Unit Ten – Isentropic Efficiencies

Mechanical Engineering 370

Thermodynamics

Larry Caretto

November 9, 2010

California State University Northridge

Outline

- Review maximum adiabatic work
- Goals for unit ten
  - Isentropic Efficiencies
  - Second law for open systems
- Problem solving with isentropic efficiencies for work input and output devices
- Second law for open systems

California State University Northridge 2

Review Maximum Adiabatic Work

- Maximum adiabatic work is for $\Delta s = 0$
  - This is also minimum adiabatic work input
- Solve using tables
  - From given inlet (initial) conditions, find the initial state properties including $s_{initial}$
  - From $s_{final} = s_{initial}$ and one other property of final state get all final state properties
- Solve for ideal gases
  - Isentropic equations for constant heat capacity or ideal gas tables give final state
    - Ideal gas tables or cΔT for energy or enthalpy

California State University Northridge 3

First Unit Ten Goal

- As a result of studying this unit you should be able to use the concept of isentropic efficiency, η_s
  - Empirical correction factor to get actual work from maximum work
  - One condition on final state (P or v) is the same for both actual and maximum work
  - The η_s equations for work input and work output are different
  - Actual and ideal processes differ in final temperature giving difference in work

California State University Northridge 4

Second Unit Ten Goal

- As a result of studying this unit you should be able to use the second law for open systems

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i \geq \frac{\dot{Q}_{cv}}{T}$$

$$\frac{1}{T}\frac{d(LW)}{dt} = \dot{S}_{irr}$$

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = \frac{\dot{Q}_{cv}}{T} + \frac{1}{T}\frac{d(LW)}{dt}$$

For steady, adiabatic, reversible systems $\sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = 0$

California State University Northridge 5

What is η_s?

- Empirical correction factor, based on past design experience, to obtain actual work output from isentropic work
- It is not a loss of energy
- Different definitions for work input and work output devices
- Compares ideal work, $|w_s|$, and actual work, $|w_a|$ or $|w|$, with same inlet state and same final pressure (or volume)
- Plot this on h-s coordinates

California State University Northridge 6

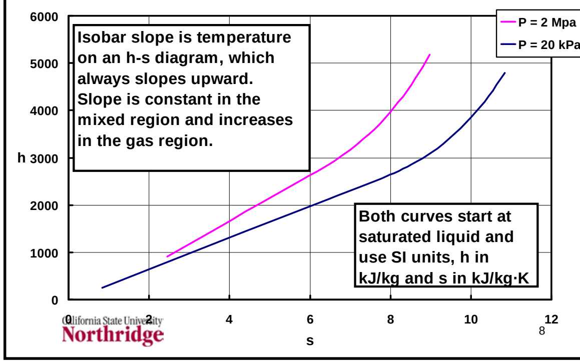
h-s Coordinates?

- We have displayed properties on P-v and T-v charts
- We can use any coordinates to display properties
- T-s coordinates used because area under curve is reversible heat transfer
- h-s coordinates illustrate actual and ideal open-system work amounts
- Slope of isobar (constant P line) on h-s diagram is the temperature, T

California State University Northridge

7

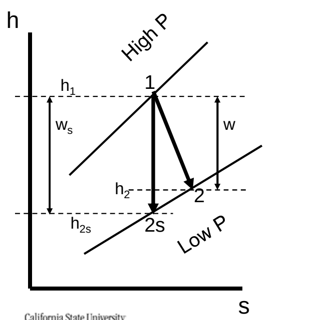
Water h-s Isobars



California State University Northridge

8

Output Isentropic Efficiency



Ideal work:
$|w_s| = |h_1 - h_{2s}|$

Actual work:
$|w| = |h_1 - h_2|$

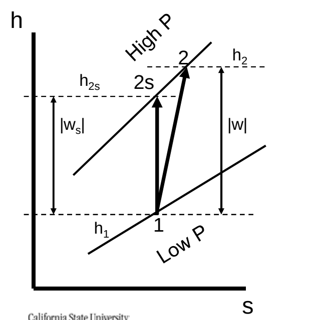
Isentropic efficiency for work output device:

$$\eta_s = |w|/|w_s|$$

California State University Northridge

9

Input Isentropic Efficiency



Ideal work:
$|w_s| = |h_1 - h_{2s}|$

Actual work:
$|w| = |h_1 - h_2|$

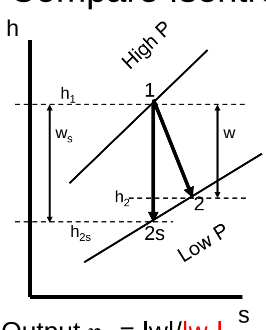
Isentropic efficiency for work input device:

$$\eta_s = |w_s|/|w|$$

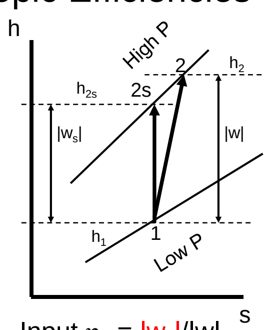
California State University Northridge

10

Compare Isentropic Efficiencies



Output $\eta_s = |w|/|w_s|$



Input $\eta_s = |w_s|/|w|$

Typical Problem

- Closed system or steady open system with one inlet and one outlet, given
  - Inlet (or initial) state
  - One property (such as pressure or specific volume) for outlet (or final) state
  - Isentropic efficiency
- Process is adiabatic
- Find work and actual outlet (final) state

California State University Northridge

12

Solution to Typical Problem

- Find maximum work, w_s, for isentropic process
  - property tables or ideal gas
  - $w_s = h_{in} - h_{out,s}$ or $w_s = u_{initial} - u_{final,s}$
- Actual work from isentropic efficiency
  - Output: $w = \eta_s w_s$ Input: $w = w_s / \eta_s$
- Actual final state from first law
  - Steady open system: $h_{out} = h_{in} - w$
  - Closed system: $u_{final} = u_{initial} - w$

California State University Northridge 13

Example Problem One

- An adiabatic steam turbine has an inlet pressure and temperature of 5 MPa and 500°C. Its outlet pressure is 50 kPa and its isentropic efficiency is 84%
- Find the actual work and the actual exit state of the turbine
- Assumptions: steady flow with negligible changes in kinetic and potential energies
- Equations: $w = h_{in} - h_{out}$ $w_s = h_{in} - h_{out,s}$

California State University Northridge 14

Solving Example One

- Find isentropic work for H_2O with P_{in} = 5 MPa, T_{in} = 500°C and P_{out} = 50 kPa
- Actual work is η_s (84%) times isentropic work for work output device
- Get actual outlet state as $h_{out} = h_{in} - w_{actual}$
- Actual outlet state important when it is input to subsequent device

California State University Northridge 15

Example One Solution

- Same first law for both maximum and actual work: $w_s = h_{in} - h_{out,s}$ $w = h_{in} - h_{out}$
- Use property tables for water:
  - At initial state of 5 MPa and 500°C, h_{in} = 3434.7 kJ/kg and s_{in} = 6.9781 kJ/kg·K
  - Outlet P_{out} = 50 kPa, $s_{out,s} = s_{in}$ is mixed
  - $x_{out,s} = (s - s_f)/s_{fg}$ = (6.9781 – 1.0912)/6.5019
  - $h_{out,s}$ = 340.54 + (0.905)2304.7 = 2427.7 kJ/kg

California State University Northridge 16

Example One Concluded

- Isentropic work, $w_s = h_{in} - h_{out,s}$ = 3434.7 kJ/kg – 2427.7 kJ/kg = 1007.0 kJ/kg
- Actual work, $w = \eta_s w_s$ = (0.84) (1007.0 kJ/kg) = 845.9 kJ/kg
- Actual outlet state
  - $h_{out} = h_{in} - w$ = 3434.7 kJ/kg – 845.9 kJ/kg = 2588.8 kJ/kg
  - $x_{out} = (h_{out} - h_f)/h_{fg}$ = (2588.8 kJ/kg– 340.54 kJ/kg)/(2304.7 kJ/kg) = 0.974 is final state (with P_{out} = 50 kPa)

California State University Northridge 17

Two Important Equations

- Second law: ds = (du + Pdv)/T
- Multiply by T: Tds = du + Pdv
- Enthalpy: h = u + Pv or u = h – Pv
- Derivative: du = dh – Pdv – vdP
- Substitute for du in Tds equation to get
- Tds = dh – vdP or dh = Tds + vdP
- For ds = 0, dh = vdP
- Slope of isobar on h-s

$$\left(\frac{\partial h}{\partial s}\right)_P = T$$

California State University Northridge 18

Integrating vdP

- dh = vdP in an isentropic process
- See last two charts (not usually covered in lecture) that integrate this for an ideal gas with isentropic constant heat capacity relation: $v = CP^{1/k}$ for ds = 0
- Main application is for liquids in which v is essentially constant
- For liquids in an isentropic flow process, $\Delta h = v\Delta p$ or $w = h_{in} - h_{out} = v_{in}(P_{in} - P_{out})$

California State University Northridge 19

Example Two

- Pump inlet saturated liquid water at 20 kPa and outlet pressure of 6 MPa has isentropic efficiency of 80%. Find work.
- $w_s = v_{in} (P_{in} - P_{out}) = (0.001017 \text{ m}^3/\text{kg})$ (20 kPa – 6000 kPa) = –6.08 kJ/kg
  - v_{in} used here is v_f(20 kPa)
- $w = w_s / \eta_s$ = (–6.08 kJ/kg) / 0.80 = –7.60 kJ/kg
- $h_{out} = h_{in} - w$ = 251.42 – (–7.60) = 259.02 kJ/kg

California State University Northridge 20

A Note on dh = vdP

- Cannot do isentropic calculation for liquid from equations
- Even computer programs have problems because of convergence
- Doing problem on previous chart as isentropic calculation on computer gives Δh = 5.77 kJ/kg instead of 6.08 kJ/kg
- Computer calculation at final state gives v = 0.001015 m^3/kg a difference of 0.2%

California State University Northridge 21

Example Three

- **Given:** adiabatic air compressor has an inlet pressure and temperature of 100 kPa and 300 K. Outlet pressure is 1.5 MPa. Isentropic efficiency of 82%.
- **Find:** work and the outlet temperature
- **Assumptions:** Steady system, one inlet and one outlet, negligible changes in kinetic and potential energies
- **Properties:** Ideal gas with variable heat capacity using tables

California State University Northridge 22

Solving Example Three

- Find minimum work input required to compress air from P_{in} = 100 kPa, T_{in} = 300 K, to P_{out} = 1.5 MPa
- Find actual work if isentropic efficiency is 82%
- Find actual outlet state for actual work and determine the temperature at that state
  - Important if used as inlet to subsequent device

California State University Northridge 23

Example Three Solution

- Use ideal gas tables for air
  - At T_{in} = 300 K, P_r = 1.3860, h = 300.19 kJ/kg
  - Maximum work in isentropic process so that $P_r(T_{out,s}) = P_r(T_{in}) P_{out}/ P_{in}$ = 20.790 => $T_{out,s}$ = 641.2 K
  - Interpolate to get $h_{out,s}$ = 654.94 kJ/kg
- First law: $w_s = h_{in} - h_{out,s}$ = –354.75 kJ/kg
- $w = w_s/\eta_s$ = –354.75/0.82 = –432.6 kJ/kg
- $h_{out} = h_{in} - w$ = 732.82 kJ/kg
- Interpolate to find T_{out} = 718.6 K

California State University Northridge 24

Example Three with Constant c_p

- Use properties at T = 450 K: c_p = 1.020 kJ/kg·K, k = 1.391
- For constant s in ideal gas with constant c_p, $T_{2,s} = T_1(P_2/P_1)^{(k-1)/k}$ = (300 K)(1500 kPa / 100 kPa)$^{(1.391 - 1)/1.391}$ = 642.26 K
- $w_s = c_p(T_1 - T_{2,s})$ = (1.020 kJ/kg·K)(300 K – 642.26) = -349.1 kJ/kg (input)
- $w = w_s/\eta_s$ = (-349.1 kJ/kg)/82% = -425.74 kJ/kg = $c_p(T_1 - T_2)$

California State University Northridge 25

Constant c_p Example II

- $T_2 = T_1 - w/c_p = T_1 - w_s/\eta_s c_p = T_1 - c_p(T_1 - T_{2,s})/\eta_s c_p = T_1 - (T_1 - T_{2,s})/\eta_s$
- T_2 = 300 K – (-425.74 kJ/kg)/(1.020 kJ/kg·K) = 717.4 K
- Actual outlet state calculations important if outlet from one device is input to another device

California State University Northridge 26

Control Volume Entropy

- General equations

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i \geq \frac{\dot{Q}_{cv}}{T}$$

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = \frac{\dot{Q}_{cv}}{T} + \frac{\dot{LW}_{cv}}{T}$$

- Lost work (LW) is lost opportunity to do work due to irreversible processes

California State University Northridge 27

Reversible Processes

- Reversible process is = part of ≥

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = \frac{\dot{Q}_{cv}}{T}$$

- Reversible adiabatic process

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = 0$$

- Steady reversible adiabatic process

$$\sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = 0$$

California State University Northridge 28

Transient Processes

- Integrate inequality over time

$$\int \left(\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i \geq \frac{\dot{Q}_{cv}}{T} \right) dt$$

$$\int \frac{dS_{cv}}{dt} dt = \int dS_{cv} = \Delta S_{cv} = [m_2 s_2 - m_1 s_1]_{cv}$$

$$\int \left(\sum \dot{m}_k s_k \right) dt = \sum \int \dot{m}_k s_k dt = \sum m_k \langle s_k \rangle$$

$$[m_2 s_2 - m_1 s_1]_{cv} + \sum_{outlet} m_o s_o - \sum_{inlet} m_i s_i \geq \int \frac{\dot{Q}_{cv}}{T} dt$$

$\langle s_k \rangle$ notation, used in derivation only, as reminder that we assume inlet and outlet entropies are constant or at some average value

California State University Northridge

Example Problem

- A steam turbine with given T_{in}, P_{in}, and $\dot{m}$ has two outlets at $P_{out,1}$ and $p_{out,2}$, with given outlet mass flow rates
- What is the maximum work?
- Find by saying that $s_{out,1} = s_{out,2} = s_{in}$
- Satisfies result that maximum adiabatic work is in isentropic process

$$\sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i = 0$$

California State University Northridge 30

Derivation Details

- Next two slides show application to isentropic work as ∫vdP
- Limited to ideal gas with constant heat capacities
- Isentropic path for v(P) generally not known
- Note general trend of vdP
  - Want large v for work output (dP < 0)
  - Want small v for work input (dP > 0)

California State University Northridge 31

Isentropic ideal gas dh = vdP

- To integrate this we must know v(P) for an isentropic process
- For an ideal gas with constant heat capacities, Pv^k = constant so v = $CP^{-1/k}$
- This gives following integration

$$\Delta h = \int dh = \left[\int vdP\right]_{const\ s} = \int_{P_1}^{P_2} CP^{-1/k}dP = \left[\frac{CP^{1-1/k}}{1-1/k}\right]_{P_1}^{P_2}$$

$$= \frac{\left(P_2^{1/k}v_2\right)P_2^{1-1/k} - P_1^{1/k}v_1P_1^{1-1/k}}{\frac{k-1}{k}} = \frac{k}{k-1}\left(P_2v_2 - P_1v_1\right)$$

California State University Northridge 32

Integrating Ideal Gas dh = vdP II

- The ratio k/(k – 1) is c_p/R

$$\frac{k}{k-1} = \frac{\frac{c_p}{c_v}}{\frac{c_p}{c_v}-1} = \frac{c_p}{c_p - c_v} = \frac{c_p}{R}$$

$$\Delta h = \frac{k}{k-1}\left(P_2v_2 - P_1v_1\right) = \frac{c_p}{R}\left(RT_2 - RT_1\right) = c_p\left(T_2 - T_1\right)$$

- The result is correct as Δh = c_pΔT, but does not predict T_2,which has to be found from another calculation

California State University Northridge 33