Unit Two – Work and Paths

Mechanical Engineering 370

Thermodynamics

Larry Caretto
September 2, 2010

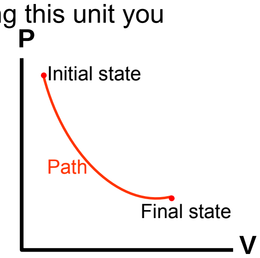
Outline

- Solution to quiz
- Assessment results
- Unit two – work and paths
  - Work, $W = \int_{path} PdV$ and $w = W/m = \int_{path} Pdv$
  - Path is equation giving P(V) for a process
  - Integral is area under path
  - Work is positive, zero, negative if volume increases, does not change, decreases
    - Work done ***by*** the system is positive

2

Unit Two Goals

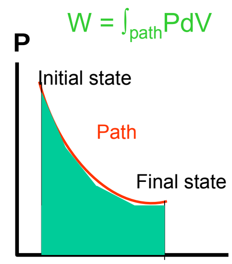
- As a result of studying this unit you should be able to
  - find properties more easily than you were able to do after completing unit one
  - use the path for a process, which is the equation that gives P(V) for the process



3

Unit Two Goals

- know the definition of work: $dW = PdV$
- find the work as the integral of PdV over the path
- find the work as the area under the path on a P-V diagram



4

More Unit Two Goals

- use the description of the path as an equation
- recognize the difference between the path equation and the equation of state
- find values of initial pressure (or volume) and final pressure (or volume) from the path equation if necessary
- use the path equation and the equation of state in a trial-and-error procedure to find the final state

5

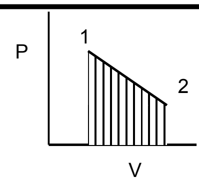
Analysis of Work

- Start with $dW = Fdx$
- $dW = (PA)d(V/A) = PdV$
- Note that dimensions are (F/L^2) times L^3
  - $Pa \cdot m^3 = (N/m^2) \cdot m^3 = N\text{-}m = J$
  - $kPa \cdot m^3 = kJ$ and $MPa \cdot m^3 = MJ$
  - $psia \cdot ft^3$ times 144 in^2/ft^2 gives $ft \cdot lb_f$; divided by 5.40395 $psia \cdot ft^3/Btu$ gives Btu
- $W = \int_{path} PdV = \int PdV$ over path
- Work depends on path

6

Simple Path

- Here we have an initial point (1), a final point (2)
- Work = area under path = trapezoid area = $(V_2 – V_1)(P_1 + P_2)/2$
- Work is positive



- Area calculation starts at P = 0

What is work for P_1 = 2 kPA, V_1 = 1 m^3, P_2 = 1 kPa, V_2 = 3 m^3?

$W = (V_2 – V_1)(P_1 + P_2)/2 = (3\ m^3 – 1\ m^3)(2\ kPa + 1\ kPa)/2 = 3\ kPa·m^3 = 3\ kJ$

California State University Northridge 7

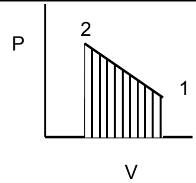
Integrating a Line

- Work = area under path = trapezoid area = $(V_2 – V_1)(P_1 + P_2)/2$
- Work is positive because if $V_2 > V_1$ (and pressure is always positive)
- Work is positive when system expands (system does work on surroundings)
- Area calculation starts at P = 0

California State University Northridge 8

Reverse Path

- Here the initial point (1) and final point (2) are reversed
- Work = area under path = trapezoid area = $(V_2 – V_1)(P_1 + P_2)/2$
- Work is negative



What is work for P_1 = 2 kPa, V_1 = 3 m^3, P_2 = 1 kPa, V_2 = 1 m^3?

$W = (V_2 – V_1)(P_1 + P_2)/2 = (1\ m^3 – 3\ m^3)(2\ kPa + 1\ kPa)/2 = –3\ kPa·m^3 = –3\ kJ$

California State University Northridge 9

Integrating a Reverse Line

- The problem on the last chart reverses the initial point (1) and final point (2), compared to the previous example
- Work = area under path = trapezoid area = $(V_2 – V_1)(P_1 + P_2)/2$
- Work is negative because $V_2 < V_1$
- Work is negative when system is compressed (work done on system)

California State University Northridge 10

Actual Integration of Path

- If we are given a mathematical equation for P(V), we can perform the integration
- Next slides gives integration details for a simple linear path: P = a + bV
- Get same results as area of trapezoid

$$P = P_1 + \frac{P_2 - P_1}{V_2 - V_1}(V - V_1) \qquad b = \frac{P_2 - P_1}{V_2 - V_1} \qquad a = P_1 - bV_1$$

California State University Northridge 11

Actual Integration of Path II

- Integrate path from last chart: P = a + bV

$$P = P_1 + \frac{P_2 - P_1}{V_2 - V_1}(V - V_1) \qquad b = \frac{P_2 - P_1}{V_2 - V_1} \qquad a = P_1 - bV_1$$

$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} (a + bV)dV = a(V_2 - V_1) + \frac{b}{2}\left(V_2^2 - V_1^2\right)$$

$$2W = 2(P_1 - bV_1)(V_2 - V_1) + b(V_2 - V_1)(V_2 + V_1) = 2P_1(V_2 - V_1)$$
$$+ b(V_2 - V_1)(V_2 + V_1 - 2V_1) = 2P_1(V_2 - V_1) + b(V_2 - V_1)^2$$

California State University Northridge 12

Actual Integration of Path III

$$2W = 2P_1(V_2 - V_1) + \frac{P_2 - P_1}{V_2 - V_1}(V_2 - V_1)^2 = 2P_1V_2 - 2P_1V_1 +$$

$$P_2V_2 - P_1V_2 - P_2V_1 + P_1V_1 = P_2V_2 - P_2V_1 + P_1V_2 - P_1V_1$$

- Factor last result and solve for W to get same formula as area of trapezoid

$$W = \frac{P_2V_2 - P_2V_1 + P_1V_2 - P_1V_1}{2} = \frac{(P_2 + P_1)(V_2 - V_1)}{2}$$

- This result gives W for P = a + bV

California State University Northridge 13

An Easier Way to Integrate

- Equation with (V – c): $P = P_1 + b(V - V_1)$
- Change integration variable to $y = V - V_1$ so that $dV = dy$, $y = 0$ when $V = V_1$ and $y = V_2 - V_1$ when $V = V_2$

$$W = \int_{V_1}^{V_2} PdV = \int_{V=V_1}^{V=V_2} [P_1 + b(V - V_1)]dV = \int_{y=0}^{y=V_2-V_1} (P_1 + by)dy = \left[P_1 y + \frac{by^2}{2}\right]_{y=0}^{V_2-V_1}$$

$$P_1(V_2 - V_1) + \frac{b}{2}(V_2 - V_1)^2 = \frac{2}{2}P_1(V_2 - V_1) + \frac{P_2 - P_1}{V_2 - V_1}\frac{(V_2 - V_1)^2}{2}$$

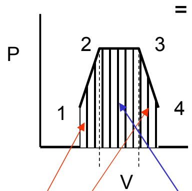
$$= \frac{2P_1V_2 - 2P_1V_1 + P_2V_2 - P_2V_1 - P_1V_2 + P_1V_1}{2} = \frac{(P_2 + P_1)(V_2 - V_1)}{2}$$

California State University Northridge 14

More Complex Path

- Here we have an initial point (1), a final point (4) and two intermediate points (2 and 3)
- Work = area under path = two trapezoid areas plus rectangle area

$P_2 = P_3 = P_{2\text{-}3}$



$P_1 = P_4$

$W = (P_1 + P_2)(V_2 – V_1)/2 + P_{2\text{-}3} (V_3 – V_2) + (P_3 + P_4) (V_4 – V_3)/2$

California State University Northridge 15

Linking Path and State

- To integrate PdV, we need path equation, P(V)
- Path equation is integrated along path in terms of P and V
- Can specify states in terms of T
- Can to use equation-of-state or tables to get v from (P,T) or P from (v,T)
- May have to use V = m v

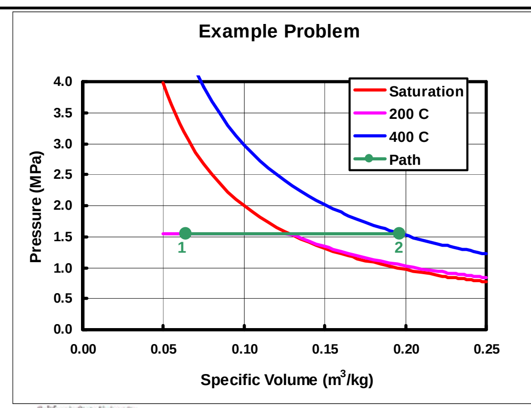
California State University Northridge 16

Example Calculation

- Given: 10 kg of H_2O at 200°C and 50% quality is expanded to 400°C at constant pressure
- Find: Work
- Equation: $W = \int_{path} PdV$ $W = P_{1\text{-}2}(V_2 – V_1)$ for constant pressure $P_1 = P_2 = P_{1\text{-}2}$
- How do we find $P_{1\text{-}2}$, V_1 and V_2 from data given?

Use property tables to find these from given data

California State University Northridge 17



California State University Northridge 18

Example Continued

- Use tables for H_2O to find P and v
- Get total volume as V = m v
- State 1 in mixed region; constant pressure, $P_{1-2} = P_1 = P_{sat}(T_1 = 200^oC) = 1554.9$ kPa (page 914)
- Get v_1 from $x_1 = 0.5$ using the equation that $v = v_f + x (v_g - v_f) = (1 - x)v_f + x v_g$
- At 200°C, v_f and v_g, respectively, = 0.001157 and 0.12721 m³/kg

Northridge 19

Example Continued

$$v_1 = (1 - x_1)v_f(T_1 = 200^oC) + x_1 v_g(T_1 = 200^oC)$$

$$v_1 = (1 - 0.5)\frac{0.001157\,m^3}{kg} + 0.5\frac{0.12721\,m^3}{kg} = \frac{0.06429\,m^3}{kg}$$

- $V_1 = m v_1 = 0.6429$ m³ for m = 10 kg
- Point 2 has $T_2 = 400^oC$ (given) and $P_2 = P_1 = 1.5549$ MPa
- Find v_2 from property tables

Northridge 20

Example Concluded

- Interpolate between P = 1.40 and 1.60 MPa to find v at $(P_2, T_2) = (1.5549$ kPa, 400°C)

Property data on page 919

$$v_2 = \frac{.21782\,m^3}{kg} + \frac{\frac{.19007\,m^3}{kg} - \frac{.21782\,m^3}{kg}}{1.60\,MPa - 1.40\,MPa} \bullet (1.5549\,MPa - 1.40\,MPa) = \frac{.1963\,m^3}{kg}$$

- Since m = 10 kg, $V_2 = 1.963$ m³
- W = (1554.9 kPa)(1.963 – 0.6429) m³

Northridge 21

Units for Work

$$W = (1554.9\,kPa)(1.963 - 0.6429)m^3\frac{1\,kJ}{kPa \cdot m^3}$$

- Basic idea is that Pa•m³ gives J, kPa•m³ gives kJ, MPa•m³ gives MJ,
- For other pressure units need to convert into kJ; here W = 2,051 kJ
- P in MPa would give work = 2.051 MPa•m³ = 2.051 MJ

Northridge 22

Do We Use V or v for Work?

- If we know the total volume V (m³) we can find the total work in J (or kJ or MJ)
- If we know v and m we can find V = mv
- If we only know v (and not m) we can find $W/m = \{\int_{path}PdV\}/m = \int_{path}Pd(V/m) = \int_{path}Pdv$ in J/kg (or kJ/kg or MJ/kg)
- Use the symbol w for W/m the work per unit mass

Northridge 23

Exercise

- Two pounds (2 lb_m) of water fills an initial volume, $V_1 = 5$ ft³, at an initial temperature of 300°F
- A process is then executed along the path $P = P_1 + b(V - V_1)$ where b = 300 lb_f/ft^5 and P_1 is the initial pressure
- The final pressure is 100 psia
- Find the work

Northridge 24

Exercise Solution

- **Given:**
  - Initial state: V_1= 5 ft^3 and T_1 = 300^{o}F
  - Final state: P_2 =100 psia
  - System mass: m = 2 lb$_m$
  - Path: P = P_1 + b(V – V_1) with b = 300 lb$_f$/ft^5
  - Substance: water
- **Solution elements:**
  - Work equation W = $\int_{path}$PdV
  - Property data: water tables
  - Path equation relates P and V at all states along path including initial and final state

California State University Northridge

25

Exercise Solution II

- Can use area under trapezoid or integrate path equation for PdV to get an equation for the work

$$W = \int_{V_1}^{V_2} PdV = \int_{V=V_1}^{V=V_2} \left[P_1 + b(V - V_1)\right]dV$$

- Let y = V – V_1 so that dy = dV, y = 0 at V = V_1 and y = V_2 – V_1 at V = V_2

$$W = \int_{y=0}^{y=V_2-V_1} (P_1 + by)dy = \left[P_1 y + \frac{by^2}{2}\right]_{y=0}^{V_2-V_1} = P_1(V_2 - V_1) + \frac{b(V_2 - V_1)^2}{2}$$

California State University Northridge

26

Exercise Solution III

- We now have an equation for W, but we do not know P_1 or V_2
- Find P_1 from initial state of T_1 = 300^{o}F and v_1 = V_1/m = (5 ft^3)/(2 lb$_m$) = 2.5 ft^3/lb$_m$
  - Find this in saturation region so P_1 = P_{sat}(300^{o}F) = 67.028 psia = 67.028 lb$_f$/in^2

data on p 964

- Find final volume, V_2, from path equation, P = P_1 + b(V – V_1)
  - Solve P_2 = P_1 + b(V_2 – V_1) for V_2

California State University Northridge

27

Exercise Solution IV

- Solving for V_2 gives V_2 – V_1 = 15.83 ft^3

$$V_2 = V_1 + \frac{P_2 - P_1}{b} = 5\ ft^3 + \frac{\frac{100\ lb_f}{in^2} - \frac{67.028\ lb_f}{in^2}}{\frac{300\ lb_f}{ft^5}\ \frac{ft^2}{144\ in^2}} = 20.83\ ft^3$$

- Find work

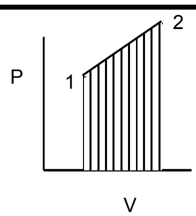
$$W = P_1(V_2 - V_1) + \frac{b(V_2 - V_1)^2}{2} = \frac{67.028\ lb_f}{in^2}\left(15.83\ ft^3\right)\frac{144\ in^2}{ft^2} + \frac{300\ lb_f}{ft^5}\frac{\left(15.83\ ft^3\right)^2}{2} = \boxed{1.904x10^5\ ft \cdot lb_f}$$

California State University Northridge

28

Exercise Solution V

- Can find work as trapezoid area = (V_2 – V_1) (P_1 + P_2)/2
- Remember that V_1 and P_2 were given
- Find V_2 and P_1 as done previously
  - P_1 from state data
  - V_2 from path equation



$$W = \frac{1}{2}\left(\frac{100\ lb_f}{in^2} + \frac{67.028\ lb_f}{in^2}\right)\bullet \left(20.83\ ft^3 - 5\ ft^3\right)\frac{144\ in^2}{ft^2} = 1.904x10^5\ ft \cdot lb_f$$

California State University Northridge

29