$ = (2510) (0.0861) / (0.287) = 753.1 K

- Air tables

v_r(300 K) = 621.2

$v_r(T_2)$ = 621.2/10

Find: T_2 = 730.0 K

$u(T_2)$ = 536.10 kJ/kg

$P_2 = RT_2/v_2$ = (0.287) (730.0) / (0.0861) = 2,433 kPa

Otto Cycle Calculations III

- Constant c_v

$T_3 = T_2 + q_H/c_v$ = 753.1 K + (2800 kJ/kg) / (0.718 kJ/kg•K) = 4653 K

$v_3 = v_2$ (both calculations)

$P_3 = RT_3/v_3$ = (0.287) (4653) / (0.0861) = 15 510 kPa

- Air tables

$u(T_3) = u(T_2) + q_H$= 536.10 kJ/kg + 2800 kJ/kg = 3336.10 kJ/kg

Find: T_3 = 3512 K

$P_3 = RT_3/v_3$ = (0.287) (3512) / (0.0861) = 11 707 kPa

Otto Cycle Calculations IV

- Constant c_v

$P_4 = P_3/(CR)^k$ = (100) /(10)$^{1.4}$ = 617.4 kPa

$v_4 = v_1$ = 0.861 m^3/kg (both calculations)

$T_4 = P_4v_4/R$ = (617.4) (0.861)/(0.287) = 1852 K

$q_L = c_v(T_4 – T_1)$

- Air tables

v_r(3512 K) = .09918

$v_r(T_4)$ = .09918*10

Find: T_4 = 1854 K

$u(T_4)$ = 1537.42 kJ/kg

$u(T_1)$ = u(300 K) = 214.36 kJ/kg

Otto Cycle Calculations V

- Constant c_v

$q_L = c_v(T_4 – T_1)$ = (0.718 kJ/kg•K) (1852 K – 300 K) = 1114.3 kJ/kg

$\eta = 1 - |q_L|/|q_H|$ = 1 – (1114.3 kJ/kg) / (2800 kJ/kg)

<u>**η = 60.2%**</u>

- Air tables

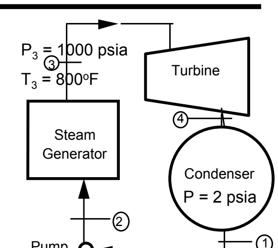
$q_L = u(T_4) – u(T_1)$ = 1537.42 kJ/kg – 214.36 kJ/kg = 1323.06 kJ/kg =

$\eta = 1 - |q_L|/|q_H|$ = 1 – (1323.06 kJ/kg) / (2800 kJ/kg)

<u>**η = 52.7%**</u>

Quiz Solution

- **Given**: Rankine cycle with data shown
- **Find:** Efficiency for (a) ideal cycle, (b) cycle with $\eta_{s,turbine}$ = 85% and $\eta_{s,pump}$ = 80%
- Point 1is saturated liquid; h_1 = 94.12 Btu/lb$_m$, v_1 = 0.016224 ft^3/lb$_m$
- $h_2 = h_1 + |w_P|$ = 94.12 Btu/lbm + 3.00 Btu/lb$_m$ = 97.12 Btu/lb$_m$



$$|w_P| = v_1(P_2 - P_1) = \frac{0.016224\ ft^3}{lb_m}(1000\ psia - 2\ psia)\frac{1\ Btu}{5.40395\ psia\cdot ft^3} = \frac{3.00\ Btu}{lb_m}$$

Quiz Solution II

- $h_3 = h(T_3,P_3) = h(800^oF, 1000\text{ psia}) = 1388.3$ Btu/lb$_m$ and $s_3 = s(T_3,P_3) = 1.5662$ Btu/lb$_m$·R
- $h_4 = h(P_{cond}, s_4 = s_3)$ is in mixed region

$$h_4 = h_f(P_{cond}) + x_4 h_{fg}(P_{cond}) = h_f(P_{cond}) + \frac{s_3 - s_f(P_{cond})}{s_{fg}(P_{cond})} h_{fg}(P_{cond}) =$$

$$\frac{94.12\ Btu}{lb_m} + \frac{\frac{1.5662\ Btu}{lb_m \cdot R} - \frac{0.1751\ Btu}{lb_m \cdot R}}{\frac{1.7445\ Btu}{lb_m \cdot R}} \frac{1021.8\ Btu}{lb_m} = \frac{908.92\ Btu}{lb_m}$$

$$\eta = \frac{w_{turb} - |w_P|}{q_{SG}} = \frac{h_3 - h_4 - |w_P|}{h_3 - h_2} = \frac{\frac{1382.4\ Btu}{lb_m} - \frac{908.92\ Btu}{lb_m} - \frac{3.00\ Btu}{lb_m}}{\frac{1382.4\ Btu}{lb_m} - \frac{97.12\ Btu}{lb_m}} = \boxed{36.6\%}$$

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Quiz Solution III

- With $\eta_s \neq 100\%$ h_1 and h_3 do not change
- Turbine work is 85% of work found above
- Pump work = 3.00 Btu/lb$_m$/80%
- $h_2 = h_1 + 3.00$ Btu/lb$_m$/80%

$$\eta = \frac{w_{turb} - |w_P|}{q_{SG}} = \frac{\eta_{s,T}(h_3 - h_4) - \frac{|w_{P,s}|}{\eta_{s,P}}}{h_3 - \left(h_1 - \frac{|w_{P,s}|}{\eta_{s,P}}\right)} = \frac{0.85\left(\frac{1382.4\ Btu}{lb_m} - \frac{908.92\ Btu}{lb_m}\right) - \frac{1}{0.80}\frac{3.00\ Btu}{lb_m}}{\frac{1382.4\ Btu}{lb_m} - \left(\frac{94.12\ Btu}{lb_m} + \frac{1}{0.80}\frac{3.00\ Btu}{lb_m}\right)}$$

$\boxed{\eta = 31.0\%}$

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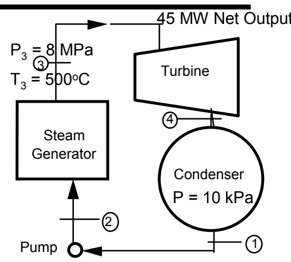
Quiz Eleven Solution

- **Given**: Rankine cycle with data shown
- **Find:** (a) Efficiency for ideal cycle
- Point 1 is saturated liquid; $h_1 = 191.78$ kJ/kg; $v_1 = 0.001010$ m³/kg
- $h_2 = h_1 + |w_P| = 191.78$ kJ/kg + 8.07 kJ/kg + 199.85 kJ/kg



$$|w_{P_1}| = v_1(P_2 - P_1) = \frac{0.001010\ m^3}{kg}(8000\ kPa - 10\ kPa)\frac{1\ kJ}{kPa \cdot m^3} = \frac{8.070\ kJ}{kg}$$

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Quiz Eleven Solution II

- $h_3 = h(T_3,P_3) = h(500^oC, 8\text{ MPa}) = 3397.8$ kJ/kg; $s_3 = s(8\text{ MPa}, 500^oC) = 6.7231$ kJ/kg·K
- $h_4 = h(P_{cond}, s_4 = s_3)$ is in mixed region

$$x_4 = \frac{s_4 - s_f(10\ kPa)}{s_{fg}(10\ kPa)} = \frac{6.7231\ kJ/kg \cdot K - 0.6487\ kJ/kg \cdot K}{7.5006\ kJ/kg \cdot K} = 0.8099$$

$h_4 = h_f + x_4 h_{fg}$ = 191.78 kJ/kg + (0.8099)(2392.4 kJ/kg) = 2129.3 kJ/kg
$q_H = h_3 - h_2$ = 3397.8 kJ/kg – 199.85 kJ/kg = 3198.0 kJ/kg
$|q_L| = |h_1 - h_4|$ = |191.81 kJ/kg – 2129.3 kJ/kg| = 1937.5 kJ/kg
Net work, $w = q_H - |q_L|$ = 3212.5 kJ/kg - 1961.8 kJ/kg = 1260.5 kJ/kg
The efficiency = w / q_H = (1260.5 kJ/kg) / (3198.0 kJ/kg)

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Quiz Eleven Solution III

- **Find:** (b) Mass flow rate of steam in cycle

$$\dot{m} = \frac{\dot{W}}{w} = \frac{45\ MW}{1260.5\ kJ/kg}\frac{1000\ kJ}{MW \cdot s} = \boxed{\frac{35.70\ kg}{s}}$$

- **Find:** (c) Cooling water temperature rise if its mass flow rate is 2000 kg/s

$$\dot{Q}_{cw} = |\dot{Q}_L| = \dot{m}_{cw}(h_{cw,out} - h_{cw,in}) = \dot{m}_{cw}c_{p,cw}(T_{cw,out} - T_{cw,in}) = \dot{m}_{cw}c_{p,cw}\Delta T_{cw}$$

$$\Delta T_{cw} = \frac{|\dot{Q}_L|}{\dot{m}_{cw}c_{p,cw}} = \frac{\dot{m}_{steam}|q_L|}{\dot{m}_{cw}c_{p,cw}} = \frac{\frac{35.702\ kg}{s}\frac{1937.5\ kJ}{kg}}{\frac{2000\ kg}{s}\frac{4.18\ kJ}{kg \cdot K}} = \boxed{8.27^oC}$$

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