College of Engineering and Computer Science
Mechanical Engineering Department

California State University Northridge

Mechanical Engineering 309
Numerical Analysis of Engineering Systems

Spring 2014 Number: 15237 Instructor: Larry Caretto

Solution to Fifth Quiz, March 5, 2014

The two sets of equations shown at the right differ only in their right-hand sides. Determine if the equations have a unique solution, an infinite solution, or no solution. If there is a unique solution, find it. It here is an infinite solution, find a general relationship for w, x, and y in terms of z

$w + 2x - y + 3z = 5$	$w + 2x - y + 3z = -5$
$2w - 3x + 5y - 2z = 9$	$2w - 3x + 5y - 2z = 20$
$4w + 4x - 2y + z = 13$	$4w + 4x - 2y + z = 4$
$5w - 8x + 11y - 12z = 16$	$5w - 8x + 11y - 12z = 60$

Combine the left hand side with the two right hand sides and apply Gauss Elimination to the six-column matrix as shown at the right.

1	2	−1	3	5	−5
2	−3	5	−2	9	20
4	4	−2	1	13	4
5	−8	11	−12	16	60

In the first step we subtract 2 times row one from row two, 4 times row 1 from row 3 and 5 times row 1 from row 4. The expressions used are shown at the left and the results are shown at the right in the table below.

1	2	−1	3	5	−5
$2-2(1)$	$-3-2(2)$	$5-2(-1)$	$-2-2(3)$	$9-2(5)$	$20-2(-5)$
$4-4(1)$	$4-4(2)$	$-2-4(-1)$	$1-4(3)$	$13-4(5)$	$4-4(-5)$
$5-5(1)$	$-8-5(2)$	$11-5(-1)$	$-12-5(3)$	$16-5(5)$	$60-5(-5)$

1	2	−1	3	5	−5
0	−7	7	−8	−1	30
0	−4	2	−11	−7	24
0	−18	16	−27	−9	85

In the next step we subtract 4/7 times row 2 from row 3 and 18/7 times row 2 from row four.

1	2	−1	3	5	−5
0	−7	7	−8	−1	30
0	$-4-(4/7)(-7)$	$2-(4/7)(7)$	$-11-(4/7)(-8)$	$-7-(4/7)(-1)$	$24-(4/7)(304)$
0	$-18-(18/7)(-7)$	$16-(18/7)(7)$	$-27-(18/7)(-8)$	$-9-(18/7)(-1)$	$85-(18/7)(30)$

The results from this step are shown at the right. We see that the final two rows for the first right hand side are exactly the same equation with different signs. Thus, this **first set of equations has an infinite solution**. If we add the last two rows for the second right-hand side, we get 0 = 89/7. Thus, **the second right hand side has no solution**. For the first set of equations, we can pick any value for z. The third and fourth equations both tell us that 2y + 45z/7 = 45/7 or **y = 45(1 – z)/14**. Substituting this into the second equation gives –7x + 7(45)(1 – z)/14 – 8z = –1; since 7(45)/14 =2, multiplying the equation by 2 gives –14x + 45 – 45z – 8z = –2. Solving this for x gives **x = 47/14 – 61z/14**. The first equation is w + 2x – y +3z = 5 so that w = 5 – 3z +(45/14 – 45z/14) – 2(47/14 – 61z/14). Getting a common factor of 1/14 gives = (70 + 45 – 94)/14 + z(–42 – 45 + 122)/14 = 21/14 + 45/z/14. Thus **w = 3/2 + 5z/2**.

1	2	−1	3	5	−5
0	−7	7	−8	−1	30
0	0	−2	−45/7	−45/7	−48/7
0	0	2	45/7	45/7	−41/7