Name: ______________________________________

College of Engineering and Computer Science
Mechanical Engineering Department

California State University
Northridge

Mechanical Engineering 309
Numerical Analysis of Engineering Systems

Spring 2014 Number: 15237 Instructor: Larry Caretto

Solution to Seventh Quiz, April 16, 2014

Gaussian quadrature is a method of numerical integration that uses specific values of x_k, rather than uniform increments ($x_k = a + kh$) to approximate the definite integral, $I = \int_a^b f(x)dx$. In this method

$I \approx \frac{(b-a)}{2}\sum_{j=1}^{n}\gamma_j f(y_j)$ **where** $y_j = \frac{(b+a)}{2} + \frac{(b-a)z_j}{2}$ **and, for n = 4, the values of γ_j and z_j are taken from the table at the right. Use this algorithm to estimate the integral cos(x) from x = –2 to x = 2. Compare your result to the exact value 2sin(2); what is the relative error?**

j	1	2	3	4
z_j	–0.86114	–0.33998	0.33998	0.86114
γ_j	0.34785	0.65215	0.65215	0.34785

Because a = -b for this integral the term (b + a)/2 in the equation for y_j will always be zero. The term (b – a)/2 = 2. We thus have the following four values for f:

$f(y_1)$ = cos[0 + 2(–0.86114)] = –0.150898

$f(y_2)$ = cos[0 + 2(–0.33998)] = 0.777597

$f(y_3)$ = cos[0 + 2(0.33998)] = 0. 777597

$f(y_4)$ = cos[0 + 2(0.86114)] = –0.150898

$I \approx \frac{(b-a)}{2}\sum_{j=1}^{n}\gamma_j f(y_j)$ = 2[(0.34785)(–0.150898) +(0.65215)(0.777597) +(0.65215) • (0.777597) +(0.65215) (0.777597) + (0.34785)(–0.150898) = **1.81846**

Relative Error =

$$\left|\frac{Numerical - Exact}{Exact}\right| = \left|\frac{1.81846 - 2\sin(2)}{2\sin(2)}\right| = \left|\frac{1.81846 - 1.81859}{1.81859}\right| = 7.3x10^{-5}$$