Unit Eight – Calculations with Entropy

Mechanical Engineering 370

Thermodynamics

Larry Caretto

October 26, 2010

California State University Northridge

Outline

- Quiz Seven Solutions
- Second law review
- Goals for unit eight
- Using entropy to calculate the maximum work in an adiabatic process
  - Example calculation
- Other calculations with entropy
  - Using heat capacities to compute entropy
  - Checking to see if a process satisfies the second law

California State University Northridge 2

The Second Law

- There exists an extensive thermo-dynamic property called the entropy, S, defined as follows (T must be absolute temperature):

$$dS = (dU + PdV)/T$$

- For any process $dS \geq dQ/T$
- For an isolated system $dS \geq 0$
- Why??? Because it's the law!

California State University Northridge 3

Second Law Notes

- Started with mathematical statement on previous chart
- Proved that heat flows from higher to lower temperature
- Proved that reversible (Carnot) cycle is most efficient for two and only two temperature reservoirs
  - Highest engine efficiency
  - Highest coefficient of performance for refrigeration cycles

California State University Northridge 4

Uses of Entropy

- We will use entropy to compute reversible processes
- Reversible processes are the most efficient
- Calculations with entropy will tell us the best possible outcome we can expect
- Efficiency may be less than 100%
- A reversible adiabatic process has constant entropy

California State University Northridge 5

Entropy is a Property

- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property
- Entropy is a Property

California State University Northridge 6

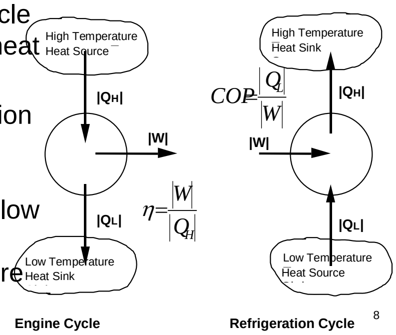
Entropy is a Property

- If we know the state of the system, we can find the entropy
- We can use the entropy as one of the properties to define the state
- Use the following if we are given a value of s and a value of T or P
  - if $s < s_f$(T or P) => compressed liquid
  - if $s > s_g$(T or P) => gas (superheat) region
  - otherwise in mixed region with saturation T and P and other properties found from quality, $x = (s – s_f)/s_{fg}$

California State University Northridge 7

Cycles with $|Q_H| = |Q_L| + |W|$

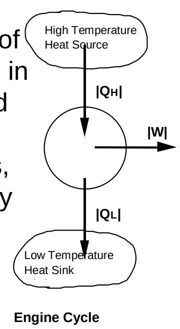
- Engine cycle converts heat to work
- Refrigeration cycle transfers heat from low to high temperature



California State University Northridge 8

What is Entropy?

- As the efficiency of processes in an isolated system decreases, the entropy of that system increases



$\Delta S_{isol\ syst} = –|Q_H|/T_H + |Q_L|/T_L + 0$ (for cycle)

For $|Q_H|$ = 100 kJ, T_H = 1000 K and T_L = 500 K. Carnot cycle efficiency is $\eta = 1 – T_H/T_H = 1 –$ 500/1000 = 50%

$|Q_L| = |Q_H| – |W| = |Q_H| – \eta|Q_H| = |Q_H|(1 – \eta)$

$\Delta S_{isol\ syst} = –|Q_H|/T_H + |Q_L|/T_L = |Q_H| [–1/T_H + (1 – \eta) / T_L]$

California State University Northridge

What is Entropy? II

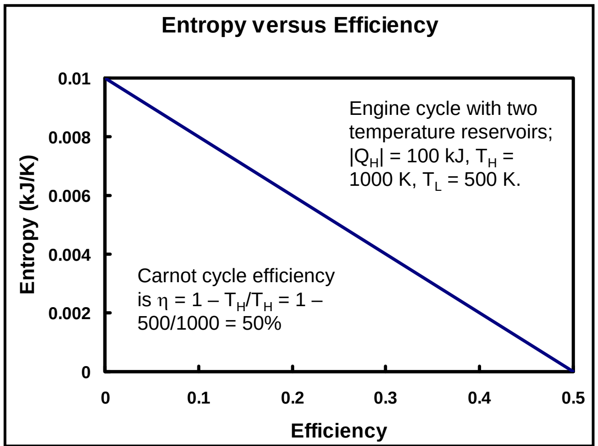
$\Delta S_{isol\ syst} = |Q_H| [–1/T_H + (1 – \eta) / T_L]$

For $|Q_H|$ = 100 kJ, T_H = 1000 K, T_L = 500 K

$\Delta S_{isol\ syst}$ = (100 kJ) [–1/(1000 K) + (1 – η) / (500 K)] = (0.1 kJ/K) (1 – 2η)

This equation gives $\Delta S_{isol\ syst} = 0$ for η = 50% (the maximum efficiency) and $\Delta S_{isol\ syst} = 0.1$ for η = 0 where there is no work.

California State University Northridge 10



Reversible Process

- In a reversible process it is possible to return an isolated collection of systems to their initial states with no changes in the surroundings
- This is an idealization; we cannot do better than a reversible process
- This is the = part of the ≥ sign in $dS \geq dQ/T$ and $dS_{isolated\ system} \geq 0$
- For a reversible process $dS = dQ/T$ and $dS_{isolated\ system} = 0$

California State University Northridge 12

Unit Eight Goals

- As a result of studying this unit you should be able to
  - feel more comfortable with the notion of entropy as property
  - find entropy values in property tables
  - use entropy and one other property to define a state that you find in a table
  - be able to pronounce entropy and enthalpy clearly to distinguish between the two

California State University Northridge 13

More Unit Eight Goals

- understand the meaning of equations for a reversible process
- recognize that the maximum work is done in a reversible process
- compute $\Delta S = \int dQ/T$ for a reversible process since $dS = dQ/T$ for reversible processes
- compute $\Delta s_{surr} = Q_{surr}/T_{surr} = -Q_{syst}/T_{surr}$
- prove that a process in an isolated system satisfies the second law because $\Delta S_{IS} \geq 0$

California State University Northridge 14

Still More Unit Eight Goals

- recognize that finding the maximum work for an adiabatic process is the same as finding the work in a constant entropy (isentropic) process
- be able to solve the following class of problems
  - **Given**: Initial Conditions, and one final property
  - **Find**: Maximum work for adiabatic process
  - **Solution:** Compute isentropic process

California State University Northridge 15

Maximum Work

- Proof that maximum work in a path between two states occurs in a reversible process
- Look at differential analysis of reversible and irreversible process
  - Initial and final state the same so any property change is the same
    - $dU_{rev} = dU_{irrev}$
    - $dS_{rev} = dS_{irrev}$

California State University Northridge 16

Maximum Work II

- Proof that maximum work in a path between two states occurs in a reversible process
- Apply second law: $dS \geq dQ/T$
  - $dS_{rev} = dQ_{rev}/T$
  - $dS_{irrev} > dQ_{irrev}/T$
- But we just said that $dS_{rev} = dS_{irrev}$
  - $dS_{rev} = dQ_{rev}/T = dS_{irrev} > dQ_{irrev}/T$
- Conclusion: $dQ_{rev} > dQ_{irrev}$

California State University Northridge 17

Maximum Work III

- Proof that maximum work in a path between two states occurs in a reversible process
- Apply first law
  - $dU_{rev} = dQ_{rev} - dW_{rev}$
  - $dU_{irrev} = dQ_{irrev} - dW_{irrev}$
  - Have shown $dU_{rev} = dU_{irrev}$; equate the first law expressions for these dU values
  - $dQ_{rev} - dW_{rev} = dQ_{irrev} - dW_{irrev}$

California State University Northridge 18

Maximum Work IV

- Proof that maximum work in a path between two states occurs in a reversible process
- Have following results from first and second law
  - $dQ_{rev} – dW_{rev} = dQ_{irrev} – dW_{irrev}$
  - $dQ_{rev} > dQ_{irrev}$
- Combining these gives $dW_{rev} – dW_{irrev} = dQ_{rev} – dQ_{irrev} > 0$

California State University Northridge 19

Maximum Work V

- Proof that maximum work in a path between two states occurs in a reversible process
- Have just shown that $dW_{rev} – dW_{irrev} > 0$ so $dW_{rev} > dW_{irrev}$
  - Remember that work output is positive and work input is negative
  - If work "output" is –200, which is larger a work output of –100 or a work output of –300?

California State University Northridge 20

Maximum Work for Input

- The "maximum work" (input or output) occurs in a reversible process
- Sign convention: work output is positive and work input is negative
- For a work input, the "maximum work" is negative; the actual work is a negative number with a larger magnitude than the "maximum work"
- For a work input, the "maximum work" is the minimum work input

California State University Northridge 21

Maximum Adiabatic Work ($\Delta S = 0$)

- From given inlet conditions, find the initial state properties including $s_{initial}$
- The maximum work in an adiabatic process occurs when $s_{final} = s_{initial}$
- From $s_{final} = s_{initial}$ and one other property of final state get all final state properties
- Find work from first law
- We have been working problems like this without realizing that we were finding the maximum work

California State University Northridge 22

Example Problem

- A steady-flow steam turbine has an inlet P and T of 10 MPa and 450°C
- What is the maximum work if Q = 0 and P_{out} = 50 kPa?
- Solution: apply first law for adiabatic steady flow with one inlet and one outlet and $\Delta KE = \Delta PE$ negligible
- For maximum work in an adiabatic process, $s_{out} = s_{in}$

California State University Northridge 23

Example Problem II

- First law reduces to $w_u = h_{in} – h_{out}$
- At 10 MPa and 450°C inlet, s_{in} = 6.4219 kJ/kg·K and h_{in} = 3242.4 kJ/kg
- At P_{out} = 50 kPa and s_{out} = 6.4219 kJ/kg·K, x_{out} = (6.4219 kJ/kg·K – 1.0912 kJ/kg·K) / 6.5019 kJ/kg·K = 0.81987
- h_{out} = 340.54 kJ/kg + 0.81987(2645.2 kJ/kg) = 2509.25 kJ/kg
- $w_u = h_{in} – h_{out}$ = 3242.4 kJ/kg – 2509.25 kJ/kg = 733.15 kJ/kg

California State University Northridge 24

ΔS from Heat Capacity

- Original definition is that $dQ = C_x dT$ in a constant x process
- For ideal gases $du = c_v dT$ and $dh = c_p dT$ for any process
  - For ideal gases, we also have $dq = c_v dT$ for constant v and $dq = c_p dT$ for constant P
- For reversible processes we have $dS = dQ/T = C_x dT/T$ or $\Delta S = \int C_x dT/T$ for a constant X process

Northridge 25

Checking the Second Law

- Ten kilograms of a fluid having a heat capacity, c_p = 3 kJ/kg•K is cooled from 500 to 300 K at constant pressure with heat rejected to surroundings at 310 K
- Is this process possible?
- Answer is found by seeing if ΔS of isolated system consisting of fluid plus surroundings is greater than zero

Northridge 26

Checking the Second Law II

- Ideas to use here
  - For reversible heat transfer $dS = dQ/T$
  - If pressure or volume is constant, $dQ = C_p dT$ or $C_v dT$
  - For a finite, reversible process, $\Delta S = \int dS = \int dQ/T$
    - For constant P, $\Delta S = \int C_p dT/T$ and $Q = \int C_p dT$
    - For constant V, $\Delta S = \int C_v dT/T$ and $Q = \int C_v dT$
  - For any process with no interactions except surroundings, $Q_{system} = -Q_{surroundings}$

Northridge 27

Checking the Second Law II

- Apply equations from previous page to see if $\Delta S_{isol\ syst} = \Delta S_{fluid} + \Delta S_{surr} \geq 0$
- $\Delta S_{fluid} = m\int c_p dT/T = mc_p \ln(T_2/T_1)$ = (10 kg)(3 kJ/ kg•K)ln(300/500) = –15.32 kJ/K
- $\Delta S_{surr} = Q_{surr}/T_{surr} = -Q_{fluid}/T_{surr}$
- $Q_{fluid} = m \int c_p dT = mc_p(T_2 - T_1)$ = (10 kg)(3 kJ/kg•K)(300 – 500)K = – 6000 kJ
- We can now find ΔS_{surr}

Northridge 28

Checking the Second Law III

- $\Delta S_{surr} = Q_{surr}/T_{surr} = -Q_{fluid}/T_{surr}$ = –(–6000 kJ)/(310 K) = 19.35 kJ/K
- $\Delta S_{isol\ syst} = \Delta S_{fluid} + \Delta S_{surr}$ = (–15.32 + 19.35) kJ/K = 4.03 kJ/K
- Since $\Delta S_{isol\ syst} \geq 0$ the process satisfies the second law
  - How can we cool a body from 500 to 300 K if the surroundings are at 310 K?
    - Transfer heat through engine and refrigerator

Northridge 29

Class Exercise

- An R-134a compressor, with zero heat transfer, has an inlet stream of saturated vapor at 10 psia. It's outlet pressure is 160 psia. What is the minimum work input to the compressor?
- **Solution:** For an adiabatic process, the maximum work output (the minimum work input) is in an isentropic process
  - Find outlet state at P_{out} and $s_{out} = s_{in}$

Northridge 30

Exercise Solution

- **Given**: R-134a compressor with Q = 0 has inlet with saturated vapor inlet at P_{in} = 10 psia and outlet with P_{out} = 160 psia
- **Find:** Minimum work input
- **Assumptions and observations:**
  - Steady process -- one inlet and one outlet
  - No heat transfer, so minimum work input is for isentropic process
  - Neglect kinetic and potential energy terms
- **Properties:** R-134a tables

California State University Northridge 31

Apply First Law

- First Law for this case
  - Steady-state open system
  - One inlet and one outlet
  - Heat transfer is given as zero
  - Assume kinetic and potential energy changes are negligible

Steady

$$\frac{dE_{system}}{dt} = \dot{Q} - \dot{W}_u + \sum_{inlet} \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) - \sum_{outlet} \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

One Inlet One Outlet

California State University Northridge 32

Get w_u and Properties

$$\dot{W}_u = \dot{m}_{in} h_{in} - \dot{m}_{out} h_{out} = \dot{m}(h_{in} - h_{out})$$

$$\Rightarrow \quad w_u = \frac{\dot{W}_u}{\dot{m}} = h_{in} - h_{out}$$

- Inlet at 10 psia is saturated vapor: s_{in} = s_g(10 psia) = 0.22948 Btu/lb_m·R; h_{in} = h_g(10 psia) = 98.68 Btu/lb_m
- At isentropic outlet, P_{out} = 160 psia, s_{out} = s_{in} = 0.22948 Btu/lb_m·R is between 120°F and 140°F; interpolate for h

California State University Northridge 33

Answer

$$h_{out} = \frac{120.06\ Btu}{lb_m} + \left(\frac{\frac{125.32\ Btu}{lb_m} - \frac{120.06\ Btu}{lb_m}}{\frac{0.23230\ Btu}{lb_m \cdot R} - \frac{0.22337\ Btu}{lb_m \cdot R}} \right) \bullet \left(\frac{0.22948\ Btu}{lb_m \cdot R} - \frac{0.22337\ Btu}{lb_m \cdot R} \right) = \frac{123.66\ Btu}{lb_m}$$

- $w_u = h_{in} - h_{out}$ = 98.68 Btu/lb_m – 123.66 Btu/lb_m = –24.98 Btu/lb_m is maximum work for isentropic process
  - For work input, this is minimum |w|

California State University Northridge 34

Another Exercise

- Steam in a piston-cylinder device has an initial P and T of 10 MPa and 450°C
- What is the maximum work if the piston expands with Q = 0 until P_{final} = 50 kPa?
- Solution: apply first law for adiabatic closed system: Q = ΔU + W = 0 so W = –ΔU or w = W/m = –ΔU/m = –Δu
- For maximum work in an adiabatic process, $s_{final} = s_{initial}$ or $s_2 = s_1$

California State University Northridge 35

Example Problem II

- First law reduces to $w = u_1 - u_2$
- Initial state 10 MPa and 450°C inlet, s_1 = 6.4219 kJ/kg·K and u_1 = 2944.5 kJ/kg
- At P_2 = 50 kPa and $s_2 = s_1$ = 6.4219 kJ/kg·K, x_2 = (6.4219 kJ/kg·K – 1.0912 kJ/kg·K) / 6.5019 kJ/kg·K = 0.81987
- u_2 = 340.49 kJ/kg + 0.81987(2142.7 kJ/kg) = 2097.22 kJ/kg
- $w = u_1 - u_2$ = 2944.5 kJ/kg – 2097.22 kJ/kg = 847.28 kJ/kg

California State University Northridge 36