College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 309
Numerical Analysis of Engineering Systems

Spring 2014 Number: 15237 Instructor: Larry Caretto

Solution to Sixth Quiz, March 19, 2014

1. Consider the following finite-difference formulas for the second derivative.

$$f_i'' = \frac{2f_i - 5f_{i-1} + 4f_{i-2} - f_{i-3}}{h^2} + O(h^2) \qquad f_i'' = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} + O(h^2)$$

(a) State the order of the error and the directionality of each finite-difference expression and (b) use an appropriate expression to find the second derivative of y at x = 0.6 and x = 1.2 for the data in the table above.

x	0	0.2	0.4	0.6	0.8	1	1.2
y	0.000	0.221	0.492	0.822	1.226	1.718	2.320

Both expressions have a **second order error**; the one on the **left is backwards**-difference and the one on the **right is a central**-difference expression.

For x = 0.6, the central difference expression gives

$$f_i'' = \frac{1.226 - 2(0.822) + 0.492}{0.2^2} = \mathbf{1.850}$$

For x = 1.2 the backwards-difference expression gives

$$f_i'' = \frac{2(2.230) - 5(1.718) + 4(1.226) - 0.822}{0.2^2} = \mathbf{3.300}$$

2. Use a quadratic Newton polynomial, p(x) = a_0 + a_1(x – x_0) a_2(x – x_0)(x – x_1), to find a value of y for x = 0.65 for the data in problem one.

A quadratic polynomial requires three points; select the three points at x = 0.4, 0.6 and 0.8 as the closest points to the desired interpolation. The divided difference table for this polynomial is shown at the right. The computations for the first divided differences (F_0 and F_1) and the second divided difference, S_0, are shown below

0.4	0.492		
		1.650	
0.6	0.822		0.925
		2.020	
0.8	1.226		

$$F_0 = \frac{0.822 - 0.492}{0.6 - 0.4} = 1.650 \qquad F_1 = \frac{1.226 - 0.822}{0.8 - 0.6} = 2.020 \qquad S_0 = \frac{2.020 - 1.650}{0.8 - 0.4} = 0.925$$

The coefficients in the Newton polynomial are found from the divided difference table with $a_0 = y_0 = 0.492$, $a_1 = F_0 = 1.650$, and $a_2 = S_0 = 0.925$. Substituting these values into the Newton Polynomial with x = 0.65 gives

p = a_0 + a_1(x – x_0) a_2(x – x_0)(x – x_1) = 0.492 + 1.650(0.65 – 0.4) + 0.925(0.65 – 0.4)(0.65 – 0.6) = **0.9161**