# Unit Six – First Law in Transient Open Systems

Mechanical Engineering 370

**Thermodynamics**

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California State University Northridge

## Outline

- Quiz five solution
- Schedule for next five meetings
- Unit six – transient open systems
  - Consider unsteady processes with mass flow across boundary
  - Start with first law for open systems
  - Use average properties across boundaries and integrated heat and work effects
  - Combine first law and mass balance

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## Schedule

- Today: Lecture on unit six
- Tuesday, October 5: Unit six group work
- Next Thursday, October 7: Quiz on unit six and lecture to review for midterm
- Tuesday, October 12: Group work on practice midterm
- Thursday, October 14: Midterm exam
  - Covers up to and including today's unit six
  - Closed book except for equation sheet

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## Unit Six Goals

- As a result of studying this unit you should be able to
  - understand the meaning of individual terms in the first law and mass balance for unsteady flows
  - apply the first law and mass balance equations to solve problems in unsteady flows

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## Unsteady Flow Terms

$m_i = \int \dot{m}_i dt$ Integrated mass flow (kg)

$Q = \int \dot{Q} dt$ Integrated heat transfer (kJ)

$W_u = \int \dot{W}_u dt$ Integrated useful work (kJ)

$\langle h_i \rangle = \frac{\int \dot{m}_i h_i dt}{\int \dot{m}_i dt}$ Average stream property

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## More Unsteady Flow Terms

Change in system energy during process (kJ)

$$\Delta E_{system} = E_2 - E_1 = \int \frac{dE_{system}}{dt} dt$$

Change in system mass during process (kg)

$$\Delta m_{system} = m_2 - m_1 = \int \frac{dm_{system}}{dt} dt$$

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## Unsteady Flow Equations

$$\left[m_2\left(u+\frac{\vec{V}^2}{2}+gz\right)_2 - m_1\left(u+\frac{\vec{V}^2}{2}+gz\right)_1\right]_{system} = Q - W_u$$

$$-\sum_{outlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)+\sum_{inlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)$$

$$[m_2 - m_1]_{system} = \sum_{inlet} m_i - \sum_{outlet} m_i$$

- Use integrated quantities, as in closed systems, not rate terms

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Derivation starts on slide 30

7

## What Process is This?

- Is their any mention of inlets, outlets or flowing streams?
  - If not, it is a closed system so Q = ΔU + W
  - If yes, it is an open system, but is it transient or steady?
    - If the problem statement mentions time, initial and final states, or changes over time it is a transient process
    - If not, it is a steady process

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8

## What Does the Problem Have?

| Kind of Problem | Inlets, outlets or mass added | Initial and final states or time |
|---|---|---|
| Closed System, Q = ΔU + W | No | Yes |
| Steady open system | Yes | No |
| Transient open system | Yes | Yes |

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9

## Example Calculation

- Given: The discharge from a compressed air cylinder with a volume of 1.5 ft³ is used to power a turbine.
- State and flow data
  - $T_1$ = 520 R, $P_1$ = 180 psia in cylinder
  - $T_2$ = 460 R, $P_2$ = 120 psia in cylinder
  - Turbine outlet $P_{out}$ = 110 psia, $T_{out}$ = 360 R
- Negligible heat transfer
- Find turbine work
- Use ideal gas tables for air

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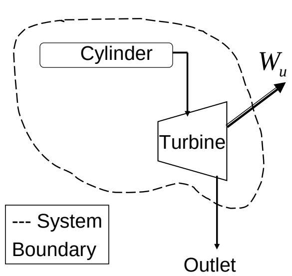
10

## Diagram and Assumptions

- Define system as cylinder and turbine
- Only one outlet
- Negligible kinetic and potential energy changes
- Q = 0 (given)

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11

## Get the First Law for the System

Negligible ΔKE and ΔPE

Adiabatic

$$\left[m_2\left(u+\frac{\vec{V}^2}{2}+gz\right)_2 - m_1\left(u+\frac{\vec{V}^2}{2}+gz\right)_1\right]_{system} = Q - W_u$$

$$-\sum_{outlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)+\sum_{inlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)$$

One outlet

No inlet

$$[m_2u_2 - m_1u_1]_{system} = -W_u - m_{out}h_{out}$$

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12

## Mass Balance plus First Law

- Mass balance for this system

$$[m_2 - m_1]_{system} = \sum_{inlet} m_i - \sum_{outlet} m_i = -m_{out}$$

- First law from previous page

$$[m_2 u_2 - m_1 u_1]_{system} = -W_u - m_{out} h_{out}$$

- Combine mass balance and first law

$$[m_2 u_2 - m_1 u_1]_{system} = -W_u + [m_2 - m_1]_{system} h_{out}$$

$$\boxed{W_u = [m_2 - m_1]_{system} h_{out} - [m_2 u_2 - m_1 u_1]_{system}}$$

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## Use Ideal Gas with Air Tables

- R = 0.06855 Btu/$lb_m$ = 0.3704 psia·$ft^3$/$lb_m$·R (Table A-1E, page 958)
- 1 Btu = 5.40395 psia·$ft^3$
- From ideal gas tables for air, Table A-17E, page 982
  - $u_1$ = u(520 R) = 88.62 Btu/$lb_m$
  - $u_2$ = u(460 R) = 78.36 Btu/$lb_m$
  - $h_{out}$ = h(360 R) = 85.97 Btu/$lb_m$

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## Use PV = mRT for $m_1$ and $m_2$

$$m_1 = \frac{P_1 V_{cyl}}{RT_1} = \frac{(180\ psia)(1.5\ ft^3)}{(520\ R)\dfrac{0.3704\ psia \cdot ft^3}{lb_m \cdot R}} = 1.402\ lb_m$$

$$m_2 = \frac{P_2 V_{cyl}}{RT_2} = \frac{(120\ psia)(1.5\ ft^3)}{(460\ R)\dfrac{0.3704\ psia \cdot ft^3}{lb_m \cdot R}} = 1.056\ lb_m$$

- Note use of R in units of psia·$ft^3$ instead of Btu here

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## Get the Answer

$$W_u = [m_1 u_1 - m_2 u_2]_{system} - [m_1 - m_2]_{system} h_{out} =$$

$$(1.402\ lb_m)\left(88.62\ {}^{Btu}\!/\!{}_{lb_m}\right) - (1.056\ lb_m)\left(78.97\ {}^{Btu}\!/\!{}_{lb_m}\right)$$

$$-(1.402\ lb_m - 1.056\ lb_m)\left(85.97\ {}^{Btu}\!/\!{}_{lb_m}\right) = 11.10\ Btu$$

- Positive work is a net work output. Value is ~0.004 hp-hr or ~0.003 kWh

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## Another Example Calculation

- Repeat calculation if flow comes from a steam line at 200 psia and 500ºF.
- Turbine exhaust flows into an initially evacuated cylinder with V = 10 $ft^3$
- At end cylinder has steam at 10 psia and 200ºF
- Negligible heat transfer
- Find turbine work

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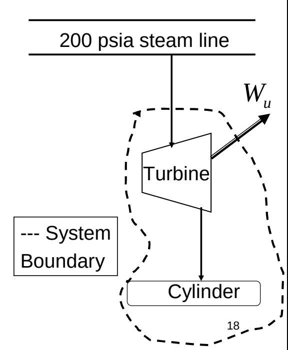
## Diagram and Assumptions

- Define system as cylinder and turbine
- Only one inlet
- Negligible kinetic and potential energy changes
- Q = 0 (given)

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## Get the First Law for the System

Negligible ΔKE and ΔPE

Adiabatic

$$\left[m_2\left(u+\frac{\vec{V}^2}{2}+gz\right)_2-m_1\left(u+\frac{\vec{V}^2}{2}+gz\right)_1\right]_{system}=Q-W_u$$

$$-\sum_{outlet}m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)+\sum_{inlet}m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)$$

No outlet

One inlet

$$\boxed{\left[m_2u_2-m_1u_1\right]_{system}=-W_u+m_{in}h_{in}}$$

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## Mass Balance plus First Law

- Mass balance for this system ($m_1$ = 0)

$$\left[m_2-m_1\right]_{system}=m_2=\sum_{inlet}m_i-\sum_{outlet}m_i=m_{in}=m$$

- First law from previous page

$$\left[m_2u_2-m_1u_1\right]_{system}=-W_u+m_{in}h_{in}$$

- Combine mass balance and first law

$$\left[m_2u_2\right]_{system}=-W_u+\left[m_2\right]_{system}h_{in}\qquad mu_2=-W_u+mh_{in}$$

$$\boxed{W_u=m\left(h_{in}-u_2\right)}$$

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## Get Properties and Answer

- $h_{in}$ = h(200 psia, 500ºF) = 1268.6 Btu/$lb_m$
- $u_2$= u(10 psia, 200ºF) = 1074.5 Btu/$lb_m$
- $v_2$= v(10 psia, 200ºF) = 38.85 $ft^3$/$lb_m$

Properties: pp 968-9

- Find m = $m_{in}$ = $m_2$ = V/$v_2$

$$m=\frac{V}{v_2}=\frac{10\ ft^3}{\dfrac{38.85\ ft^3}{lb_m}}=0.2574\ lb_m$$

Do not use $v_{in}$ to find m

$$W_u=m\left(h_{in}-u_2\right)=\left(0.2574\ lb_m\right)\left(\frac{1268.6\ Btu}{lb_m}-\frac{1074.5\ Btu}{lb_m}\right)$$

$W_u$ = 49.96 Btu =.020 hp-hr = 0.015 kWh

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## How to Handle Constant $c_v$

- How to handle both h and u in this equation:

$$W_u=\left[m_1u_1-m_2u_2\right]_{system}-\left[m_1-m_2\right]_{system}h_{out}=$$

- Answer: Pick an arbitrary reference temperature, $T_0$, and write u = $c_v$(T – $T_0$) and h = u + RT

$$W_u=m_1c_v(T_1-T_0)-m_2c_v(T_2-T_0)$$
$$-(m_1-m_2)\left[c_v(T_{out}-T_0)+RT_{out}\right]$$

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## $T_0$ Terms Cancel

- Can pick any temperature for $T_0$
- Easiest to pick $T_0$ = $T_1$ or $T_0$ = 0 (in absolute or relative temperature)
- See calculations on next chart
- Can also pick arbitrary $T_0$ when integrating equations for $c_v$(T) or $c_p$(T)
  – Choose $T_0$ = 0 so h = $c_p$T
  – Choose $T_0$ = Initial or final T to set u = 0

$$h=\int_{T_0}^{T}c_p(T)dT\qquad u=h-RT$$

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## Example Calculations

Calculate for $T_0$ = 0 and $T_0$ = $T_1$= 520 R

$$W_u=m_1c_v(T_1-T_0)-m_2c_v(T_2-T_0)$$
$$-(m_1-m_2)\left[c_v(T_{out}-T_0)+RT_{out}\right]$$

$$=\left(1.402\ lb_m\right)\frac{0.1715\ Btu}{lb_m\cdot R}\left(520\ R\right)-\left(1.056\ lb_m\right)\frac{0.1715\ Btu}{lb_m\cdot R}\left(460\ R\right)$$

$$-\left(1.402\ lb_m-1.056\ lb_m\right)\left[\frac{0.1715\ Btu}{lb_m\cdot R}\left(360\ R\right)+\frac{0.06855\ Btu}{lb_m\cdot R}\left(360\ R\right)\right]=11.82\ Btu$$

$$=-\left(1.056\ lb_m\right)\frac{0.1715\ Btu}{lb_m\cdot R}\left(460\ R-520\ R\right)-\left(1.402\ lb_m-1.056\ lb_m\right)\bullet$$

$$\left[\frac{0.1715\ Btu}{lb_m\cdot R}\left(360\ R-520\ R\right)+\frac{0.06855\ Btu}{lb_m\cdot R}\left(360\ R\right)\right]=11.82\ Btu$$

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## Class Exercise

- A scuba tank with a volume of 1.5 ft[3] is filled from a line with air at 1000 psia and 70ºF. The process is so rapid that heat transfer is negligible. The final pressure in the tank is 950 psia. If the cylinder is evacuated initially, what is the final temperature in the tank?

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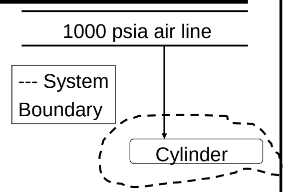
## Diagram and Assumptions

- Define system as cylinder and turbine
- Only one inlet
- Q = 0 (given)
- Negligible kinetic and potential energy changes
- No useful work (no physical work crosses boundary

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## Get the First Law for the System

Negligible ΔKE and ΔPE

Adiabatic

$$\left[m_2\left(u+\frac{\vec{V}^2}{2}+gz\right)_2-m_1\left(u+\frac{\vec{V}^2}{2}+gz\right)_1\right]_{system}=Q-W_u$$

$$-\sum_{outlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)+\sum_{inlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)$$

No useful work

No outlet

One inlet

$$\left[m_2u_2-m_1u_1\right]_{system}=m_{in}h_{in}$$

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## Mass Balance plus First Law

- Mass balance for this system ($m_1$ = 0 because cylinder evacuated initially)

$$\left[m_2-m_1\right]_{system}=m_2=\sum_{inlet} m_i-\sum_{outlet} m_i=m_{in}=m$$

- First law from previous page

$$\left[m_2u_2-m_1u_1\right]_{system}=m_{in}h_{in}$$

- Combine mass balance and first law

$$\left[m_2u_2\right]_{system}=\left[m_2\right]_{system}h_{in} \qquad mu_2=mh_{in}$$

$$h_{in}=u_2$$

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## Use Ideal Gas Relations

$$h_{in}=u_2 \quad\Rightarrow\quad h_{in}=h_2-RT_2 \quad\Rightarrow\quad RT_2=\int_{T_{in}}^{T_2}c_p dT$$

- Assume constant heat capacity, $c_p$ = 0.240 Btu/lb$_m$·R (page 959)
  - For air, R = 0.06855 Btu/lb$_m$·R

$$RT_2=\int_{T_{in}}^{T_2}c_p dT=c_p\left(T_2-T_{in}\right) \quad\Rightarrow\quad T_2=T_{in}\frac{c_p}{c_p-R}$$

$$T_2=(529.67\ R)\frac{c_p}{c_p-R}\frac{\frac{0.240\ Btu}{lb_m\cdot R}}{\frac{0.240\ Btu}{lb_m\cdot R}-\frac{0.06855\ Btu}{lb_m\cdot R}}=741.4\ R=281.7^o\ F$$

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## Derivations

- Derivation of transient open system provided here (not covered in class)
- Start with general open system first law and mass balance
- Multiply by dt and integrate between two time points, 1 and 2
- Use terms defined on slide 5

$$m_i=\int \dot{m}_i dt \qquad Q=\int \dot{Q} dt \qquad W_u=\int \dot{W}_u dt$$

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## General Open System

– First Law

$$\frac{dE_{system}}{dt} = +\sum_{inlet} \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

$$+\dot{Q} - \dot{W}_u - \sum_{outlet} \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

– Mass balance

$$\frac{dm_{system}}{dt} = \sum_{inlet} \dot{m}_i - \sum_{outlet} \dot{m}_i$$

– Multiply by dt and integrate from 1 to 2

31

## Integrate Mass Balance

$$\int_1^2 \frac{dm_{system}}{dt} dt = \int_1^2 \sum_{inlet} \dot{m}_i dt - \int_1^2 \sum_{outlet} \dot{m}_i dt$$

$$\int_1^2 \frac{dm_{system}}{dt} dt = \int_1^2 dm_{system} = m_{system,2} - m_{system,1}$$

$$\int_1^2 \sum \dot{m}_i dt = \sum \int_1^2 \dot{m}_i dt = \sum m_i$$

$$\boxed{m_{system,2} - m_{system,1} = \sum_{inlet} m_i - \sum_{outlet} m_i}$$

32

## Integrate First Law

$$\int_1^2 \left\{ \frac{dE_{system}}{dt} = +\sum_{inlet} \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \dot{Q} - \dot{W}_u - \sum_{outlet} \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) \right\} dt$$

$$\int_1^2 \frac{dE_{system}}{dt} dt = \int_1^2 dE_{system} = E_{system,2} - E_{system,1}$$

$$= \left[ m_2 \left( u + \frac{\vec{V}^2}{2} + gz \right)_2 - m_1 \left( u + \frac{\vec{V}^2}{2} + gz \right)_1 \right]_{system}$$

33

## Integrate First Law II

- Use definitions: $Q = \int \dot{Q} dt \qquad W_u = \int \dot{W}_u dt$

$$\int_1^2 \left\{ \sum \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) \right\} dt = \sum \int_1^2 \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) dt$$

- Definition of average property: $\langle h_i \rangle = \dfrac{\int \dot{m}_i h_i dt}{\int \dot{m}_i dt} = \dfrac{\int \dot{m}_i h_i dt}{m_i}$

$$\sum \int_1^2 \dot{m}_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) dt = \sum m_i \left( \langle h_i \rangle + \left\langle \frac{\vec{V}_i^2}{2} \right\rangle + \langle gz_i \rangle \right)$$

34

## Integrate First Law III

- Combine all results from integration of individual terms
  – Write terms like $\langle h_i \rangle$ as $h_i$, but remember that there are averages during process

$$\left[ m_2 \left( u + \frac{\vec{V}^2}{2} + gz \right)_2 - m_1 \left( u + \frac{\vec{V}^2}{2} + gz \right)_1 \right]_{system} = Q - W_u$$

$$- \sum_{outlet} m_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \sum_{inlet} m_i \left( h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

35