Unit Eleven – Cycle Analysis

Mechanical Engineering 370

Thermodynamics

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November 18, 2010

California State University Northridge

Outline

- Quiz solution
- Analysis of thermodynamic cycles
  - Assumptions for cycle analysis: no line losses, isentropic work, isobaric heat with no added work, saturated fluids
- Rankine cycle analysis
  - Simple cycles
  - Cycles with corrections to assumptions
  - Cycles with additional components

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Schedule Reminder

- November 23 – Unit 11 Group Work
- November 25 – Give Thanks
- November 30 – Unit 11 Quiz (Unit 12 Lecture)
- December 2 – Unit 12 Group Work
- December 7 – Unit 12 Quiz (Review Lecture with questions on sample final questions)
- December 14 – Final exam (10:15 am to 12:15 pm)

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Unit Eleven Goals

- As a result of studying this unit you should be able to
  - understand and apply the assumptions of simple cycle analysis
    - No line losses (output state of one device is input to the next device)
    - Work devices are isentropic
    - Heat transfer has no work and $\Delta P = 0$
    - Exit from two-phase device is saturated

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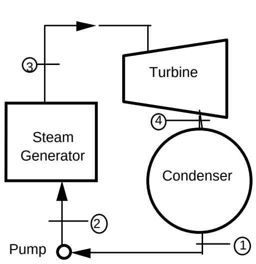
More Unit Eleven Goals

- Note that basic assumptions are not used if actual data are available
- Recognize the components of a simple Rankine cycle shown in the diagram to the right



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More Unit Eleven Goals

- Apply the equation below for Rankine cycle efficiency given only T_3, P_3 and P_{cond}

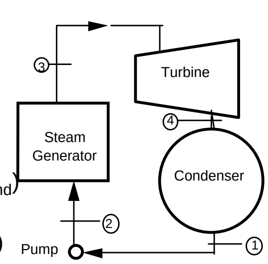
$$\eta = \frac{(h_3 - h_4) - |h_2 - h_1|}{h_3 - h_2}$$

$h_1 = h_f(P_{cond})$ & $v_1 = v_f(P_{cond})$

$h_2 = h_1 + v_1(P_3 - P_{cond})$

$h_3 = h(T_3,P_3)$; $s_3 = s(T_3,P_3)$

$h_4 = h(P_{cond}, s_4 = s_3)$



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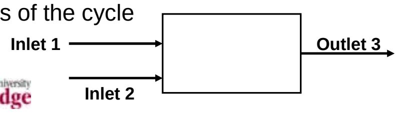
Unit Eleven Goals (concluded)

- Analyze a mixing device with two inlets (1 and 2) and one outlet (3) to get mass flow rate ratios

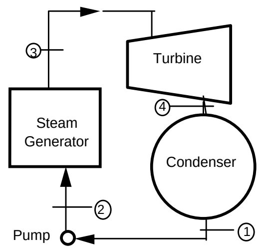
$$\frac{\dot{m}_2}{\dot{m}_3} = 1 - \frac{\dot{m}_1}{\dot{m}_3} = \frac{h_3 - h_1}{h_2 - h_1}$$

- Analyze variations on the Rankine cycle in which the mass flow rate differs in different parts of the cycle



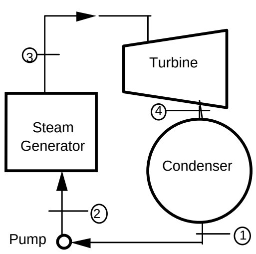
Sample Problem

Given: T_3 = 600°C;
P_3 = 10 MPa;
P_{cond} = 50 kPa
Find: Cycle efficiency
First get h values
$h_1 = h_f(P_{cond}) = h_f$(50 kPa) = 340.54 kJ/kg
$|w_P| = h_2 - h_1 = v_1(P_3 - P_1)$
$v_1 = v_f(P_{cond}) = v_f$(50 kPa)



Sample Problem Continued

$|w_P|$ = (0.001030 m³/kg) (10000 – 50) kPa = 10.25 kJ
$h_2 = h_1 + |w_P|$ = 340.54 + 10.25 = 350.79 kJ/kg
$h_3 = h(T_3,P_3)$; $s_3 = s(T_3,P_3)$
T_3 = 600°C; P_3 = 10 MPa
h_3 = 3625.8 kJ/kg
s_3 = 6.9045 kJ/kg·K
$h_4 = h(P_{cond}, s_4 = s_3)$



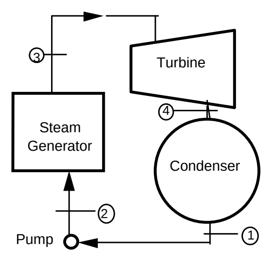
Answer to Example Problem

x_4 = 89.37%
h_4 = 2400.9 kJ/kg

$$\eta = \frac{(h_3 - h_4) - |h_2 - h_1|}{h_3 - h_2}$$

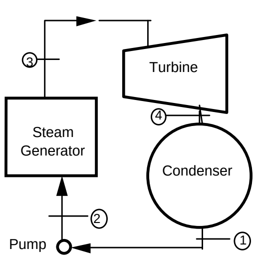
$$\eta = \frac{(3625.8 - 2400.9) - 10.25}{3625.8 - 350.79}$$

η = 37.1%



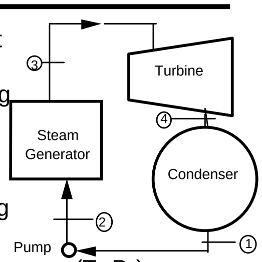
Exercise

Given: T_3 = 500°C;
P_3 = 15 MPa;
P_{cond} = 40 kPa
Find: Cycle efficiency
Find property values
Compute work in turbine and pump
Compute heat transfer in steam generator
Compute efficiency



Exercise Solution

State 1 is saturated liquid at P_1 = 40 kPa h_1 = 317.62 kJ/kg; v_1 = 0.001026 m³/kg
$|w_P|$ = (0.001027 m³/kg) (15000 – 40) kPa = 15.40 kJ/kg
$h_2 = h_1 + |w_P|$ = 333.02 kJ/kg
T_3 = 500°C; P_3 = 15 MPa
$h_3 = h(T_3,P_3)$ = 3310.8 kJ/kg; $s_3 = s(T_3,P_3)$ = 6.3480 kJ/kg·K
$h_4 = h(P_{cond}, s_4 = s_3)$ = 2173.68 (mixed region)



Exercise Solution II

$h_4 = h(P_{cond}, s_4 = s_3)$

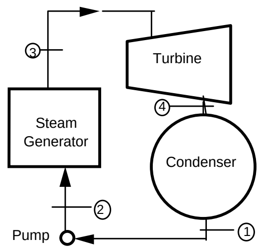
$x_4 = 80.03\%$

$h_4 = 2173.68$ kJ/kg

$$\eta = \frac{(h_3 - h_4) - |h_2 - h_1|}{h_3 - h_2}$$

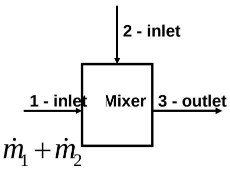
$$\eta = \frac{(3310.8 - 2173.68) - 15.40}{3310.8 - 333.02}$$

η = 37.6%



Mixing of Two Streams

- Steady, adiabatic, no useful work, negligible changes in kinetic and potential energies

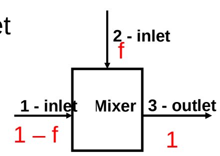


$$\dot{m}_3 h_3 = \dot{m}_1 h_1 + \dot{m}_2 h_2 \qquad \dot{m}_3 = \dot{m}_1 + \dot{m}_2$$

$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2 = \left(1 - \frac{\dot{m}_2}{\dot{m}_3}\right) h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2 \quad \Rightarrow \quad \frac{\dot{m}_2}{\dot{m}_3} = \frac{h_3 - h_1}{h_2 - h_1}$$

Mixing of Two Streams II

- Easier way to handle mass flow rate ratios: set the total flow rate to 1, and the component flow rates to f and 1 – f

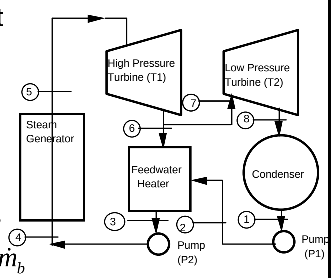


$$\dot{m}_3 = 1 \quad \dot{m}_2 = f \quad \dot{m}_1 = 1 - f \quad (1)h_3 = (1 - f)h_1 + fh_2$$

$$h_3 = \frac{\dot{m}_1}{\dot{m}_3} h_1 + \frac{\dot{m}_2}{\dot{m}_3} h_2 = (1 - f)h_1 + fh_2 \quad \Rightarrow \quad f = \frac{h_3 - h_1}{h_2 - h_1}$$

Feedwater Heater Cycle

- There are different mass flow rates in different parts of the cycle



1 $\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_a$

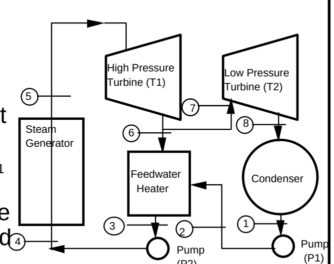
f $\dot{m}_7 = \dot{m}_8 = \dot{m}_1 = \dot{m}_2 = \dot{m}_b$

$\dot{m}_6 = \dot{m}_3 - \dot{m}_2 = \dot{m}_a - \dot{m}_b$

1 – f

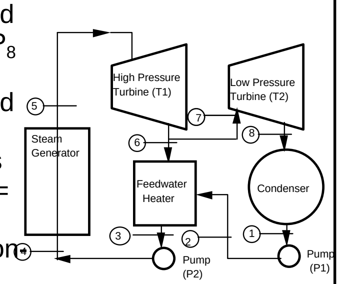
Feedwater Cycle Assumptions

- How do we apply cycle assumptions here?
  - No line losses
  - Constant P for heat
    - $P_2 = P_3 = P_6 = P_7$
    - $P_4 = P_5$ and $P_8 = P_1$
  - Isentropic work
  - Exit from two phase devices is saturated
    - Points 1 and 3 are saturated liquid



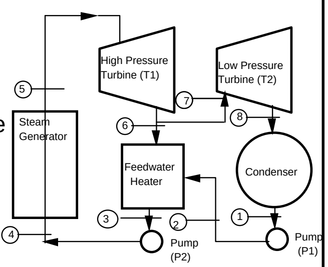
Feedwater Cycle Assumptions

- State 1 is saturated liquid: $P_{cond} = P_1 = P_8$
- Pump work = vΔP
- State 3 is saturated liquid at P_{FWH}
- Isentropic turbines
- $P_7 = P_6 = P_3 = P_2 = P_{FWH}$; $P_4 = P_5$
- $W_u = 0$ in FWH, condenser and SG



Feedwater Heater Cycle Data

- The cycle efficiency can be calculated from the following data
  - Condenser pressure
  - Steam generator outlet temperature and pressure
  - Feedwater heater pressure



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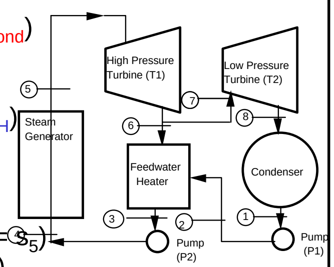
Feedwater Cycle Enthalpy

- $h_1 = h_f(P_{cond})$
- $w_{P1} = v_1(P_{FWH} - P_{cond})$
- $h_2 = h_1 + |w_{P1}|$
- $h_3 = h_f(P_{FWH})$
- $w_{P2} = v_3(P_{SG} - P_{FWH})$
- $h_4 = h_3 + |w_{P2}|$
- $h_5 = h(P_{SG}, T_{SG})$
- $s_5 = s(P_{SG}, T_{SG})$
- $h_7 = h_6 = h(P_{FWH}, s_6 = s_5)$
- $h_8 = h(P_{cond}, s_8 = s_5)$



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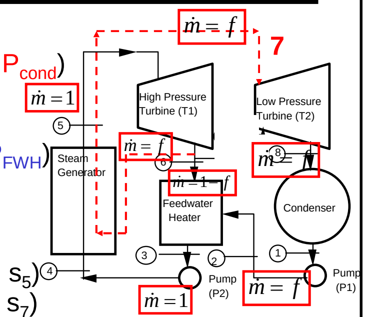
Reheat Feedwater Cycle

- $h_1 = h_f(P_{cond})$
- $w_{P1} = v_1(P_{FWH} - P_{cond})$
- $h_2 = h_1 + |w_{P1}|$
- $h_3 = h_f(P_{FWH})$
- $w_{P2} = v_3(P_{SG} - P_{FWH})$
- $h_4 = h_3 + |w_{P2}|$
- $h_5 = h(P_{SG}, T_{SG})$
- $s_5 = s(P_{SG}, T_{SG})$
- $h_6 = h(P_{FWH}, s_6 = s_5)$
- $h_8 = h(P_{cond}, s_8 = s_7)$



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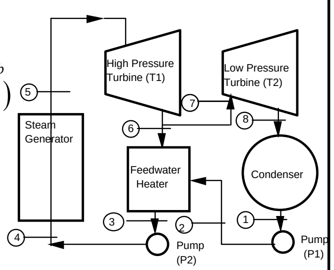
Heat and Work Rates

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_5 = \dot{m}_a$$
$$\dot{m}_7 = \dot{m}_8 = \dot{m}_1 = \dot{m}_2 = \dot{m}_b$$
$$\dot{m}_6 = \dot{m}_3 - \dot{m}_2 = \dot{m}_a - \dot{m}_b$$
$$\dot{Q}_H = \dot{Q}_{SG} = \dot{m}_a(h_5 - h_4)$$
$$\dot{W}_T = \dot{m}_a(h_5 - h_6) + \dot{m}_b(h_7 - h_8)$$
$$|\dot{W}_P| = \dot{m}_a|w_{P_2}| + \dot{m}_b|w_{P_1}|$$
$$\eta = \left(\dot{W}_T - |\dot{W}_P|\right) \Big/ \dot{Q}_{SG}$$



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Combine Previous Equations

$$\eta = \left(\dot{W}_T - |\dot{W}_P|\right) \Big/ \dot{Q}_{SG} \qquad |\dot{W}_P| = \dot{m}_a|w_{P_2}| + \dot{m}_b|w_{P_1}|$$

$$\dot{W}_T = \dot{m}_a(h_5 - h_6) + \dot{m}_b(h_7 - h_8) \qquad \dot{Q}_{SG} = \dot{m}_a(h_5 - h_4)$$

$$\eta = \frac{\dot{m}_a(h_5 - h_6) + \dot{m}_b(h_7 - h_8) - \dot{m}_a|w_{P_2}| - \dot{m}_b|w_{P_1}|}{\dot{m}_a(h_5 - h_4)}$$

$$\eta = \frac{(h_5 - h_6) + \frac{\dot{m}_b}{\dot{m}_a}(h_7 - h_8) - |w_{P_2}| - \frac{\dot{m}_b}{\dot{m}_a}|w_{P_1}|}{(h_5 - h_4)}$$

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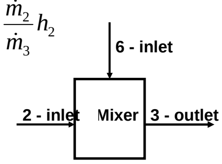
Apply Mixer Equation to FWH

$$\dot{m}_3 h_3 = \dot{m}_6 h_6 + \dot{m}_2 h_2 \qquad \dot{m}_3 = \dot{m}_6 + \dot{m}_2$$

$$h_3 = \frac{\dot{m}_6}{\dot{m}_3}h_6 + \frac{\dot{m}_2}{\dot{m}_3}h_2 = \left(1 - \frac{\dot{m}_2}{\dot{m}_3}\right)h_6 + \frac{\dot{m}_2}{\dot{m}_3}h_2$$

$$\boxed{\frac{\dot{m}_2}{\dot{m}_3} = \frac{h_3 - h_6}{h_2 - h_6} = \frac{\dot{m}_b}{\dot{m}_a} = f}$$



$$\eta = \frac{(h_5 - h_6) + \frac{\dot{m}_b}{\dot{m}_a}(h_7 - h_8) - |w_{P_2}| - \frac{\dot{m}_b}{\dot{m}_a}|w_{P_1}|}{(h_5 - h_4)}$$

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Condenser Analysis

- Usually simpler than work analysis

$$\left|\dot{Q}_L\right| = \left|\dot{Q}_{cond}\right| = \left|\dot{m}_b\left(h_1 - h_8\right)\right| = \dot{m}_b\left(h_8 - h_1\right)$$

$$\left|\dot{Q}_H\right| = \left|\dot{Q}_{SG}\right| = \left|\dot{m}_a\left(h_5 - h_4\right)\right| = \dot{m}_a\left(h_5 - h_4\right)$$

$$\eta = \frac{\left|\dot{W}_{net}\right|}{\left|\dot{Q}_H\right|} = \frac{\left|\dot{Q}_H\right| - \left|\dot{Q}_L\right|}{\left|\dot{Q}_H\right|} = 1 - \frac{\dot{m}_b\left(h_8 - h_1\right)}{\dot{m}_a\left(h_5 - h_4\right)}$$

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Cycle Analysis Summary

- Use the following cycle idealizations
  - No line losses – the state entering a device is the exit state from the previous device
  - All work devices are isentropic
  - Useful work and $\Delta P = 0$ in heat-transfer devices
  - The exit state of two-phase devices is saturated (liquid or vapor)
- Can have combination of idealizations and actual data; use data when available
- Solve for mass flow rate ratios if necessary

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