Review for Final Examination

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Mechanical Engineering 370

Thermodynamics

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California State University Northridge

Outline

- Quiz 12 solution
- Review for final
  - Property tables and ideal gases
  - First law for closed and open (steady and unsteady) systems
  - Entropy and maximum work calculations
  - Isentropic efficiencies
  - Cycle calculations (Rankine, refrigeration, air standard) with mass flow rate ratios

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Review, but, first a word about units

- Units and dimensions
- SI units and engineering units
- Extensive, intensive and specific
  - E is extensive, e.g., V, U, H, S, Q, W
  - T and P are intensive
  - e = E/m is specific (e.g. kJ/kg, Btu/lb_m)
- Unit conversions (kPa·m^3 = kJ) (m^2/s^2 = J/kg) (lb_f & lb_m) (psia·ft^3, Btu, $lb_m \cdot ft^2/s^2$)
  - Factors on equation sheet

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Property Data and Relations I

- Find properties from tables
  - Given T and P, $T < T_{sat}(P)$ or $P > P_{sat}(T)$ is liquid; $T > T_{sat}(P)$ or $P < P_{sat}(T)$ is gas
  - Liquid at P, T approximately saturated liquid at given T
  - When given P or T and e where e may be v, u, h, s, compare e to saturation properties
    - $e < e_f$(P or T) is liquid; $e > e_g$ (P or T) is gas
    - otherwise compute $x = (e - e_f) / (e_g - e_f)$
  - Forget all this if you find the state point

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Property Data and Relations II

- Ideal gas equations and properties
  - $Pv = RT$, $du = c_v dT$, $dh = c_p dT$, $ds = c_v dT/T + R dv/v = c_p dT/T - R dP/P$, $c_p = c_v + R$, $h = u + RT$
  - u, h, c_v and c_p = f(T) only ($k = c_p/c_v$)
  - Pick constant heat capacity at average T
  - Handle variable heat capacities by equations or use ideal gas tables for u(T), h(T) and $s^o(T)$
  - Isentropic relations for constant and variable heat capacities, e.g $P_1 v_1^k = P_2 v_2^k$, $P_2/P_1 = P_r(T_2)/P_r/(T_1)$, $v_2/v_1 = v_r(T_2)/v_r/(T_1)$

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Basic First Law Terms

- Energy terms (per unit mass): internal, u, kinetic, $\mathcal{V}^2/2$, and potential, gz
- Heat, Q (or q), is energy in transit due only to a temperature difference
- Work, W, is action of force over distance
  - $W = \int_{path} PdV$ or $w = \int_{path} Pdv$
- Heat added to a system is positive, heat removed from a system is negative
- Work done by a system is positive, work done on a system is negative

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Energy Balances

- System energy change = Heat added to system – work done by system + Energy from inflows – Energy outflows
- Usually in kJ (or Btu), but open systems can use power (kW or Btu/hr)
- Can use q = Q/m and w = W/m or equivalent rates: $q = \dot{Q}/\dot{m}$; $w = \dot{W}/\dot{m}$
- Flowing stream terms include flow work to give h = u + Pv

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Closed Systems

- Q = ΔU + W = $m(u_{final} - u_{initial}) + \int PdV$
- Integral is area under path
  - Path equation gives P(V) for process
  - Integrate equation or find area
  - Watch sign
- Internal energy depends on state
  - Tables, may have to use u = h – Pv
  - Ideal gases: du = c_vdT or u(T) in ideal-gas tables

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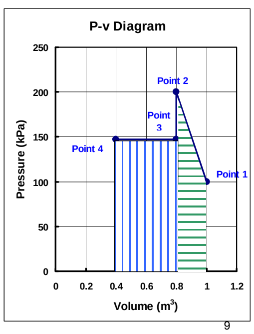
Work as Area Under Path

- This works if the path has a simple shape
- Here we have a path with three components
- $W = W_{1\text{-}2} + W_{2\text{-}3} + W_{3\text{-}4}$
- $W = (P_1 + P_2)(V_2 - V_1)/2 + 0 + P_{3\text{-}4}(V_4 - V_3)$
- W is zero if V is constant and is negative when volume decreases



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Formal Integration of Path

- Analytical path equation examples
  - Isothermal ideal gas: P = RT/v
  - Polytropic process: Pv^n = const (n ≠ k)
  - Arbitrary: $P = P_1 + a(V - V_1)^2 + \ldots$
- Evaluate ∫PdV from V_1 to V_2
- Use P(V)dV for work in kJ (or Btu) or use P(v)dv for kJ/kg (or Btu/lb_m)
- Path equation includes initial and final states and can be solved for these

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Open Systems/Assumptions

- General energy and mass balances

$$\frac{dE_{system}}{dt} = \dot{Q} - \dot{W}_u - \sum_{outlet} \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + \sum_{inlet} \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

$$\frac{dm_{system}}{dt} = \sum_{inlet} \dot{m}_i - \sum_{outlet} \dot{m}_i$$

- Steady flow: $\frac{dm_{system}}{dt} = \frac{dE_{system}}{dt} = 0$
- One inlet and one outlet
- Negligible kinetic and potential energies

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Steady-Flow Systems

$$\dot{Q} = \dot{W}_u + \sum_{outlet} \dot{m}_o \left(h_0 + \frac{\vec{V}_o^2}{2} + gz_o \right) - \sum_{inlet} \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)$$

Mass balance

$$\sum_{outlet} \dot{m}_o = \sum_{inlet} \dot{m}_i$$

First law for ΔKE = ΔPE = 0

$$\dot{Q} = \dot{W}_u + \sum_{outlet} \dot{m}_o h_o - \sum_{inlet} \dot{m}_i h_i$$

For ΔKE = ΔPE = 0, one inlet and one outlet

$$\dot{Q} = \dot{W}_u + \dot{m}(h_{out} - h_{in}) \qquad q = w_u + h_{out} - h_{in}$$

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Unsteady Flow Equations

$$\left[m_2\left(u+\frac{\vec{V}^2}{2}+gz\right)_2 - m_1\left(u+\frac{\vec{V}^2}{2}+gz\right)_1\right]_{system} = Q - W_u$$

$$-\sum_{outlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)+\sum_{inlet} m_i\left(h_i+\frac{\vec{V}_i^2}{2}+gz_i\right)$$

$$\left[m_2 - m_1\right]_{system} = \sum_{inlet} m_i - \sum_{outlet} m_i$$

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First Law Comparisons

- Closed system: $Q = m(u_{final} - u_{initial}) + W$
- Steady open system, $\Delta KE = \Delta PE = 0$, one inlet and one outlet

$$\dot{Q} = \dot{m}\left(h_{out} - h_{in}\right) + \dot{W}_u$$

- Transient open system, $\Delta KE = \Delta PE = 0$, one inlet and one outlet
- $m_{final} = m_{initial} + m_{in} - m_{out}$
- $Q = m_{final}u_{final} - m_{initial}u_{initial} + W_u + m_{out}h_{out} - m_{in}h_{in}$

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Which First Law?

- If the problem mentions the following terms use the first law indicated

Flows, inlet, outlet?	Initial and final states?	Use this first law
No	Yes	Closed System
Yes	No	Steady open system
Yes	Yes	Transient open system

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The Second Law

- There exists an extensive thermo-dynamic property called the entropy, S, defined as follows:

$$dS = (dU + PdV)/T$$

- For any process $dS \geq dQ/T$
- For an isolated system $dS \geq 0$
- T must be absolute temperature

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Entropy as a Property

- Dimensions of entropy are energy divided by temperature (kJ/K or Btu/R for S, kJ/kg·K or Btu/lb_m·R for s = S/m)
- If we know the state we can find the entropy (tables or ideal gas relations)
- If we know the entropy, we can use it to find the state (tables or ideal gases)
- Use in tables similar to specific volume

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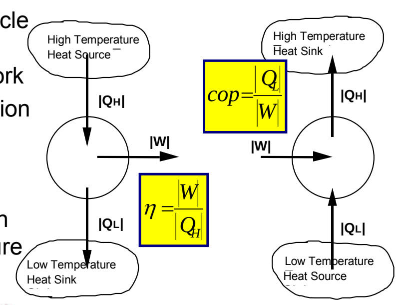
Cycles with $|Q_H| = |Q_L| + |W|$

- Engine cycle converts heat to work
- Refrigeration cycle transfers heat from low to high temperature



Engine Cycle Refrigeration Cycle

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Cycle Parameters

- Engine cycle efficiency $\eta = \frac{|W|}{|Q_H|}$
- Refrigeration cycle COP (coefficient of performance) $cop = \beta = \frac{|Q_L|}{|W|}$
- General definitions, valid for any cycle
- Engine efficiency always less than one
- COP can be greater than one

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From Blobs to Real Cycles

- Analyze each device or step in the cycle
  - Can use per-unit-mass quantities or rate quantities
  - Complex cycles have different mass flow rates
- In an engine cycle
  - Add all heat transfers that are positive to get the total Q_H $|Q_H| = Q_H$
  - Add all heat transfers that are negative to get the total Q_L $|Q_L| = -Q_L$
  - The work W is the algebraic sum of all the work terms in the cycle $|W| = W$
- Engine cycle efficiency $\eta = |W| / |Q_H|$

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From Blobs to Real Cycles II

- In a refrigeration cycle
  - Add all heat transfers that are positive to get the total Q_L $|Q_L| = Q_L$
  - Add all heat transfers that are negative to get the total Q_H $|Q_H| = -Q_H$
  - The work W is the algebraic sum of all the work terms in the cycle $|W| = -W$
- Coefficient of performance = $|Q_L| / |W|$
- $|Q_H| = |Q_L| + |W|$ for all cycles

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Reversible Process

- Idealization (the = of the ≥ sign), cannot do better than a reversible process
- Internal reversibility $dS = dQ/T$
  - For a reversible process, $Q = \int TdS$, $\Delta S = \int dQ/T = \int mc_x dT/T$
  - Reversible isothermal process: $\Delta S = Q/T$
- External reversibility $dS_{\text{isolated system}} = 0$
- Maximum work in a reversible process
  - Minimum work input for work input device
  - For adiabatic process $\Delta s = 0$ for maximum

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Ideal Gas Entropy

- For constant heat capacity

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{v_2}{v_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

- For constant heat capacity with $s_2 = s_1$
  - $T_2/T_1 = (P_2/P_1)^{(k-1)/k}$ => $T/P^{(k-1)/k}$ = constant
  - $T_2/T_1 = (v_1/v_2)^{(k-1)}$ => Tv^{k-1} = constant
  - $P_2/P_1 = (v_1/v_2)^k$ => Pv^k = constant

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Ideal Gas Entropy II

- For variable heat capacity

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R\ln\left(\frac{P_2}{P_1}\right) = s^o(T_2) - s^o(T_1) - R\ln\left(\frac{P_2}{P_1}\right)$$

- For variable heat capacity with $s_2 = s_1$
  - $P_r(T_2) = P_r(T_1)\,(P_2/P_1)$
  - $v_r(T_2) = v_r(T_1)\,(v_2/v_1)$

$$s^o(T_2) = s^o(T_1) + R\ln\left(P_2/P_1\right)$$

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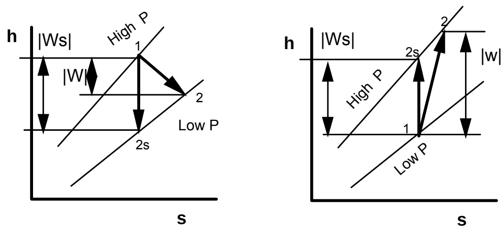
Isentropic Efficiencies



Work output: $w = \eta_s w_s$ Work input: $w = w_s/\eta_s$

Isentropic Efficiency Problems

- Find ideal work from given inlet state and one outlet state property: $\Delta s = 0$
- e.g., $w_s = h_{in} - h_{out,s}$
- Actual work = $\eta_s w_s$ for work output or w_s / η_s for work input
- Actual outlet state: $h_{out} = h_{in} - w$
  - Use sign convention for work in $h_{in} - w$
- Note that h_{out} is different from $h_{out,s}$

Control Volume Entropy

- General (use = part of ≥ for reversible)

$$\frac{dS_{cv}}{dt} + \sum_{outlet} \dot{m}_o s_o - \sum_{inlet} \dot{m}_i s_i \geq \frac{\dot{Q}_{cv}}{T}$$

- Reversible adiabatic process $$\frac{dS_{cv}}{dt} = \sum_{inlet} \dot{m}_i s_i - \sum_{outlet} \dot{m}_o s_o$$
- Steady reversible adiabatic process $$\sum_{outlet} \dot{m}_o s_o = \sum_{inlet} \dot{m}_i s_i$$
- Transient reversible adiabatic process $$\left[m_2 s_2 - m_1 s_1\right]_{cv} = \sum_{inlet} m_i s_i - \sum_{outlet} m_o s_o$$

Cycle Idealizations

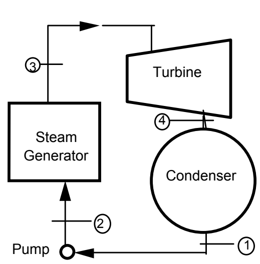
- Use these idealizations in lieu of data
  - No line losses (output state of one device is input to the next device)
  - Work devices are isentropic
  - Heat transfer and mixing devices have no work and $\Delta P = 0$
  - Exit from two-phase device is saturated
  - Air standard cycles assume air (an ideal gas) as working fluid with combustion modeled as heat transfer into fluid

Rankine Cycle

Compute Rankine cycle efficiency given only T_3, P_3 and P_{cond}

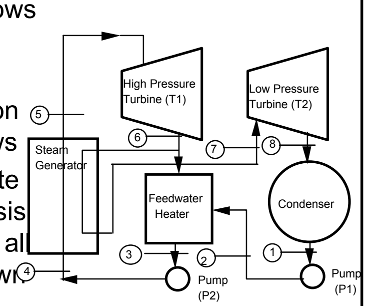
$$\eta = \frac{(h_3 - h_4) - |h_2 - h_1|}{h_3 - h_2}$$

$h_1 = h_f(P_{cond})$
$h_2 = h_1 + v_1(P_3 - P_1)$
$h_3 = h(T_3,P_3)$; $s_3 = s(T_3,P_3)$
$h_4 = h(P_{cond}, s_4 = s_3)$



Modified Rankine Cycle

- Different mass flows rates in different parts
- Results depend on ratio of mass flows
- Get mass flow rate ratios from analysis of devices where all h values are known

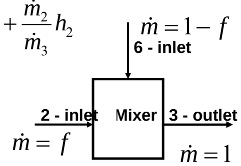


Mixing Heat and Mass Balance

$$\dot{m}_3 h_3 = \dot{m}_6 h_6 + \dot{m}_2 h_2 \qquad \dot{m}_3 = \dot{m}_6 + \dot{m}_2$$

$$h_3 = \frac{\dot{m}_6}{\dot{m}_3} h_6 + \frac{\dot{m}_2}{\dot{m}_3} h_2 = \left(1 - \frac{\dot{m}_2}{\dot{m}_3}\right) h_6 + \frac{\dot{m}_2}{\dot{m}_3} h_2$$

$$\frac{\dot{m}_2}{\dot{m}_3} = \frac{h_3 - h_6}{h_2 - h_6} = \frac{\dot{m}_b}{\dot{m}_a} = f$$



$$\eta = 1 - \frac{|\dot{Q}_L|}{|\dot{Q}_H|} = 1 - \frac{\dot{m}_b(h_8 - h_1)}{\dot{m}_a(h_5 - h_4) + \dot{m}_b(h_7 - h_6)} = 1 - \frac{f(h_8 - h_1)}{(h_5 - h_4) + f(h_7 - h_6)}$$

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Condenser Analysis

- Usually simpler than work analysis

$$|\dot{Q}_L| = |\dot{Q}_{cond}| = |\dot{m}_b(h_1 - h_8)| = \dot{m}_b(h_8 - h_1)$$

$$|\dot{Q}_H| = |\dot{Q}_{SG}| = |\dot{m}_a(h_5 - h_4)| = \dot{m}_a(h_5 - h_4)$$

$$\eta = \frac{|\dot{W}_{net}|}{|\dot{Q}_H|} = \frac{|\dot{Q}_H| - |\dot{Q}_L|}{|\dot{Q}_H|} = 1 - \frac{\dot{m}_b(h_8 - h_1)}{\dot{m}_a(h_5 - h_4)}$$

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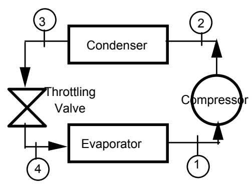
Refrigeration Cycles

- $P_{evaporator} = P_1 = P_4 = P_{sat}(T_4 = T_1)$
- $P_{condenser} = P_2 = P_3 = P_{sat}(T_3 < T_2)$

State 1: $h_1 = h_g(P_1)$
State 2: $h_2 = h(P_2, s_2 = s_1 = s_g(P_1))$
State 3: $h_3 = h_f(P_3)$
State 4: $h_4 = h_3$
cop = $(h_1 - h_4) / (h_2 - h_1)$



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Air-Standard Cycle Analysis

- Use air properties as ideal gas with variable or constant heat capacity
- Model chemical energy release as heat addition (~1,200 Btu/lb_m or 2,800 kJ/kg)
- Heat addition at constant pressure, volume or temperature
- Isentropic work
- Closed system except Brayton Cycle

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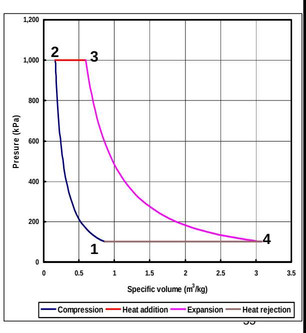
Constant c_p Brayton Cycle

- **Brayton Cycle**
- **Given:** PR, P_1, T_1, q_H, **Find:** η
- $P_2 = P_1/PR$
- Isentropic compression to P_2
- $T_2 = T_1(PR)^{(k-1)/k}$
- $T_3 = T_2 + q_H / c_p$



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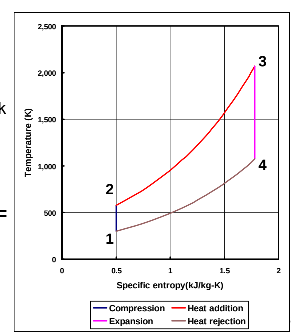
Brayton Example (const c_p)

- Isentropic expansion from $P_3 = P_2$ to $P_4 = P_1$
- $T_4 = T_3/(PR)^{(k-1)/k}$
- $|q_L| = c_p|T_1 - T_4|$
- $\eta = 1 - |q_L| / |q_H|$
- Can show that $\eta = 1 - 1 / (PR)^{(k-1)/k}$ for constant c_p



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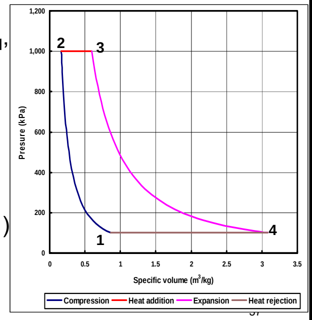
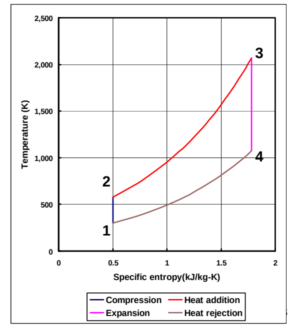
Variable c_p Brayton Cycle

- **Brayton Cycle**
- **Given:** PR, P_1, T_1, q_H, **Find:** η
- Get $h(T_1)$
- Isentropic compression to P_2
- $P_r(T_2) = P_r(T_1)\,(PR)$
- Find $h(T_2)$ from $P_r(T_2)$
- $h(T_3) = h(T_2) + q_H$



Brayton Cycle Example

- Isentropic expansion to $P_4 = P_1$
- $P_r(T_4) = P_r(T_3)/(PR)$
- $|q_L| = h(T_1) - h(T_4)$
- $\eta = 1 - |q_L| / |q_H|$
- $w_{net} = [h(T_3) - h(T_4)] - [h(T_2) - h(T_1)]$
- $\eta = 1 - w_{net} / |q_H|$



And, in conclusion

- Need to know property relations (tables and ideal gases) to work problems
- First law energy balances in a variety of systems (closed, steady and unsteady)
- Main application of second law is isentropic work and efficiencies
- Cycle analysis looks at groups of devices to get overall efficiency or coefficient of performance

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The Final

- Tuesday, December 14, 10:15 am to 12:15 pm
- Similar to the homework, midterm and quiz problems
- Use a clean equation sheet (with no writing)
  - Can add equation dh = Tds + vdP
- More credit for showing how to get solution than for details of algebra/arithmetic
- Sample final exam questions online
  - Will post solutions on Thursday

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The Final Continued

- Like four quiz problems
  - Cycles: air-standard, refrigeration, Rankine
  - At least one problem with different mass flow rates
  - Isentropic efficiencies in at least one problem
  - Properties for liquid, mixed and gas
  - At least one problem with open and one with closed systems
    - Low, but non-zero, probability of transient problem (may involve second law)

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